Lesson Topics

**Fixed Costs of Production** are those production costs that are present whenever production is positive. The simplest model of fixed costs in a linear program restricts decision variables to binary (0 or 1).

**Fixed Costs of Assignment** are modeled like Fixed Costs of Production. The simplest model of fixed costs restricts decision variables (the fraction of work completed) to be binary.

**Location Covering (3)** Problems are a special kind of Linear Programming problem when outputs are fixed because the firm has only established customers, who require coverage by service centers.

**Worker Covering (2)** Problems are like Location Covering Problems, where customers require scheduling workers so that at each time period is covered by a prescribed minimal number of workers.

**Transportation Problems with New Origins (2)** are Transportation Problems extended so that new origins may be added, at a fixed cost. They choose the best plant locations and how much to ship.

**Transshipment Problems with New Nodes (4)** are Transshipment Problems extended so that new transshipment nodes may be added, at a fixed cost. They choose the best transshipment locations.
Location Covering

Question. The Bayside Art Gallery is considering installing a video camera security system to reduce its insurance premiums. A diagram of the eight display rooms that Bayside uses for exhibitions is shown below; the openings between the rooms are numbered 1 through 13. A security firm proposed that two-way cameras be installed at some room openings. Each camera has the ability to monitor the two rooms between which the camera is located. For example, if a camera were located at opening number 4, rooms 1 and 4 would be covered; if a camera were located at opening 11, rooms 7 and 8 would be covered; and so on. Management decided not to locate a camera system at the entrance to the display rooms. The objective is to provide security coverage for all eight rooms using the minimum number of two-way cameras.

a. Formulate a binary linear programming model that will enable Bayside’s management to determine the locations for the camera systems.

b. Use Management Scientist to solve the model formulated in part (a) to determine how many two-way cameras to purchase and where they should be located. (You need only find one solution, not all alternative solutions.)

c. Suppose that management wants to provide additional security coverage for room 7. Specifically, management wants room 7 to be covered by two cameras. How would your model formulated in part (a) have to change to accommodate this policy restriction?

d. With the policy restriction specified in part (c), use Management Scientist to determine how many two-way camera systems will need to be purchased and where they will be located. (You need only find one solution, not all alternative solutions.)
FIGURE 7.13 DIAGRAM OF DISPLAY ROOMS FOR BAYSIDE ART GALLERY
Hint: Use the variables

\[ x_i = \begin{cases} 
1 & \text{if a camera is located at opening } i \\
0 & \text{if not} 
\end{cases} \]

Then, \( x_7 + x_8 + x_9 + x_{10} \geq 1 \) is the constraint that Room 5 is covered by at least one camera.
Answer to Question:

Let \( x_i = \begin{cases} 
1 & \text{if a camera is located at opening } i \\
0 & \text{if not} 
\end{cases} \)

\[
\begin{align*}
\text{min } & x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} \\
\text{s.t. } & 
\begin{align*}
x_1 + x_4 + x_6 & \geq 1 \quad \text{Room 1} \\
x_6 + x_8 + x_{12} & \geq 1 \quad \text{Room 2} \\
x_1 + x_2 + x_3 & \geq 1 \quad \text{Room 3} \\
x_3 + x_4 + x_5 + x_7 & \geq 1 \quad \text{Room 4} \\
x_7 + x_8 + x_9 + x_{10} & \geq 1 \quad \text{Room 5} \\
x_{10} + x_{12} + x_{13} & \geq 1 \quad \text{Room 6} \\
x_2 + x_5 + x_9 + x_{11} & \geq 1 \quad \text{Room 7} \\
x_{11} + x_{13} & \geq 1 \quad \text{Room 8} 
\end{align*}
\end{align*}
\]

b. \( x_1 = x_5 = x_8 = x_{13} = 1 \). Thus, cameras should be located at 4 openings: 1, 5, 8, and 13. An alternative optimal solution is \( x_1 = x_7 = x_{11} = x_{12} = 1 \).

c. Change the constraint for room 7 to \( x_2 + x_5 + x_9 + x_{11} \geq 2 \)

d. \( x_3 = x_6 = x_9 = x_{11} = x_{12} = 1 \). Thus, cameras should be located at openings 3, 6, 9, 11, and 12. An alternate optimal solution is \( x_2 = x_4 = x_6 = x_{10} = x_{11} = 1 \). Optimal Value = 5
**Location Covering**

**Question.** Sam’s wholesale club is drawing up new zones for the location of distribution centers to service retail stores in four counties. Sam’s is considering ten possible locations for distribution centers (numbered 1 through 10). The list of which distribution center could be reached easily from each county is listed below.

<table>
<thead>
<tr>
<th>County</th>
<th>Can be Served by Distribution Center Locations:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Los Angeles</td>
<td>1, 4, 6, 8, 9</td>
</tr>
<tr>
<td>San Diego</td>
<td>2, 4, 8, 9, 10</td>
</tr>
<tr>
<td>Ventura</td>
<td>1, 3, 5, 7, 9</td>
</tr>
<tr>
<td>Santa Barbara</td>
<td>2, 3, 5, 7, 9</td>
</tr>
</tbody>
</table>

Formulate a model to minimize the number of distribution center locations yet make sure that Los Angeles county is served by at least four distribution centers, and each other county is served by at least three distribution centers. Formulate the model, but you need not solve for the optimum.

Finally, why would you want to minimize the number of distribution center locations?

Tip: Your written answer should define the decision variables, and formulate the objective and constraints.
**Answer to Question:** Define the Decision Variables:

$x_1 = 1$ if there is a distribution center in location 1, and $= 0$ otherwise. And so on.

Min $\Sigma_i x_i$

s.t. $x_1 + x_4 + x_6 + x_8 + x_9 \geq 4$

$x_2 + x_4 + x_8 + x_9 + x_{10} \geq 3$

$x_1 + x_3 + x_5 + x_7 + x_9 \geq 3$

$x_2 + x_3 + x_5 + x_7 + x_9 \geq 3$
Location Covering

Question. New England Trucking provides service from Boston to Richmond using 6 regional offices located in Boston, New York, Philadelphia, Baltimore, Washington, and Richmond. The number of miles between each of the regional offices is provided in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Boston</th>
<th>New York</th>
<th>Philadelphia</th>
<th>Baltimore</th>
<th>Washington</th>
<th>Richmond</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boston</td>
<td>0</td>
<td>221</td>
<td>320</td>
<td>424</td>
<td>459</td>
<td>565</td>
</tr>
<tr>
<td>New York</td>
<td>221</td>
<td>0</td>
<td>109</td>
<td>213</td>
<td>248</td>
<td>354</td>
</tr>
<tr>
<td>Philadelphia</td>
<td>320</td>
<td>109</td>
<td>0</td>
<td>104</td>
<td>139</td>
<td>245</td>
</tr>
<tr>
<td>Baltimore</td>
<td>424</td>
<td>213</td>
<td>104</td>
<td>0</td>
<td>35</td>
<td>141</td>
</tr>
<tr>
<td>Washington</td>
<td>459</td>
<td>248</td>
<td>139</td>
<td>35</td>
<td>0</td>
<td>106</td>
</tr>
<tr>
<td>Richmond</td>
<td>565</td>
<td>354</td>
<td>245</td>
<td>141</td>
<td>106</td>
<td>0</td>
</tr>
</tbody>
</table>

The company’s expansion plans involve constructing service facilities in some of the cities where a regional office is located. Each regional office must be within 250 miles of a service facility. For instance, if a service facility is constructed in Richmond, it can provide service to regional offices located in Philadelphia and Baltimore and Washington. Management would like to determine the minimum number of service facilities needed and where they should be located.

Formulate a linear program that can be used to determine the minimum number of service facilities needed and their location.

Tip: Your written answer should define the decision variables, and formulate the objective and constraints.
B.4 Binary Fixed Costs

**Answer to Question:**

Let \( x_i = \begin{cases} 
1 & \text{if a service facility is located in city } i \\
0 & \text{otherwise} 
\end{cases} \)

where 1 is Boston, 2 is New York, 3 is Philadelphia, 4 is Baltimore, 5 is Washington, and 6 is Richmond.

\[
\begin{align*}
\text{min} & \quad x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \\
\text{s.t.} & \quad \begin{align*}
(Boston) & \quad x_1 + x_2 \geq 1 \\
(New \ York) & \quad x_1 + x_2 + x_3 + x_4 + x_5 \geq 1 \\
(Philadelphia) & \quad x_2 + x_3 + x_4 + x_5 + x_6 \geq 1 \\
(Baltimore) & \quad x_2 + x_3 + x_4 + x_5 + x_6 \geq 1 \\
(Washington) & \quad x_2 + x_3 + x_4 + x_5 + x_6 \geq 1 \\
(Richmond) & \quad x_3 + x_4 + x_5 + x_6 \geq 1 \\
\end{align*}
\end{align*}
\]

\( x_i = 0, 1 \)
Worker Covering

Question. Amazon.Com is open 24 hours a day. The number of phone operators need in each four hour period of a day is listed below.

<table>
<thead>
<tr>
<th>Period</th>
<th>Operators Needed</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 p.m. to 2 a.m.</td>
<td>8</td>
</tr>
<tr>
<td>2 a.m. to 6 a.m.</td>
<td>4</td>
</tr>
<tr>
<td>6 a.m. to 10 a.m.</td>
<td>7</td>
</tr>
<tr>
<td>10 a.m. to 2 p.m.</td>
<td>12</td>
</tr>
<tr>
<td>2 p.m. to 6 p.m.</td>
<td>10</td>
</tr>
<tr>
<td>6 p.m. to 10 p.m.</td>
<td>15</td>
</tr>
</tbody>
</table>

Suppose phone operators work for eight consecutive hours. Formulate and solve the company’s problem of determining how many should be scheduled to begin working in each period in order to minimize the number of phone operators needed? (Hint: Phone operators can work from 6 p.m. to 2 a.m.)

Tip: Your written answer should define the decision variables, and formulate the objective and constraints, and solve for the optimum.
Answer to Question: Define the decision variables.

Let

\[ \begin{align*}
TNP & = \text{the number of phone operators who begin working at 10 p.m.} \\
TWA & = \text{the number of phone operators who begin working at 2 a.m.} \\
SX A & = \text{the number of phone operators who begin working at 6 a.m.} \\
TNA & = \text{the number of phone operators who begin working at 10 a.m.} \\
TWP & = \text{the number of phone operators who begin working at 2 p.m.} \\
SXP & = \text{the number of phone operators who begin working at 6 p.m.}
\end{align*} \]

Min \[ TNP + TWA + SXA + TNA + TWP + SXP \]

s.t. \[ \begin{align*}
TNP + TWA & \geq 4 \\
TWA + SXA & \geq 7 \\
SX A + TNA & \geq 12 \\
TNA + TWP & \geq 10 \\
TWP + SXP & \geq 15 \\
SXP + TNP & \geq 8 \\
\text{all variables} & \geq 0
\end{align*} \]
B.4 Binary Fixed Costs

**Review Questions**

![Optimal Solution](image)

**Objective Function Value** = 31,000

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>TNF</td>
<td>4.000</td>
</tr>
<tr>
<td>TVA</td>
<td>0.000</td>
</tr>
<tr>
<td>SXA</td>
<td>12.000</td>
</tr>
<tr>
<td>TNA</td>
<td>0.000</td>
</tr>
<tr>
<td>TWF</td>
<td>11.000</td>
</tr>
<tr>
<td>SXF</td>
<td>4.000</td>
</tr>
</tbody>
</table>
Worker Covering

Question. Flying J truck stop, travel plaza, and deluxe bistro is open 24 hours a day. The number of cashiers need in each four hour period of a day is listed below.

<table>
<thead>
<tr>
<th>Period</th>
<th>Cashiers Needed</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 p.m. to 2 a.m.</td>
<td>8</td>
</tr>
<tr>
<td>2 a.m. to 6 a.m.</td>
<td>4</td>
</tr>
<tr>
<td>6 a.m. to 10 a.m.</td>
<td>7</td>
</tr>
<tr>
<td>10 a.m. to 2 p.m.</td>
<td>12</td>
</tr>
<tr>
<td>2 p.m. to 6 p.m.</td>
<td>10</td>
</tr>
<tr>
<td>6 p.m. to 10 p.m.</td>
<td>15</td>
</tr>
</tbody>
</table>

Suppose cashiers work either part time for four consecutive hours, or full time for eight consecutive hours. Suppose it costs $30 to hire a cashier part time for four consecutive hours, and it costs $50 to hire a cashier full time for eight consecutive hours.

Formulate the company’s problem of determining how many part-time and how many full-time cashiers should be scheduled to begin working in each period in order to minimize cost? (Hint: Workers can work from 6 p.m. to 2 a.m.) Formulate the problem, but you need not solve for the optimum.

Tip: Your written answer should define the decision variables, and formulate the objective and constraints.
Answer to Question: Let
F1 = the number of full-time cashiers who begin working at 10 p.m.
P1 = the number of part-time cashiers who begin working at 10 p.m.
F2 = the number of full-time cashiers who begin working at 2 a.m.
P2 = the number of part-time cashiers who begin working at 2 a.m.
F3 = the number of full-time cashiers who begin working at 6 a.m.
P3 = the number of part-time cashiers who begin working at 6 a.m.
F4 = the number of full-time cashiers who begin working at 10 a.m.
P4 = the number of part-time cashiers who begin working at 10 a.m.
F5 = the number of full-time cashiers who begin working at 2 p.m.
P5 = the number of part-time cashiers who begin working at 2 p.m.
F6 = the number of full-time cashiers who begin working at 6 p.m.
P6 = the number of part-time cashiers who begin working at 6 p.m.

Min \ 30(P1+P2+P3+P4+P5+P6) + 50(F1+F2+F3+F4+F5+F6)

s.t.
F6+F1+P1 \geq 8 \ (working \ in \ the \ first \ period, \ 10p.m. \ to \ 2a.m.)
F1+F2+P2 \geq 4 \ (working \ in \ the \ second \ period, \ 2a.m. \ to \ 6a.m.)
F2+F3+P3 \geq 7 \ (working \ in \ the \ third \ period, \ 6a.m. \ to \ 10a.m.)
F3+F4+P4 \geq 12 \ (working \ in \ the \ fourth \ period, \ 10a.m. \ to \ 2p.m.)
F4+F5+P5 \geq 10 \ (working \ in \ the \ fifth \ period, \ 2p.m. \ to \ 6p.m.)
F5+F6+P6 \geq 15 \ (working \ in \ the \ sixth \ period, \ 6p.m. \ to \ 10p.m.)
all \ variables \ integers \ and \ \geq \ 0
**Transportation Problems with New Origins**

**Question.** Linksys operates a plant to produce its wireless routers in St. Louis with an annual capacity of 30,000 units. Product is shipped to regional distribution centers in Boston, Atlanta, and Houston. Because of an anticipated increase in demand, Linksys plans to increase capacity by constructing a new plant in one or more of the following cities: Detroit, Toledo, Denver, or Kansas City. The estimated annual fixed cost and annual capacity for the four proposed plants are as follows:

<table>
<thead>
<tr>
<th>Proposed Plant</th>
<th>Annual Fixed Cost</th>
<th>Annual Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detroit</td>
<td>$250,000</td>
<td>15,000</td>
</tr>
<tr>
<td>Toledo</td>
<td>$350,000</td>
<td>25,000</td>
</tr>
<tr>
<td>Denver</td>
<td>$375,000</td>
<td>30,000</td>
</tr>
<tr>
<td>Kansas City</td>
<td>$500,000</td>
<td>40,000</td>
</tr>
</tbody>
</table>

The company’s long-range planning group forecasts of the anticipated annual demand at the distribution centers are as follows:

<table>
<thead>
<tr>
<th>Distribution Center</th>
<th>Annual Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boston</td>
<td>35,000</td>
</tr>
<tr>
<td>Atlanta</td>
<td>45,000</td>
</tr>
<tr>
<td>Houston</td>
<td>20,000</td>
</tr>
</tbody>
</table>

The shipping cost per unit from each plant to each distribution center is as follows:

<table>
<thead>
<tr>
<th>Plant\Distribution</th>
<th>Boston</th>
<th>Atlanta</th>
<th>Houston</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detroit</td>
<td>$3</td>
<td>$7</td>
<td>$9</td>
</tr>
<tr>
<td>Toledo</td>
<td>$4</td>
<td>$3</td>
<td>$4</td>
</tr>
<tr>
<td>Denver</td>
<td>$9</td>
<td>$7</td>
<td>$5</td>
</tr>
<tr>
<td>Kansas City</td>
<td>$10</td>
<td>$4</td>
<td>$2</td>
</tr>
<tr>
<td>St. Louis</td>
<td>$8</td>
<td>$4</td>
<td>$3</td>
</tr>
</tbody>
</table>

Formulate the problem of minimizing the cost of meeting all demands.
Answer to Question:
Define binary variables for plant construction,
\[ Y_1 = 1 \text{ if a plant is constructed in Detroit; } 0, \text{ if not} \]
\[ Y_2 = 1 \text{ if a plant is constructed in Toledo; } 0, \text{ if not} \]
\[ Y_3 = 1 \text{ if a plant is constructed in Denver; } 0, \text{ if not} \]
\[ Y_4 = 1 \text{ if a plant is constructed in Kansas City; } 0, \text{ if not} \]
Define integer shipment variables just as in transportation problems,
\[ X_{ij} = \text{the units shipped (in thousands) from plant } i \ (i = 1, 2, 3, 4, 5) \text{ to distribution center } j \ (j = 1, 2, 3) \text{ each year}. \]

The objective is minimize total cost. From cost data, shipping costs (in thousands of dollars) are
\[ 3X_{11} + 7X_{12} + 9X_{13} + 4X_{21} + 3X_{22} + 4X_{23} + 9X_{31} + 7X_{32} + 5X_{33} + 10X_{41} + 4X_{42} + 2X_{43} + 8X_{51} + 4X_{52} + 3X_{53} \]
From cost data, plant construction costs (in thousands of dollars) are
\[ 250Y_1 + 350Y_2 + 375Y_3 + 500Y_4 \]
Hence, the objective to minimize total costs is
\[ \text{Min } 3X_{11} + 7X_{12} + 9X_{13} + 4X_{21} + 3X_{22} + 4X_{23} + 9X_{31} + 7X_{32} + 5X_{33} + 10X_{41} + 4X_{42} + 2X_{43} + 8X_{51} + 4X_{52} + 3X_{53} + 250Y_1 + 350Y_2 + 375Y_3 + 500Y_4 \]

From capacity data,
Detroit capacity constraint is \[ X_{11} + X_{12} + X_{13} \leq 15Y_1 \]
Toledo capacity constraint is \[ X_{21} + X_{22} + X_{23} \leq 25Y_2 \]
Denver capacity constraint is \[ X_{31} + X_{32} + X_{33} \leq 30Y_3 \]
Kansas City capacity constraint is \[ X_{41} + X_{42} + X_{43} \leq 40Y_4 \]
And St. Louis capacity constraint is \[ X_{51} + X_{52} + X_{53} \leq 30 \]
From demand data,
Boston demand constraint is \[ X_{11} + X_{21} + X_{31} + X_{41} + X_{51} = 35 \]
Atlanta demand constraint is \[ X_{12} + X_{22} + X_{32} + X_{42} + X_{52} = 45 \]
Houston demand constraint is \[ X_{13} + X_{23} + X_{33} + X_{43} + X_{53} = 20 \]
Non-negativity constraints complete the linear programming formulation.
Transportation Problems with New Origins

Question. Linksys operates a plant to produce its wireless routers in St. Louis with an annual capacity of 65,000 units. Product is shipped to regional distribution centers in Boston, Atlanta, and Houston. Because of an anticipated increase in demand, Linksys plans to increase capacity by constructing a new plant in one or more of the following cities: Detroit or Kansas City. The estimated annual fixed cost and annual capacity for the four proposed plants are as follows:

<table>
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<tr>
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<th>Annual Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detroit</td>
<td>$300,000</td>
<td>45,000</td>
</tr>
<tr>
<td>Kansas City</td>
<td>$400,000</td>
<td>55,000</td>
</tr>
</tbody>
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The company’s long-range planning group forecasts of the anticipated annual demand at the distribution centers are as follows:

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<tr>
<td>Atlanta</td>
<td>20,000</td>
</tr>
<tr>
<td>Houston</td>
<td>30,000</td>
</tr>
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The shipping cost per unit from each plant to each distribution center is as follows:

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<th>Atlanta</th>
<th>Houston</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detroit</td>
<td>$1</td>
<td>$2</td>
<td>$3</td>
</tr>
<tr>
<td>Kansas City</td>
<td>$4</td>
<td>$5</td>
<td>$6</td>
</tr>
<tr>
<td>St. Louis</td>
<td>$7</td>
<td>$8</td>
<td>$9</td>
</tr>
</tbody>
</table>

Formulate the problem of minimizing the cost of meeting all demands.

But you need not compute an optimum.
Answer to Question:
Define binary variables for plant construction,
Y1 = 1 if a plant is constructed in Detroit; 0, if not
Y2 = 1 if a plant is constructed in Kansas City; 0, if not
Define integer shipment variables just as in transportation problems,
Xij = the units shipped (in thousands) from plant i (i = 1, 2, 3) to distribution center j (j = 1, 2, 3) each year.

The objective is minimize total cost. From cost data, shipping costs (in thousands of dollars) are
X11 + 2X12 + 3X13 + 4X21 + 5X22 + 6X23 + 7X31 + 8X32 + 9X33
From cost data, plant construction costs (in thousands of dollars) are
300Y1 + 400Y2
Hence, the objective to minimize total costs is
Min X11 + 2X12 + 3X13 + 4X21 + 5X22 + 6X23 + 7X31 + 8X32 + 9X33 + 300Y1 + 400Y2

From capacity data,
Detroit capacity constraint is X11 + X12 + X13 ≤ 45Y1
Kansas City capacity constraint is X21 + X22 + X23 ≤ 55Y2
And St. Louis capacity constraint is X31 + X32 + X33 ≤ 65

From demand data,
Boston demand constraint is
X11 + X21 + X31 = 10
Atlanta demand constraint is
X12 + X22 + X32 = 20
Houston demand constraint is
X13 + X23 + X33 = 30

Non-negativity constraints complete the linear programming formulation.
Transshipment Problems with New Nodes

Question. Zeron Industries supplies three stores (Albertsons, Best Buy, Cookie Cutters) with customized shelving for its offices. Zeron orders shelving from the same two manufacturers, Arnold Manufacturers and Supershelf, Inc. Currently, weekly demands by the users are 20 for Albertsons, 30 for Best Buy, and 40 for Cookie Cutters. Arnold can supply up to 55 units of shelving, and Supershelf can supply up to 65 units of shelving.

Zeron currently ships from its Northside facilities, but it can develop Westside facilities for a weekly fixed cost of 700, and Southside facilities for a weekly fixed cost of 800, and Eastside facilities for a weekly fixed cost of 900. Unit costs from the manufacturers to the shipment centers are:

<table>
<thead>
<tr>
<th></th>
<th>North</th>
<th>West</th>
<th>South</th>
<th>East</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arnold</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Supershelf</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>2</td>
</tr>
</tbody>
</table>

The costs to ship and install the shelving at the various locations are:

<table>
<thead>
<tr>
<th></th>
<th>Albertsons</th>
<th>Best Buy</th>
<th>Cookie Cutters</th>
</tr>
</thead>
<tbody>
<tr>
<td>North</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>West</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>South</td>
<td>8</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>East</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Formulate the linear program that allows Zeron to minimize the cost of meeting all demands. But you need not compute an optimum.

Tip: Your written answer should define the decision variables, and formulate the objective and constraints.
Answer to Question:
List origin, transshipment, and destination nodes in this transshipment problem:
1 = Arnold, 2 = Supershelf
3 = Northside, 4 = Westside, 5 = Southside, 6 = Eastside
7 = Albertsons, 8 = Best Buy, 9 = Cookie Cutters.

Define binary variables for transshipment nodes,
\[ Y_4 = 1 \text{ if a facility is constructed in the Westside; 0, if not} \]
\[ Y_5 = 1 \text{ if a facility is constructed in the Southside; 0, if not} \]
\[ Y_6 = 1 \text{ if a facility is constructed in the Eastside; 0, if not} \]

Define integer shipment variables just as in any transshipment problem,
\[ X_{ij} = \text{the units shipped from node } i \text{ (} i = 1, 2, 3, 4, 5, 6 \text{) to node } j \text{ (} j = 3, 4, 5, 6, 7, 8, 9 \text{) each week}. \]

The objective is minimize total cost. From cost data, shipping costs are
\[ 3X_{13} + 4X_{14} + 5X_{15} + 6X_{16} + \]
\[ 7X_{23} + 8X_{24} + 9X_{25} + 2X_{26} + \]
\[ 2X_{37} + 3X_{38} + 4X_{39} + \]
\[ 5X_{47} + 6X_{48} + 7X_{49} + \]
\[ 8X_{57} + 9X_{58} + 2X_{59} + \]
\[ 2X_{67} + 3X_{68} + 4X_{69} + \]

From cost data, facility development costs are
\[ 700Y_4 + 800Y_5 + 900Y_6 \]
Hence, the objective to minimize total costs is
Min
\[3X_{13} + 4X_{14} + 5X_{15} + 6X_{16} + 7X_{23} + 8X_{24} + 9X_{25} + 2X_{26} + 2X_{37} + 3X_{38} + 4X_{39} + 5X_{47} + 6X_{48} + 7X_{49} + 8X_{57} + 9X_{58} + 2X_{59} + 2X_{67} + 3X_{68} + 4X_{69} + 700Y_{4} + 800Y_{5} + 900Y_{6}\]

From capacity data,
Arnold capacity constraint is \(X_{13} + X_{14} + X_{15} + X_{16} \leq 55\)
Supershelf capacity constraint is \(X_{23} + X_{24} + X_{25} + X_{26} \leq 65\)

From demand data,
Albertsons demand constraint is \(X_{37} + X_{47} + X_{57} + X_{67} \geq 20\)
Best Buy demand constraint is \(X_{38} + X_{48} + X_{58} + X_{68} \geq 30\)
Cookie Cutters demand constraint is \(X_{39} + X_{49} + X_{59} + X_{69} \geq 40\)

Transshipment constraints are
\(X_{37} + X_{38} + X_{39} \leq X_{13} + X_{23}, \text{ through Northside}\)
\(X_{47} + X_{48} + X_{49} \leq X_{14} + X_{24}, \text{ through Westside}\)
\(X_{57} + X_{58} + X_{59} \leq X_{15} + X_{25}, \text{ through Southside}\)
\(X_{67} + X_{68} + X_{69} \leq X_{16} + X_{26}, \text{ through Eastside}\)

And setup indicator constraints are
\(X_{47} + X_{48} + X_{49} \leq 120Y_{4}, \text{ through Westside}\)
\(X_{57} + X_{58} + X_{59} \leq 120Y_{5}, \text{ through Southside}\)
\(X_{67} + X_{68} + X_{69} \leq 120Y_{6}, \text{ through Eastside}\)
Transshipment Problems with New Nodes

Question. Zeron Industries supplies three stores (Albertsons, Best Buy, Cookie Cutters) with customized shelving for its offices. Zeron orders shelving from the same two manufacturers, Arnold Manufacturers and Supershelf, Inc. Currently, weekly demands by the users are 20 for Albertsons, 30 for Best Buy, and 40 for Cookie Cutters. Arnold can supply up to 65 units of shelving, and Supershelf can supply up to 75 units of shelving.

Zeron currently ships from its Northside facilities, but it can develop Westside facilities for a weekly fixed cost of 700, and Southside facilities for a weekly fixed cost of 800. Unit costs from the manufacturers to the shipment centers are:

<table>
<thead>
<tr>
<th></th>
<th>North</th>
<th>West</th>
<th>South</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arnold</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Supershelf</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

The costs to ship and install the shelving at the various locations are:

<table>
<thead>
<tr>
<th></th>
<th>Albertsons</th>
<th>Best Buy</th>
<th>Cookie Cutters</th>
</tr>
</thead>
<tbody>
<tr>
<td>North</td>
<td>5</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>West</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>South</td>
<td>8</td>
<td>9</td>
<td>2</td>
</tr>
</tbody>
</table>

Formulate the linear program that allows Zeron to minimize the cost of meeting all demands.

Tip: Your written answer should define the decision variables, and formulate the objective and constraints.
Answer to Question:
List origin, transshipment, and destination nodes in this transshipment problem:
1 = Arnold, 2 = Supershelf
3 = Northside, 4 = Westside, 5 = Southside
6 = Albertsons, 7 = Best Buy, 8 = Cookie Cutters.

Define binary variables for transshipment nodes,
Y4 = 1 if a facility is constructed in the Westside; 0, if not
Y5 = 1 if a facility is constructed in the Southside; 0, if not

Define integer shipment variables just as in any transshipment problem,
Xij = the units shipped from node i (i = 1, 2, 3, 4, 5) to node j (j = 3, 4, 5, 6, 7, 8) each week.

The objective is minimize total cost. From cost data, shipping costs are
2X13 + 2X14 + 3X15 +
4X23 + 5X24 + 6X25 +
5X36 + 2X37 + 5X38 +
5X46 + 6X47 + 7X48 +
8X56 + 9X57 + 2X58 +

From cost data, facility development costs are
700Y4 + 800Y5
Hence, the objective to minimize total costs is
\[
\text{Min} \quad 2X_{13} + 2X_{14} + 3X_{15} + \\
4X_{23} + 5X_{24} + 6X_{25} + \\
5X_{36} + 2X_{37} + 5X_{38} + \\
5X_{46} + 6X_{47} + 7X_{48} + \\
8X_{56} + 9X_{57} + 2X_{58} + \\
700Y_4 + 800Y_5
\]

From capacity data,
Arnold capacity constraint is \( X_{13} + X_{14} + X_{15} \leq 65 \)
Supershelf capacity constraint is \( X_{23} + X_{24} + X_{25} \leq 75 \)

From demand data,
Albertsons demand constraint is \( X_{36} + X_{46} + X_{56} \geq 20 \)
Best Buy demand constraint is \( X_{37} + X_{47} + X_{57} \geq 30 \)
Cookie Cutters demand constraint is \( X_{38} + X_{48} + X_{58} \geq 40 \)

Transshipment constraints are
\[
X_{36} + X_{37} + X_{38} \leq X_{13} + X_{23}, \text{ through Northside} \\
X_{46} + X_{47} + X_{48} \leq X_{14} + X_{24}, \text{ through Westside} \\
X_{56} + X_{57} + X_{58} \leq X_{15} + X_{25}, \text{ through Southside}
\]

And setup indicator constraints are
\[
X_{46} + X_{47} + X_{48} \leq 140Y_4, \text{ through Westside} \\
X_{56} + X_{57} + X_{58} \leq 140Y_5, \text{ through Southside}
\]
Transshipment Problems with New Nodes

Question. Google Inc. supplies Target and Walmart with computers. Google orders computers from manufacturers in California and China. Each day’s demand by the distribution centers are 500 for Target and 600 for Walmart. California can supply up to 550 computers and China can supply up to 800 computers.

Google can ship from either Nevada or from New York (or from both). If any units are shipped from Nevada, there is a fixed cost of 100 per day. If any units are shipped from New York, there is a fixed cost of 70 per day.

Unit costs from the manufacturers to the shipment centers are:

<table>
<thead>
<tr>
<th></th>
<th>Nevada</th>
<th>New York</th>
</tr>
</thead>
<tbody>
<tr>
<td>California</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>China</td>
<td>9</td>
<td>8</td>
</tr>
</tbody>
</table>

The unit costs to distribution centers are:

<table>
<thead>
<tr>
<th></th>
<th>Target</th>
<th>Walmart</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nevada</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>New York</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

Formulate and solve the linear program that minimizes the cost of meeting all demands. What is that minimum cost?

Tip: Your written answer should define the decision variables, formulate the objective and constraints, solve for the decision variables, and report all of the values (at least, all of the values that are positive).
Answer to Question:
List origin, transshipment, and destination nodes in this transshipment problem:
1 = California, 2 = China
3 = Nevada, 4 = New York,
5 = Target, 6 = Walmart.

Define binary variables for transshipment nodes,
Y3 = 1 if the Nevada facility is used; 0, if not
Y4 = 1 if the New York facility is used; 0, if not

Define integer shipment variables just as in any transshipment problem,
Xij = the units shipped from node i (i = 1, 2, 3, 4) to node j (j = 3, 4, 5, 6) this year.

The objective is minimize total cost. From cost data, shipping costs are
2X13 + 5X14 +
9X23 + 8X24 +
3X35 + 4X36 +
5X45 + 6X46 +

From cost data, facility use fixed costs are
100Y3 + 70Y4
Hence, the objective to minimize total costs is

Min
2X_{13} + 5X_{14} +
9X_{23} + 8X_{24} +
3X_{35} + 4X_{36} +
5X_{45} + 6X_{46} +
100Y_{3} + 70Y_{4}

From capacity data,
California capacity constraint is \( X_{13} + X_{14} \leq 550 \)
China capacity constraint is \( X_{23} + X_{24} \leq 800 \)

From demand data,
Target demand constraint is \( X_{35} + X_{45} \geq 500 \)
Walmart demand constraint is \( X_{36} + X_{46} \geq 600 \)

Transshipment constraints are

\( X_{35} + X_{36} \leq X_{13} + X_{23}, \text{ through Nevada} \)
\( X_{45} + X_{46} \leq X_{14} + X_{24}, \text{ through New York} \)

And setup indicator constraints are

\( X_{35} + X_{36} \leq 1350Y_{3}, \text{ through Nevada} \)
\( X_{45} + X_{46} \leq 1350Y_{4}, \text{ through New York} \)
B.4 Binary Fixed Costs

This module solves the following types of Integer Linear Programs: All-Integer Linear Programs and Mixed Integer Programs. The program can handle problems containing up to 100 decision variables and 50 constraints.

**Problem Features**

- **Number of Decision Variables:** 10
- **Number of Constraints:** 8 (Not Including Nonnegativity)
- **Optimization Type:** Minimize

OK  Cancel
B.4 Binary Fixed Costs

Review Questions

[Image of optimization problem with variables and constraints]

[Image of integer variable identification choice]
That solution is the same if, instead of $X_{ij}$ variables being integer, they were continuous.

In that solution, Nevada transshipment node is used, but New York is not used.

Finally, the minimum cost is $10,050 per day.
Transshipment Problems with New Nodes

Question. Apple supplies distribution centers in California, Colorado, Florida, and New York with smart phones. Apple orders computers from manufacturers in Arizona and China. Each day’s demand for phones are 400 for California, 200 for Colorado, 300 for Florida, and 500 for New York. Arizona can supply up to 800 computers and China can supply up to 1200 computers.

Apple can ship from Chicago, or from Las Vegas, or from Newark (or from any combination). If any units are shipped from Chicago, there is a fixed cost of 100 per day. If any units are shipped from Las Vegas, there is a fixed cost of 70 per day. If any units are shipped from Newark, there is a fixed cost of 80 per day.

Unit costs from the manufacturers to the shipment centers are:

<table>
<thead>
<tr>
<th></th>
<th>Chicago</th>
<th>Las Vegas</th>
<th>Newark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arizona</td>
<td>2</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>China</td>
<td>9</td>
<td>8</td>
<td>6</td>
</tr>
</tbody>
</table>

The unit costs to distribution centers are:

<table>
<thead>
<tr>
<th></th>
<th>California</th>
<th>Colorado</th>
<th>Florida</th>
<th>New York</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chicago</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>Las Vegas</td>
<td>5</td>
<td>6</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>Newark</td>
<td>2</td>
<td>5</td>
<td>8</td>
<td>3</td>
</tr>
</tbody>
</table>

Formulate the linear program that minimizes the cost of meeting all demands at minimum cost.

Tip: Your written answer should define the decision variables, and formulate the objective and constraints.
**Answer to Question:**
List origin, transshipment, and destination nodes in this transshipment problem:
1 = Arizona, 2 = China
3 = Chicago, 4 = Las Vegas, 5 = Newark,
6 = California, 7 = Colorado, 8 = Florida, 9 = New York.

Define binary variables for transshipment nodes,
Y3 = 1 if the Chicago facility is used; 0, if not
Y4 = 1 if the Las Vegas facility is used; 0, if not
Y5 = 1 if the Newark facility is used; 0, if not

Define integer shipment variables just as in any transshipment problem,
Xij = the units shipped from node i (i = 1, 2, 3, 4, 5) to node j (j = 3, 4, 5, 6, 7) on a particular day.

The objective is minimize total cost. From cost data, shipping costs are

\[
2X_{13} + 5X_{14} + 4X_{15} +
9X_{23} + 8X_{24} + 6X_{25} +
3X_{36} + 4X_{37} + 6X_{38} + 5X_{39} +
5X_{46} + 6X_{47} + 4X_{48} + 7X_{49} +
2X_{56} + 5X_{57} + 8X_{58} + 3X_{59} +
\]

From cost data, facility use fixed costs are

\[
100Y_3 + 70Y_4 + 80Y_5
\]
Hence, the objective to minimize total costs is

\[
\text{Min} \\
2X_{13} + 5X_{14} + 4X_{15} + \\
9X_{23} + 8X_{24} + 6X_{25} + \\
3X_{36} + 4X_{37} + 6X_{38} + 5X_{39} + \\
5X_{46} + 6X_{47} + 4X_{48} + 7X_{49} + \\
2X_{56} + 5X_{57} + 8X_{58} + 3X_{59} + \\
100Y_{3} + 70Y_{4} + 80Y_{5}
\]

From capacity data,

Arizona capacity constraint is \(X_{13} + X_{14} + X_{15} \leq 800\)

China capacity constraint is \(X_{23} + X_{24} + X_{25} \leq 1200\)

From demand data,

California demand constraint is \(X_{36} + X_{46} + X_{56} \geq 400\)

Colorado demand constraint is \(X_{37} + X_{47} + X_{47} \geq 200\)

Florida demand constraint is \(X_{38} + X_{48} + X_{58} \geq 300\)

New York demand constraint is \(X_{39} + X_{49} + X_{49} \geq 500\)

Transshipment constraints are

\(X_{36} + X_{37} + X_{38} + X_{39} \leq X_{13} + X_{23}, \text{through Chicago}\)

\(X_{46} + X_{47} + X_{48} + X_{49} \leq X_{14} + X_{24}, \text{through Las Vegas}\)

\(X_{56} + X_{57} + X_{58} + X_{59} \leq X_{15} + X_{25}, \text{through Newark}\)

And fixed-cost indicator constraints are

\(X_{36} + X_{37} + X_{38} + X_{39} \leq 2000 Y_{3}, \text{through Chicago}\)

\(X_{46} + X_{47} + X_{48} + X_{49} \leq 2000 Y_{4}, \text{through Las Vegas}\)

\(X_{56} + X_{57} + X_{58} + X_{59} \leq 2000 Y_{5}, \text{through Newark}\)