Optimal Incentive Contracts under Relative Income Concerns*

Levon Goukasian†
Xuhu Wan‡

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Abstract

We are studying in this paper an interplay between workers in organizations under the assumption that workers exhibit behavioral biases: envy, jealousy, or admiration towards the other co-workers' compensations. We assume workers care about their relative position, and we study the impact of this assumption on their efforts and on their optimal incentive contracts. We explicitly solve for the optimal incentive contract of moral hazard a la Holmstrom Milgrom (1987) when there is team production, each agent's effort generates an observable signal that depends on efforts of all. One of the important findings is that a worker's optimal effort is less than it would be in the absence of any behavioral biases in workers' judgments. That is, envious workers exert less effort. We also show that the value of the firm is less in the presence of envy or jealousy; that is, envious behavior is destructive for organizations. The sensitivity of the workers' pay to their own performance is studied. Consistent with Tirole (2001), in the presence of agency problems (induced by envy or jealousy), the optimal compensation exhibits high pay-for-performance sensitivity.

Key Words and Phrases: Multi-Agent Problems, Hidden Action, Envy, Jealousy, Behavioral Contract Theory, Optimal Effort, Pay for Performance Sensitivity.

JEL Classification: C65, D23, D62, J22, J31, J33, M52.

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†Business Division of Seaver College, Pepperdine University, Malibu, CA, 90263, USA. Ph: (310)506-4425. Fax: (310)506-4696. E-mail: Levon.Goukasian@Pepperdine.edu.

‡Department of Information and Systems Management, HKUST Business School, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, HK. E-mail: imwan@ust.hk.

1 Introduction

Optimal contracting and incentive problems arise in many economic situations, and the standard principal-agent problems have addressed them in many forms. Optimal contracts tie compensation to performance of an individual or of a team. In most of the literature, employees are viewed as rational economic agents. In this paper, employees are viewed as emotional beings rather than just economic agents. We solve an optimal contracting and compensation policy problem here, taking the presence of some "irrational sensibilities" of agents into account. Namely, we model the envy or jealousy of employees towards their coworkers' compensation and show the impact of that on the optimal effort that employees exert and their optimal compensation contracts.[1]

In general, firms tend to avoid potential conflicts with their employees and among their employees, since conflicts, which could be result of disappointment regarding compensation or promotion, are costly to the firms. Within organizations, compensation or promotion often are based on performance measures that are difficult to gauge. Examples of those are teamwork, initiative, etc. Sometimes subjective evaluation is adopted to measure employee performance, such as supervisor or peer evaluation. Of course, using subjective methods of evaluation, the principal is faced with the problem of disagreement on the employees' part and distortion of incentives in times when the principal's and employees' evaluations are not in line. As argued in MacLeod (2005), if the principal's and employees' subjective evaluations concur, then it is possible to implement the optimal contract, just as if the evaluations were objective and verifiable. An employee may exert less effort if he/she does not agree with a low evaluation by the principal. Anticipating this, the principal will be reluctant to provide a low evaluation unless he truly believes it to be deserved.[2]

Having the above in mind, we propose to study optimal incentive contracting in the presence of certain distortionary biases in agents' assessments, such as envy, jealousy, or admiration towards the compensation of the others.[3] The idea that the utility of an individual

[1]Neoclassical economic theory assumes that an agent's utility depends solely on the absolute level of consumption; however, recent findings show that utility depends at least in part on the comparison of one's own consumption to that of others.

[2]see Prendergast (1999) for a similar argument. Here it is argued that principals tend to have equal valuations among employees when it comes to compensation. Frank (1985) claims that positional concerns can explain many real-world phenomena. For example, he claims that wage profiles within firms are flatter than standard theory predicts. That is, lower productivity workers are paid more than their marginal product, to compensate them for their low status, and higher productivity workers earn less.

[3]We are incorporating envy and jealousy in employees' behavior in that they are envious or jealous of the compensation levels of the other coworkers. We are not using subjective evaluations here and the disutility is derived solely from the well-being of the others and may have a negative impact on the worker's effort and output.

depends not only upon his/her own consumption, but also the consumption of the others, is an important feature of our social existence.[4]

This idea has been used in the analysis of government policies,[5] stock market behavior, historical equity premium explanations, etc. Recent research in finance, in particular in asset pricing, has explored the idea to explain some pricing anomalies.[6]

We are studying in this paper an interplay between workers in organizations under the assumption of envy, jealousy, and moral hazard:[7] employees' utilities are increasing in their own compensation and decrease in the compensation of the others and the amount of effort exerted.[8] In our model, principals can't observe the agents' efforts and agents can't observe each other's effort; they exert efforts simultaneously.

Since the seminal work of Holmstrom and Milgrom (1987), continuous-time models have been applied to various principal-agent problems. The continuous-time approach offers tractable ways to solve the discrete-time problems with unavoidable technical difficulties, and generate relatively simple forms of optimal contracts. With the use of the methods of backward stochastic differential equation(BSDE), the agents' optimal efforts are determined explicitely. To the best of our knowledge, contracting problems with multiple agents and in continuous-time have not been studied yet. In this paper, we extend the continuous-time stochastic model of Holmstrom and Milgrom (1987), by allowing multiple agents and a principal in the main case.[9] We also solve the problem in the case of multiple agents and

[4]The envy/jealousy of others' consumption is not directly modeled here. We use compensation as a proxy of the consumption. This is a noisy but a reasonable proxy for the consumption. Jealousy is typically over what one possesses and fears to lose, while envy may be over something one has never possessed (and may never hope to possess). Thus, envy is typically towards the other person, rather than the particular thing or quality one is envious over. Our discussion of envy relates to concern about relative income, as studied by Frank (1984) and Frank (1985). The above papers argue that a worker may prefer a job at firm A which pays less than a job at firm B, if the wage firm A offers is high compared to what it pays others. Fershtman, Hvide, and Weiss (2003) examine how such concern about relative status affects workers effort and affects the pay package a firm should offer.

[5]E.g. Dupor and Liu (2003) examine the implications of jealousy and "keeping up with the Joneses" preferences on equilibrium consumption.

[6]The list of papers is too long to present here. See the work of Abel(1990) on Asset Pricing under Habit Formation, where individuals' utility is greater if they consume more than the others. Constantinides (1991) studies the impact of "internal" habit formation on asset prices, where the individuals' utility is greater if at every period they consume more than they did in the recent past. Campbell and Cochrane (1999) study general habit formation models of consumption to explain observed asset pricing anomalies. All of these papers try to explain the equity premium puzzle, by arguing that individuals get more utility of their own consumption above and beyond certain benchmarks. The benchmark is sometimes the others' consumption.

[7]Pingle, Mitchell (2005) state that about 70% of their survey participants exhibited some type of positional concern.

[8]The idea that individuals act based on where they see themselves relative to others is very important and has recently been used in sociology and in psychology.

[9]Holmstrom and Milgrom (1987) show that the optimal contract is linear, by assuming that the principal

multiple principals. We develop a new technique which applies the comparison theorem for backward stochastic differential equations to solve for the optimal incentive contract that can implement desired/feasible effort levels, as opposed to the first order approach used in the literature.[10] Our results have implications for the design of incentives, particularly incentives that reward relative performance. An incentive scheme where a worker's income depends upon his performance rank would motivate workers with no positional concern for income in that the worker would provide effort to obtain a high ranking so that his own income would be high.

The outline of the paper is the following : In Section 2 we present a literature survey about envy and jealousy at the workplace. Sections 3 lays out the continuous-time model setting and related technical assumptions. The maximization problem in the setting of one principal and multiple agents is formalized in Section 3. In Section 4 we derive and study the first best contract, the solution of a problem in which the effort is observable by the principal. The latter is used as a benchmark. Sections 5, 6, and 7 study the second best contracts, the case in which the agents exhibit behaviorial biases (envy, jealousy, or admiration) towards the pay levels of their coworkers. We conclude in Section 8. Section 9, the Appendix, contains the proofs of all the results outlined in the body of the paper.

2 Envy and Jealousy at the Workplace

Men do not desire to be rich, but richer than other men.[11]

J.S. Mills

Organizations are not emotion-free, nor are workplaces emotion free zones. Dealing with emotionality is important from a productivity perspective: negative emotions affect productivity and thus deserve an attention. Since one of the major goals of management is to increase productivity, it is of paramount importance to understand the sources of and the impact of negative emotions and take them into account[12].

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and the agent have exponential utilities; Schattler and Sung (1993) generalize the results of Holmstrom and Milgrom (1987),using a dynamic programming and martingales approach of Stochastic Control Theory, Sung (1995) showes that the linearity of the optimal contract still holds even if the agent can control the volatility, too. Williams (2004) uses the stochastic maximum principle to characterize the optimal contract in the principal-agent problems with hidden action, modeling the disutility of an agent's effort to be separable from the utility of pay. Sung (2001) provides a nice survey of the literature.

[10]Several recent papers study optimal incentive contracting problems, under the assumption that workers feel envious/jealous towards the well-being of their coworkers. See Bartling and Von Siemens (2003), Biel (2002), Demougin and Fluet (2003), Grund and Sliwka (2003) and Itoh (2004).

[11]The quote is taken from Jorgensen and Herby (2004).

[12]Survey data suggest that positional concerns are extremely important and that well-being is affected by

In traditional economic models, individual utility depends only on absolute consumption. Recent years have seen renewed interest in economic models in which individual utility depends not only on absolute consumption, but also on relative consumption. In contrast to traditional models, these models identify a fundamental conflict between individual and social welfare.

Many papers have empirically examined the relationship between relative position and well-being. Based on their findings, individuals appear to care about their relative position. Neumark and Postlewaite (1998) and Bowles and Park (2002) relate relative position to labor supply decisions.[13] Luttmer (2005) documents a strong negative relationship between an individual's reported happiness measures and average neighborhood income. Frank (1985) and Luttmer (2005) have found evidence of the positional aspects of income.[14]

In recent years social scientists have begun to study particular types of emotions - interpersonal jealousy and envy (JE from now on). Veccio (1995) shows that the majority of employees report experiencing JE at work. Another important finding is that those employees believe that their managers are not effectively managing those situations. Of course, not all of these feel JE towards the compensation of the others.[15]

It has been shown that employees compare coworkers' salaries and performances in the firm with their own. Bewley (1999) shows that 69% of firms' managers interviewed offer formal pay structures because it creates internal equity. Most of those managers (78%) view internal equity as an important factor in keeping high morale and harmony in their firms. Also, 49% of the managers view it as important in job performance.

Having the above facts in mind, our model shows how Behavioral Contract Theory could be useful to study organizational issues in firms. We believe that comparisons or relative income concerns among workers are important enough to be part of the design of the optimal incentive contracts.

Neumark and Postlewaite (1998) introduced a relative income concern into a choice theory model and used it to explain why, over some periods of time, women's employment rose faster than can be accounted for by a standard neoclassical model. Aronsson, Bloomquist, and Slacklen (1999) find empirical evidence that the individual work hour choices are influenced by the work hour average of relevant social reference groups. More recently, compelling

relative, rather than absolute, income levels (Easterlin (1973); Easterlin (1995)). Thus, ignoring positional concerns may lead to incorrect descriptive explanation.

[13]Using data on sisters, Neumark and Postlewaite (1993) tested a hypothesis that a woman's labor supply decision was affected by the family's concern about its relative income. They found that women's employment decisions were influenced by the employment decisions of their sisters.

[14]Neumark and Postlewaite (1993) found evidence that income is more positional than leisure. Frey and Stutzer (2002) provide a detailed review of this literature.

[15]Veccio (2000) defines Jealousy/Envy (JE) as patterns of thoughts, emotions, and behaviors that result from an employee's loss of self-esteem and/or loss of outcomes associated with a working relationship.

evidence has accumulated that people tend to evaluate their own consumption in the light of the consumption of others. For example, starting from Richard Easterlin (1974), a number of studies have found that self-reported happiness may be more sensitive to relative than to absolute income.

Miner (1990) conducted a survey to determine the part JE plays in organizations. He found that more than 50% of respondents indicate they were directly involved in situations, in which JE was expressed by other coworkers. The data show that the jealous person typically will try to bring coworkers to their side (72%).[16] Miner writes, "Although it's natural that people will talk, the data show that these people are doing more than conveying information. For example, in more than one-third of the situations, jealous people try to undermine (spread rumors, act destructively, and so on) the co-workers they're jealous of: a quarter of the time they try to undermine the position of the benefit provider." Often, managers tried to solve these problems by redistributing the benefits to the jealous worker, but most of the time it created even more problems. Miner then suggests that managers need to be aware that when they give out promotions and other benefits to their employees, jealousy can backfire in their groups. As a part of this situation, their behaviors can have a dramatic affect on the degree of jealousy in the work environment. Second, managers should consider the underlying issues at the root of the jealousy. If someone is passed over for a promotion, he or she might be furious with the boss and cause trouble for the new promotee. Although anger is the obvious emotion, the employee's self-worth may be at the heart of it. Ignoring the basic human emotions at work here will only cause bigger problems later. Thus, these findings suggest that organizations should consider the existence and impact of emotions at workplaces. Vaccio (1995) found out that jealousy and envy seemed to arise less frequently in large workplaces than small. For this, he suggests a possible explanation: people tend to feel less possessive in larger divisions. Veccio also suggests that being envied or feeling envious create different outcomes: being the subject of envy is independent of gender, age, or education; however, it explains the job longevity. On the other hand, employees who claimed to have felt envious/jealous toward their coworkers also claimed lower job satisfaction and lower-quality working relationships with their supervisors. An important description of JE in workplaces is the reduction of self-worth that occurs as a result of social comparison of workers.[17]

[16] Frank (1985) is the most comprehensive recent exploration of concerns about relative standing. In his work, positional externalities are said to occur when "one person's action alters an important frame of reference for others". Frank (1985) claims that "someone whose close associates all earn $50,000 a year is likely to feel actively dissatisfied with his material standard of living if his own salary is only $40,000. Yet that same person would likely be content if his closest associates earned not $50,000 but $30,000 a year."

[17] For more references see Mumford (1983) and Ambrose, Harland, and Kulik (1991). Veccio (2000) argues that although there is a tendency to view employee JE as dysfunctional, it can be very functional; JE can energize behavior and focus attention to protect relationships in organizations.

Is it possible to detect JE at workplaces? In which situations does it arise? Employees are likely to express JE in the presence of competitive reward systems of compensation. Reward systems that are based on win/lose situations promote higher levels of JE, as a result of competitiveness. This has been demonstrated by Veccio (2000). This also implies that if employees work independently of coworkers and perform their tasks by not involving the others, they will not experience JE towards the (compensation of) other coworkers. There are other explanations for and relationships with JE, in terms of manager considerateness, employees' sense of lack of control, propensity to quit, etc. We refer interested readers to Veccio (2000) for more information on the latter.

3 The Model

Consider an organization with a principal and multiple agents. Without loss of generality, we assume the economy consists of one principal and two agents indexed by $i = 1, 2$. The principal delegates a project to the agent i whose value dynamics S_t^i are given by the following[18]

$$\begin{aligned} dS_t^1 &= \left(\delta_{11}u_t^1 + \delta_{12}u_t^2\right) dt + \sigma_1 dW_t^1 \\ dS_t^2 &= \left(\delta_{21}u_t^1 + \delta_{22}u_t^2\right) dt + \sigma_2(\rho dW_t^1 + \sqrt{1-\rho^2} dW_t^2) \end{aligned}$$

where W_t^1, W_t^2 are two independent Brownian motions in probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, P)$. Note that the projects' outcomes are correletad and the correlation between them is ρ. $\mathcal{F}_t$ is the augmented filtration generated by (W_t^1, W_t^2) and u_t^i is the effort exerted by the agent i. The structure of our model implies that each agent's effort affects both projects' returns and it can be generalized into a multi-task model. The principal can't observe the efforts of both agents and the sources of uncertainty that impact the outcomes of the projects assigned to agents. So, neither the effort nor the uncertainty can be incorporated into the contracts. The principal's information flow is generated by processes (S_t^1, S_t^2), and we denote it by $\mathcal{G}_t$. In a certain probability space $(\Omega, \mathcal{G}_t, Q^0)$, we can find $\mathcal{G}_t$-measurable Brownian motions (B_t^1, B_t^2), satisfying the following weak formulation setting (as in Holmstrom and Milgrom (1987))

$$dS_t = \Sigma dB_t$$

where $S_t = \begin{bmatrix} S_t^1 \\ S_t^2 \end{bmatrix}$, $B_t = \begin{bmatrix} B_t^1 \\ B_t^2 \end{bmatrix}$ two independent dimensional Brownian motions on space $(\Omega, Q^0, \mathcal{G}, \mathcal{G}_t)$ and $\Sigma = \begin{bmatrix} \sigma_1 & 0 \\ \sigma_2\rho & \sigma_2\sqrt{1-\rho^2} \end{bmatrix}$. With this construction, the contracts can be

[18]We take linear drift functions of efforts for simplicity of notations and results, which can be easily extended to nonlinear form. However, the simplicity of our model doesn't lose any important implications in contracting with JE. The way to model two correlated brownian motions is widely used in the literatures of continuous-time financial and economic modeling.

written on B_t processes. The agents' efforts can change the measure from Q^0 to Q^u, with the following dynamics:

$$\frac{dQ^u}{dQ^0} = K_T = exp\left\{-\frac{1}{2}\int_0^T |\Sigma^{-1}Mu_s|^2 ds + \int_0^T (\Sigma^{-1}Mu_s)' dB_s\right\}$$

Where $M = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}$, $(\ )'$ denotes the transpose of a matrix. It follows from the Girsonov Theorem that

$$B_t^u = B_t - \int_0^t \Sigma^{-1}Mu_s ds$$

is a vector of Brownian motion processes under the measure Q^u. Then

$$dS_t = Mu_t + \Sigma dB_t^u$$

That is

$$\begin{aligned} dS_t^1 &= \left(\delta_{11}u_t^1 + \delta_{12}u_t^2\right) dt + \sigma_1 dB_t^{u,1} \\ dS_t^2 &= \left(\delta_{21}u_t^1 + \delta_{22}u_t^2\right) dt + \sigma_2(\rho dB_t^{u,1} + \sqrt{1-\rho^2} dB_t^{u,2}) \end{aligned}$$

This is the so-called weak formulation of the hidden action model in continuous time. All random processes and effort processes now are adapted to $\mathcal{G}_t$, which is generated by observable processes (S_t^1, S_t^2). Here, we denote E^{u^1,u^2} the expectation, defined on the space $\left(\Omega, Q^{u^1,u^2}\right)$. u_t^i is the effort of the worker i and the outcomes of the projects are increasing in levels of efforts of both workers. So here we are modeling collaboration between workers in that the outcome of each project depends on the effort levels of both workers. The parameters δ_{ij} for $i, j = 1, 2$ measure the relative importance of the effort of worker i on project j. This is a very realistic assumption and general enough to enable us to learn about the interplay between workers when their actions have an impact on the outcomes of their own and other coworkers' projects.

In addition, we assume the assigned projects are, in general, correlated. This is also a general enough assumption to learn about interplays in organizations where employees are assigned to projects that are dependent on each other.

Note that by taking $\rho = 0$, we will be in a case of two projects that are independent. Along with the assumption that $\delta_{12} = \delta_{21} = 0$, we will have the case that the two agents are delegated with two independent projects. If $\rho = 1$, the two projects are affected by a common source of uncertainty. Σ is not invertible. To find the optimal contracts when $\rho = 0$, we first assume $\rho \neq 0$, find the optimal contracts, and then find the limit as ρ converges to 0. The total output generated by the group (of two employees) is $S_t^1 + S_t^1$ at time t.

At time 0, the principal offers take-it-or-leave-it compensation contracts (C_T^1, C_T^2) to agents 1 and 2. These are payable at time T in the future and both are $\mathcal{G}_T-$measurable.

We assume that each agent i knows the other agent j's contract payment C_T^j, $j = 3 - i$. Only when both agents accept the contracts at time 0, the principal and two agents enter into the contract; otherwise each agent has outside opportunity with a certain reservation utility level.

We assume that the principal is risk neutral, so his objective is to maximize the expected utility of terminal wealth[19]:

$$E^u \left(S_T^1 + S_T^2 - C_T^1 - C_T^2 \right)$$

We assume workers are risk averse with utility of compensation, and that they demonstrate envy/jealousy or admiration towards the compensation of the other workers in their groups. Worker i's utility function is

$$U^i \left(C_T^i, C_T^j, G_T^i \right)$$

where

$$G_T^i = \int_0^T g_i \left(u_s^i \right) ds$$

is the cumulative disutility of effort for agent i.

Thus, our model of utility is such that it depends on one's own compensation and the compensation of the others in the group.[20]

We make the following assumptions about utility functions of the agents:

1. U^i is twice differentiable,, $U^i_{C^i_T} > 0, U^i_{G^i_T} < 0$ for all C_T^i, C_T^j, G_T^i, and $U^i_{G^i_T} + U^i_{G^j_T} > 0$ if $C_T^i = C_T^j$.

2. if $U^i_{C^j_T} < 0$, the preferences exhibit **jealousy/envy (JE)**; if $U^i_{C^j_T} > 0$, the preferences exhibit **admiration (AD)**.

That is, workers, are less happy to see other coworkers getting higher compensation under the jealousy/envy assumption, and they are happier to see other coworkers get higher compensation under the admiration assumption.[21]

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[19]Our method also works well when the principal has exponential utility:

$$E^u \left(-\frac{1}{R} exp(-R(S_T^1 + S_T^2 - C_T^1 - C_T^2)) \right)$$

[20]Economists generally assume that utility is a function of the individual's endowment, independent of his relative position. Envy (or jealousy) is one reason individuals care about their relative status. Following Holmstrom and Milgroms (1991), optimal contracts must account for everything employees care about. Thus, we develop a model in which a principal has to design a reward scheme for two agents who dislike the other's pay.

[21]Bewley (1999) reports that 87% of managers interviewed think that their employees know each other's wages. Agell and Lundborg (1999), Bewley (1999), Blinder and Choi (1990), Campbell and Kamlani (1997)) show that employees purport to care for the well being of their coworkers and not only for their own.

Throughout this paper, in order to get closed-form analytic results, we make the following assumption on the utility functions:

$$U_i\left(C_T^i, C_T^j, G_T^i\right) = -\frac{1}{r^i}\exp\left\{-r_i\left(C_T^i - \alpha_i C_T^i - G_T^i\right)\right\}$$

with $-1 < \alpha_i < 1$.[22]

Remark 3.1 *If $0 < \alpha_i < 1$ then $U_j^i < 0$ and thus the preferences exhibit jealousy/envy. Otherwise, if $\alpha_i < 0$, then $U_j^i > 0$ and the preferences exhibit admiration. Thus, α_i is the measure of jealousy/envy or admiration.*

Technically, we only consider $(C_T^i, u_t^i) \in \mathcal{A}$, such that $\forall (C_T^i, u_t^i) \in \mathcal{A}$

i. $E^u\left\{\exp\left\{-r_i\left(C_T^i - \alpha_i C_T^i - G_T^i\right)\right\}\right\} < \infty$;

ii. $Eexp\{N\int_0^T |\Sigma^{-1}Mu_s|^2 ds\} < \infty$ for some large enough constant N which depends on constant parameters $\delta_{i,j}, k_i, r_i$;

iii. $E^u\left\{\left(C_T^i - \alpha_i C_T^i\right)^2 + \left(\int_0^T g^i(u_s^i)ds\right)^2\right\} < \infty$

Remark 3.2 *(ii) is a strong condition for K_t to be a martingale so that Q^u is well defined as a measure by the Girsanov theorem. This condition could be relaxed if the agents' efforts u_s^i can only be chosen from a closed convex set (i.e. $|u_s^i| \le b$, where b is a constant). Conditions (i) and (iii) are necessary for uniqueness and existence of quadratic BSDEs, which we deal with later in the paper.*

Let $\mathbf{S}^2$ be the space of all $\mathcal{G}_T$- measurable random variables $X : (\Omega \times [0,T] \to \mathbf{R}$ satisfying $E^0[\sup_{0\le t\le T}|X_t|^2] < \infty$. Also, $\mathbf{H}^2$ will denote the space of all predictable processes $\phi : \Omega \times [0,T] \to \mathbf{R}^2$ such that $E\int_0^T |\phi_s|^2 ds < \infty$.

3.1 The Agents' Problem

Both agents exert efforts simultaneously. So, given their contracts (C_T^1, C_T^2), we say that $(\bar{u}_t^1, \bar{u}_t^2)$ is Nash-Equilibrium-Effort choice if

$$\overline{u}_i \in \arg\max_{u_i} V_i\left(u_i, \overline{u}_j\right)$$

[22]With exponential utility, our methods here can also be applied to a more complicated model, i.e., a utility function of the form $-\frac{1}{r^i}\exp\left\{-r_i\left(H(C_T^i, C_T^j) - G_T^i\right)\right\}$, where H can be any nonlinear function that captures the envy/jealousy in groups , that is $[\frac{\partial H(x,y)}{\partial x} + \frac{\partial H(x,y)}{\partial y}]|_{x=y} > 0$ and $\frac{\partial H(x,y)}{\partial x} > 0$. In addition, if $\frac{\partial H(x,y)}{\partial y} > 0$, the preferences exhibit **admiration**; if $\frac{\partial H(x,y)}{\partial y} < 0$, the preferences exhibit **envy/jealousy**.

given $\left(\{u_t^j\}_{t\geq 0}^T, C_T^i, C_T^j\right)$, where

$$V_i\left(u^i, u^j\right) = E^{u^i,u^j}\left[-\frac{1}{r^i}\exp\left\{-r_i\left(C_T^i - \alpha_i C_T^j - G_T^i\right)\right\}\right]$$

3.2 The Principal's Problem

The principal's problem is to maximize expected utility of terminal wealth, by choosing the optimal amount of effort and compensation for workers: $\{u_t^1\}_{t\geq 0}^T$, $\{u_t^2\}_{t\geq 0}^T$, C_T^1 and C_T^2:

$$\max_{C_T^1, C_T^2, \{u_t^1\}_{t\geq 0}^T, \{u_t^2\}_{t\geq 0}^T}\left\{V_P\left(u, C_T\right) = E^{u^i,u^j}\left(S_T^1 + S_T^2 - C_T^1 - C_T^2\right)\right\}$$

under the following two constraints:

1. Individual Reservation constraints (IR) are satisfied for both agents:

$$V_i\left(u^i, u^j\right) \geq -\frac{1}{r_i}\exp\left\{-r_i L_i\right\}$$

and

2. Incentive Compatibility constraints (IC) are satisfied:

$$\overline{u}_i \in \arg\max_{u_i} V_i\left(u_i, \overline{u}_j\right)$$

We first study a case in which the agents' efforts are observable by the principal. In this case we derive the first-best incentive contract and use it later as a benchmark.

4 Benchmark: First-Best Solutions

As a benchmark, in this section we find the first-best solutions of a case with multiple agents, in which the principal can contract on both agents' efforts. That is

$$\max_{C_T^1, C_T^2, \{u_t^1\}_{t\geq 0}^T, \{u_t^2\}_{t\geq 0}^T} E^{u^i,u^j}\left(S_T^1 + S_T^2 - C_T^1 - C_T^2\right)$$

such that

$$E^{u^1,u^2}\left\{-\frac{1}{r^i}\exp\left\{-r_i\left(C_T^i - \alpha_i C_T^j - G_T^i\right)\right\}\right\} \geq -\frac{1}{r_i}\exp\left\{-r_i L_i\right\}$$

where $-\frac{1}{r_i}\exp\left\{-r_i L_i\right\}$ is the reservation utility for the agent i.

We have the following result:

Theorem 4.1 *With symmetric and complete information (first-best case), the principal will pay agents a linear combination of the costs of efforts:*

$$C_T^{1,FB} = ln\left[\left(\frac{1+\alpha_1}{(1-\alpha_1\alpha_2)\lambda_2}\right)^{\frac{\alpha_1}{r_2(\alpha_1\alpha_2-1)}} \left(\frac{1+\alpha_2}{(1-\alpha_1\alpha_2)\lambda_1}\right)^{\frac{1}{r_1(\alpha_1\alpha_2-1)}} \right] + \frac{G_T^1+\alpha_1 G_T^2}{1-\alpha_1\alpha_2}$$

$$C_T^{2,FB} = ln\left[\left(\frac{1+\alpha_2}{(1-\alpha_1\alpha_2)\lambda_1}\right)^{\frac{\alpha_2}{r_1(\alpha_1\alpha_2-1)}} \left(\frac{1+\alpha_1}{(1-\alpha_1\alpha_2)\lambda_2}\right)^{\frac{1}{r_2(\alpha_1\alpha_2-1)}} \right] + \frac{G_T^2+\alpha_2 G_T^1}{1-\alpha_1\alpha_2}$$

and the first-best optimal efforts $(u^{1,FB}, u^{2,FB})$ *(which are constant over time) satisfy the following first-order conditions*

$$\delta_{11}+\delta_{21} = \frac{1+\alpha_2}{1-\alpha_1\alpha_2} g_1'(u^{1,FB}), \delta_{12}+\delta_{22} = \frac{1+\alpha_1}{1-\alpha_1\alpha_2} g_2'(u^{2,FB})$$

and

$$G_T^i = \int_0^T g_i(u_s^{i,FB})ds$$

Proof: See in Appendix.

Remark 4.1 *The first-best effort of each agent depends on the other agent's JE level. If there is no jealousy,* $\alpha_1 = \alpha_2 = 0$*, agents' first best efforts are reduced to the solutions of the first best results of the classic multiple agents problem. If the cost function is of quadratic form* $g_i(u) = \frac{k_i}{2}(u)^2$*, the first-best effort is* $u^{i,FB} = \frac{(\delta_{1i}+\delta_{2i})(1-\alpha_1\alpha_2)}{k_i(1+\alpha_{3-i})}$*,* $i = 1, 2$*.*

From now on, we will assume agents have quadratic cost functions for the tractability of the results.

Remark 4.2 *Notice that the level of the optimal effort here, in the case of symmetric and complete information, is less than the one we would get in the absence of envy or jealousy; and the optimal effort is more than what it would be in the absence of envy or jealousy if the coworker is an admirer. That is, for* $0 < \alpha_{3-i} < 1$

$$u^{i,FB} = \frac{(\delta_{1i}+\delta_{2i})(1-\alpha_1\alpha_2)}{k_i(1+\alpha_{3-i})} < \frac{(\delta_{1i}+\delta_{2i})}{k_i} := \hat{u}^i$$

Next we derive expressions for the lagrange multipliers λ_i and the optimal compensation contract:

Corollary 4.1 *The lagrange multipliers of the previous problem are*

$$\lambda_1 = e^{r_1 L_1}\frac{1+\alpha_2}{1-\alpha_1\alpha_2}, \lambda_2 = e^{r_2 L_2}\frac{1+\alpha_1}{1-\alpha_1\alpha_2}$$

and the first-best contract is

$$C_T^{1,FB} = \frac{L_1+G_T^1}{1-\alpha_1\alpha_2} + \frac{\alpha_1(L_2+G_T^2)}{1-\alpha_1\alpha_2}, C_T^{2,FB} = \frac{L_2+G_T^2}{1-\alpha_1\alpha_2} + \frac{\alpha_2(L_1+G_T^1)}{1-\alpha_1\alpha_2}$$

The expected utility of the principal is

$$
\begin{aligned}
V_P^{FB}(\alpha_1,\alpha_2) &= S_0^1 + S_0^2 - \frac{1+\alpha_2}{1-\alpha_1\alpha_2}L_1 - \frac{1+\alpha_1}{1-\alpha_1\alpha_2}L_2 \\
&+ \frac{T(\delta_{11}+\delta_{21})^2(1-\alpha_1\alpha_2)}{2k_1(1+\alpha_2)} + \frac{T(\delta_{12}+\delta_{22})^2(1-\alpha_1\alpha_2)}{2k_2(1+\alpha_1)}
\end{aligned}
$$

Without loss of generality, we normalize the two agents' reservation utility to be 0, that is $L_1 = L_2 = 0$.

Corollary 4.2 *(i) If both agents exhibit envy or jealousy, that is* $0 < \alpha_1, \alpha_2 < 1$*, then we have*

$$V_P^{FB}(\alpha_1,\alpha_2) < V_P^{FB}(0,0)$$

(ii) On the other hand, if an agent exhibits admiration toward his/her colleague, the principal's expected utility will be higher. That is, if $-1 < \alpha_1, \alpha_2 < 0$*, then we have*

$$V_P^{FB}(\alpha_1,\alpha_2) > V_P^{FB}(0,0)$$

In case where the workers exhibit envy or jealousy, the principal's expected utility with this first best contracting is less than the one in the situation when agents exhibit no envy or jealousy ($\alpha_1 = \alpha_2 = 0$).

Thus, if both agents' efforts can be contracted on, the principal's expected utility will be lower if agents exhibit envy/jealousy and it will be higher if agents exhibit admiration.

5 Incentive Compatibility and Implementable Contracts

To solve the problem in Section 3, we propose a three-step process:

1. Define conditional Nash-Equilibrium-Efforts: for given compensation levels (C_T^1, C_T^2), define equilibrium amount of efforts/actions of workers $(\overline{u}_t^1, \overline{u}_t^2)$ that they will optimally exert and find necessary and sufficient conditions for optimality.

2. Based on the necessary and sufficient conditions in the first step, fixing efforts u_t^1, u_t^1, find the space of contracts (C_T^1, C_T^2), under which $(\overline{u}_t^1, \overline{u}_t^2)$ are Nash-Equilibrium-Efforts for the workers. The contract that implements the $(\overline{u}_t^1, \overline{u}_t^2)$ is to be used in the next step.

3. Maximize principal's utility with the implementable contract of step 2, subject to agents' IR constraints. In the second step, the implementable contract is a functional of efforts. In this step, the principal only needs to choose the effort levels to maximize his utility, subject to both workers' individual reservation (**IR**) constraints.

Definition 5.1 *We say $(\overline{u}^1, \overline{u}^2)$ is Nash-Equilibrium-Effort(NEE), given (C_T^1, C_T^2) if*

$$
\begin{aligned}
\overline{u}^1 &\in \arg\max_{\{u_t^1\}_{t\geq 0}^T} E^{u^1,\overline{u}^2}\left\{-\tfrac{1}{r_1}\exp\left\{-r_1\left(C_T^1 - \int_0^T g_1\left(u_s^1\right)ds - \alpha_1 C_T^2\right)\right\}\right\} \\
\overline{u}^2 &\in \arg\max_{\{u_t^2\}_{t\geq 0}^T} E^{\overline{u}^1,u^2}\left\{-\tfrac{1}{r_2}\exp\left\{-r_2\left(C_T^2 - \int_0^T g_2\left(u_s^2\right)ds - \alpha_2 C_T^1\right)\right\}\right\}
\end{aligned}
$$

where $g_1 = \frac{k_1}{2}\left(u_t^1\right)^2$ and $g_2 = \frac{k_2}{2}\left(u_t^2\right)^2$

To find the NEE, we need a result that we state in the appendix as a Theorem NEE. By the monotonicity of exponential function, the NEE can be obtained as follows:[23]

Lemma 5.1 *$(\overline{u}_t^1, \overline{u}_t^2)$ is Nash Equilibrium of efforts if and only if*

$$\bar{u}_t^i \in arg\max_{u^i}\; Y_0^{i,u^i,\bar{u}^{3-i}}$$

for $i = 1, 2.$, where $Y_0^{i,u^i,\bar{u}^{3-i}}$ is defined in Theorem NEE in the appendix.

6 Implementable Contracts

Now we answer the following question: given the optimal effort levels $(\bar{u}^1, \bar{u}^2)$ what kind of contract (C_T^1, C_T^2) can implement the Nash-Equilibrium efforts $(\bar{u}^1, \bar{u}^2)$? From the results of the last section, it is equivalent to asking the following question: given $f_1(Z^1)$ and $f_2(Z^2)$, what contract (C_T^1, C_T^2) can implement them? We have the following result to answer the question:

Theorem 6.2 *The Nash-Equilibrium-Efforts (NEE) $(u_t^1 = f_1(Z_t^1), u_t^2 = f_2(Z_t^2))$ for which (9.2) is satisfied can be achieved by the following contracts (C_T^1, C_T^2):*

$$
\begin{aligned}
C_T^1 &= \frac{\widetilde{L}_1 + \alpha_1\widetilde{L}_2}{1-\alpha_1\alpha_2} + \frac{1}{1-\alpha_1\alpha_2}\int_0^T \left((\frac{k_1}{2}|f_1(Z_s^1)|^2 + \frac{\alpha_1 k_2}{2}|f_2(Z_s^2)|^2) + (\frac{|Z_s^1|^2}{2r_1} + \frac{\alpha_1|Z_s^2|^2}{2r_2}) \right) ds \\
&+ \frac{1}{1-\alpha_1\alpha_2}\int_0^T \left(\frac{Z_s^1}{r_1} + \frac{\alpha_1 Z_s^2}{r_2} \right) dB_s^Z
\end{aligned}
$$

$$
\begin{aligned}
C_T^2 &= \frac{\widetilde{L}_2 + \alpha_2\widetilde{L}_1}{1-\alpha_1\alpha_2} + \frac{1}{1-\alpha_1\alpha_2}\int_0^T \left((\frac{k_2}{2}|f_2(Z_s^2)|^2 + \frac{\alpha_2 k_1}{2}|f_1(Z_s^1)|^2) + (\frac{|Z_s^2|^2}{2r_2} + \frac{\alpha_2|Z_s^1|^2}{2r_1}) \right) ds \\
&+ \frac{1}{1-\alpha_1\alpha_2}\int_0^T \left(\frac{Z_s^2}{r_2} + \frac{\alpha_2 Z_s^1}{r_1} \right) dB_s^Z
\end{aligned}
$$

Under this contract, there exists a unique pair of equilibrium efforts $u_t^1 = f_1(Z^1), u_t^2 = f_2(Z^2)$ and the agent i's expected utility is $-\frac{1}{r_i}exp\{-\frac{\tilde{L}_i}{r_i}\}$

[23] This could also be used as a definition of NEE.

Proof. See in the Appendix.

■

Remark 6.3 *Since in this paper we consider multi-dimensional noise, the implementable contracts are functional of multi-dimensional adjoint processes, satisfying certain constraints. In the next section, we will incorporate this contract into the principal's problem, and the principal will choose the best intensity processes* (Z^i) *and the utility levels of the workers* $(\widetilde{L}_i)$ *to write optimal contracts.*

7 The Principal's Problem

We assume that the principal is risk neutral. The principal's problem then is to choose the best intensity processes (Z^1, Z^2) to maximize the expected profit:[24]

$$
\begin{aligned}
&V_P^{SB}(Z^1, Z^2, \widetilde{L}_1, \widetilde{L}_2) = E^Z\left(S_T^1 + S_T^2 - C_T^1 - C_T^2\right)\\
&= E^Z\Big\{S_0^1 + S_0^2 + \int_0^T ((\delta_{11}+\delta_{21})\bar{u}_s^1 + (\delta_{12}+\delta_{22})\bar{u}_s^2)ds - \frac{1}{1-\alpha_1\alpha_2}\Big((1+\alpha_2)\,\widetilde{L}_1 + (1+\alpha_1)\,\widetilde{L}_2\Big)\\
&-\frac{1}{1-\alpha_1\alpha_2}\int_0^T \left((\frac{k_1}{2}|f_1(Z_s^1)|^2 + \frac{\alpha_1 k_2}{2}|f_2(Z_s^2)|^2) + (\frac{|Z_s^1|^2}{2r_1} + \frac{\alpha_1|Z_s^2|^2}{2r_2})\right) ds\\
&-\frac{1}{1-\alpha_1\alpha_2}\int_0^T \left((\frac{k_2}{2}|f_2(Z_s^2)|^2 + \frac{\alpha_2 k_1}{2}|f_1(Z_s^1)|^2) + (\frac{|Z_s^2|^2}{2r_2} + \frac{\alpha_2|Z_s^1|^2}{2r_1})\right) ds\Big\}
\end{aligned}
$$

subject to **IR** constraints

$$\widetilde{L}_1 \geq L_1, \widetilde{L}_2 \geq L_2$$

Since $\widetilde{L}_i$ is separable from choices of Z_t^i, then we have $\widetilde{L}_1 = L_1$ and $\widetilde{L}_2 = L_2$.

Furthermore , $V_P^{SB}(Z^1, Z^2, L_1, L_2)$ can be simplified into:

$$
\begin{aligned}
&V_P^{SB}(Z^1, Z^2, L_1, L_2)\\
&= E^Z\Big\{S_0^1 + S_0^2 - \frac{1}{1-\alpha_1\alpha_2}\left((1+\alpha_2)\,L_1 + (1+\alpha_1)\,L_2\right)\\
&+ \frac{1}{2r_1^2k_1^2(1-\alpha_1\alpha_2)}\int_0^T \Gamma_1(Z_s^{1,1}, Z_s^{1,2})ds + \frac{1}{2r_2^2k_2^2(1-\alpha_1\alpha_2)}\int_0^T \Gamma_2(Z_s^{2,1}, Z_s^{2,2})ds\Big\}
\end{aligned}
$$

with

$$
\begin{aligned}
\Gamma_1(Z_t^{1,1}, Z_t^{1,2}) \quad &= 2k_1r_1(1-\alpha_1\alpha_2)(\delta_{11}+\delta_{21})(M_{11}Z_t^{1,1} + M_{12}Z_t^{1,2})\\
&-(1+\alpha_2)\left[(M_{11}Z_t^{1,1} + M_{12}Z_t^{1,2})^2 + r_1k_1^2(|Z_t^{1,1}|^2 + |Z_t^{1,2}|^2)\right]
\end{aligned}
$$

[24]The SB label is used for the Second-Best.

$$\Gamma_2(Z_t^{2,1}, Z_t^{2,2}) \quad = 2k_2r_2(1-\alpha_1\alpha_2)(\delta_{12}+\delta_{22})(M_{21}Z_t^{2,1}+M_{22}Z_t^{2,2}) \\ -(1+\alpha_1)\left[(M_{21}Z_t^{2,1}+M_{22}Z_t^{2,2})^2+r_2k_2^2(|Z_t^{2,1}|^2+|Z_t^{2,2}|^2)\right]$$

The principal's maximization problem is then reduced to the following problem:

$$\max_{Z^{1,1},Z^{1,2}} \Gamma_1(Z_t^{1,1}, Z_t^{1,2}), \max_{Z^{2,1},Z^{2,2}} \Gamma_2(Z_t^{2,1}, Z_t^{2,2})$$

It is easy to check that the Hessian matrix of $\Gamma_i(Z_t^{i,1}, Z_t^{i,2})$ is negatively definite, so the first-order conditions give the optimal solution. Thus, we have the following result:

Proposition 7.1 *The optimal $\left(\bar{Z}_t^1, \bar{Z}_t^2\right)$ for the principal are constants and given by*

$$\bar{Z}^{1,1} = \frac{M_{11}k_1r_1(1-\alpha_1\alpha_2)(\delta_{11}+\delta_{21})}{(1+\alpha_2)(r_1k_1^2+M_{11}^2+M_{12}^2)}, \bar{Z}^{1,2} = \frac{M_{12}k_1r_1(1-\alpha_1\alpha_2)(\delta_{11}+\delta_{21})}{(1+\alpha_2)(r_1k_1^2+M_{11}^2+M_{12}^2)}$$

$$\bar{Z}^{2,1} = \frac{M_{21}k_2r_2(1-\alpha_1\alpha_2)(\delta_{12}+\delta_{22})}{(1+\alpha_1)(r_2k_2^2+M_{12}^2+M_{22}^2)}, \bar{Z}^{2,2} = \frac{M_{22}k_2r_2(1-\alpha_1\alpha_2)(\delta_{12}+\delta_{22})}{(1+\alpha_1)(r_2k_2^2+M_{12}^2+M_{22}^2)}$$

The optimal efforts the principal wants to implement are constants and given by

$$\bar{u}^1 = \frac{(M_{11}^2+M_{12}^2)(1-\alpha_1\alpha_2)(\delta_{11}+\delta_{21})}{(1+\alpha_2)(r_1k_1^2+M_{11}^2+M_{12}^2)}, \bar{u}^2 = \frac{(M_{21}^2+M_{22}^2)(1-\alpha_1\alpha_2)(\delta_{12}+\delta_{22})}{(1+\alpha_1)(r_2k_2^2+M_{12}^2+M_{22}^2)}$$

Proof.

See in the Appendix.

■

A worker's utility may increase in his own income, but envy or jealousy can make his utility decline with other coworkers' income. We will see next how envy/jealousy changes the effect of incentives on effort and the optimal incentive pay. We show that JE has negative impact on the incentive pay and on optimal effort, so that optimal incentive pay structure is different when workers are envious or jealous.

The following is a corollary of the previous result:

Lemma 7.2 *The sensitivities of optimal efforts with respect to the "measure of envy/jealousy" are:*

$$\frac{\partial \bar{u}^1}{\partial \alpha_1} = -\frac{\alpha_2}{(1+\alpha_2)} \frac{\left(M_{11}^2+M_{12}^2\right)(\delta_{11}+\delta_{21})}{\left(r_1k_1^2+M_{11}^2+M_{12}^2\right)}$$

$$\frac{\partial \bar{u}^1}{\partial \alpha_2} = -\frac{1+\alpha_1}{(1+\alpha_2)^2} \frac{(M_{11}^2+M_{12}^2)(\delta_{11}+\delta_{21})}{(1+\alpha_2)^2(r_1k_1^2+M_{11}^2+M_{12}^2)}$$

$$
\begin{aligned}
\frac{\partial \bar{u}^1}{\partial \alpha_1 \partial \alpha_2} &= -\tfrac{1}{(1+\alpha_2)^2} \tfrac{\left(M_{11}^2+M_{12}^2\right)(\delta_{11}+\delta_{21})}{\left(r_1 k_1^2+M_{11}^2+M_{12}^2\right)} \\
\frac{\partial \bar{u}^2}{\partial \alpha_2} &= -\tfrac{\alpha_1}{(1+\alpha_1)} \tfrac{\left(M_{21}^2+M_{22}^2\right)(\delta_{12}+\delta_{22})}{\left(r_2 k_2^2+M_{21}^2+M_{22}^2\right)} \\
\frac{\partial \bar{u}^2}{\partial \alpha_1} &= -\tfrac{(1+\alpha_2)}{(1+\alpha_1)^2} \tfrac{(M_{21}^2+M_{22}^2)(\delta_{12}+\delta_{22})}{(r_2 k_2^2+M_{12}^2+M_{22}^2)} \\
\frac{\partial \bar{u}^2}{\partial \alpha_2 \partial \alpha_1} &= -\tfrac{\alpha_1}{(1+\alpha_1)} \tfrac{\left(M_{21}^2+M_{22}^2\right)(\delta_{12}+\delta_{22})}{\left(r_2 k_2^2+M_{21}^2+M_{22}^2\right)}
\end{aligned}
$$

Proof. Follows directly from the formulas for $\bar{u}^i$. ■

The last results show that the optimal effort is negatively correlated with the "measure of envy/jealousy". The more a worker is envious/jealous towards the compensation of the other coworker, the less effort he/she will exert. Also, if the worker 1 is envious/jealous of worker 2's compensation, then worker 2's optimal effort will decrease with his own level of envy/jealousy. That is, if a worker knows the other is jealous of his pay, his effort will decrease with envy. On the other hand, if a worker knows the other admires him (and is happy with his pay), he will exert more effort as his own jealousy measure gets higher. This will help to increase the total output of the projects.

From the weak formulation, $dS_t = \Sigma dB_t$, so we have

$$
dB_t^1 = \frac{1}{\sigma_1} dS_t^1, dB_t^2 = -\frac{\rho}{\sigma_1\sqrt{1-\rho^2}} dS_t^1 + \frac{1}{\sigma_2\sqrt{1-\rho^2}} dS_t^2
$$

With this representations we have the following result:

Proposition 7.2 *The second-best contract with hidden actions, and under JE or AD, is given by:*

$$
\begin{aligned}
C_T^{1,SB} &= \frac{L_1+\alpha_1 L_2}{1-\alpha_1\alpha_2} \\
&+ \left(\frac{k_1(r_1k_1+M_{11}^2+M_{12}^2)}{2(M_{11}^2+M_{12}^2)} |\bar{u}_{SB}^1|^2 + \frac{\alpha_1 k_2(r_2k_2+M_{21}^2+M_{22}^2)}{2(M_{21}^2+M_{22}^2)} |\bar{u}_{SB}^2|^2 - \sigma^{-1} M \bar{u}_{SB} \right) \frac{T}{1-\alpha_1\alpha_2} \\
&+ \left(\frac{M_{11}k_1\bar{u}_{SB}^1}{M_{11}^2+M_{12}^2} + \frac{\alpha_1 M_{21}k_2\bar{u}_{SB}^2}{M_{21}^2+M_{22}^2} - \frac{\rho}{\sqrt{1-\rho^2}} \left(\frac{M_{12}k_1\bar{u}_{SB}^1}{M_{11}^2+M_{12}^2} + \frac{\alpha_1 M_{22}k_2\bar{u}_{SB}^2}{M_{21}^2+M_{22}^2} \right) \right) \frac{S_T^1}{\sigma_1(1-\alpha_1\alpha_2)} \\
&+ \left(\frac{M_{12}k_1\bar{u}_{SB}^1}{M_{11}^2+M_{12}^2} + \frac{\alpha_1 M_{22}k_2\bar{u}_{SB}^2}{M_{21}^2+M_{22}^2} \right) \frac{S_T^2}{\sigma_2\sqrt{1-\rho^2}(1-\alpha_1\alpha_2)}
\end{aligned}
$$

$$
\begin{aligned}
C_T^{2,SB} &= \frac{L_2+\alpha_2 L_1}{1-\alpha_1\alpha_2} \\
&+ \left(\frac{k_2(r_2k_2+M_{21}^2+M_{22}^2)}{2(M_{21}^2+M_{22}^2)} |\bar{u}_{SB}^2|^2 + \frac{\alpha_2 k_1(r_1k_1+M_{11}^2+M_{12}^2)}{2(M_{11}^2+M_{12}^2)} |\bar{u}_{SB}^1|^2 - \sigma^{-1} M \bar{u}_{SB} \right) \frac{T}{1-\alpha_1\alpha_2}
\end{aligned}
$$

$$+\left(\frac{M_{21}k_2\bar{u}^2_{SB}}{M^2_{21}+M^2_{22}}+\frac{\alpha_2 M_{11}k_1\bar{u}^1_{SB}}{M^2_{11}+M^2_{12}}-\frac{\rho}{\sqrt{1-\rho^2}}\left(\frac{M_{22}k_2\bar{u}^2_{SB}}{M^2_{21}+M^2_{22}}+\frac{\alpha_2 M_{12}k_1\bar{u}^1_{SB}}{M^2_{11}+M^2_{12}}\right)\right)\frac{S^1_T}{\sigma_1(1-\alpha_1\alpha_2)}$$
$$+\left(\frac{M_{22}k_2\bar{u}^2_{SB}}{M^2_{21}+M^2_{22}}+\frac{\alpha_2 M_{12}k_1\bar{u}^1_{SB}}{M^2_{11}+M^2_{12}}\right)\frac{S^2_T}{\sigma_2\sqrt{1-\rho^2}(1-\alpha_1\alpha_2)}$$

where

$$\bar{u}_{SB}=\begin{bmatrix}\bar{u}^1_{SB}\\ \bar{u}^2_{SB}\end{bmatrix}=\begin{bmatrix}\frac{(M^2_{11}+M^2_{12})(1-\alpha_1\alpha_2)(\delta_{11}+\delta_{21})}{(1+\alpha_2)(r_1k^2_1+M^2_{11}+M^2_{12})}\\ \\ \frac{(M^2_{21}+M^2_{22})(1-\alpha_1\alpha_2)(\delta_{12}+\delta_{22})}{(1+\alpha_1)(r_2k^2_2+M^2_{12}+M^2_{22})}\end{bmatrix}$$

Remark 7.4 *Since the $\bar{u}^i_{SB}$s are constant, it is easy to check that $(C^{i,SB}_T,\bar{u}^i_{SB})\in\mathcal{A}$*

We can incorporate the results of the previous proposition into the principal's utility function and obtain the following result:

Corollary 7.3 *The optimal utility of the principal in the case of hidden action and in the presence of JE or AD is given by:*

$$V^{SB}_P(\alpha_1,\alpha_2)=\left(S^1_0-\frac{L_1(1+\alpha_2)}{1-\alpha_1\alpha_2}+\frac{T(\delta_{11}+\delta_{21})^2(M^2_{11}+M^2_{21})(1-\alpha_1\alpha_2)}{2(1+\alpha_2)(r_1k^2_1+k_1(M^2_{11}+M^2_{21}))}\right)$$
$$+\left(S^2_0-\frac{L_2(1+\alpha_1)}{1-\alpha_1\alpha_2}+\frac{T(\delta_{12}+\delta_{22})^2(M^2_{12}+M^2_{22})(1-\alpha_1\alpha_2)}{2(1+\alpha_1)(r_2k^2_2+k_2(M^2_{12}+M^2_{22}))}\right)$$

If we normalize the agents' reservation utilities to 0 , that is $L_i=0$, the principal's utility is

$$V^{SB}_P(\alpha_1,\alpha_2)=S^1_0+S^2_0$$
$$+\frac{T(\delta_{11}+\delta_{21})^2(M^2_{11}+M^2_{21})(1-\alpha_1\alpha_2)}{2(1+\alpha_2)(r_1k^2_1+k_1(M^2_{11}+M^2_{21}))}+\frac{T(\delta_{12}+\delta_{22})^2(M^2_{12}+M^2_{22})(1-\alpha_1\alpha_2)}{2(1+\alpha_1)(r_2k^2_2+k_2(M^2_{12}+M^2_{22}))}$$

and the principal's expected utility in the benchmark case (without envy or jealousy) is

$$V^{SB}_P(0,0)=S^1_0+S^2_0$$
$$+\frac{T(\delta_{11}+\delta_{21})^2(M^2_{11}+M^2_{21})}{2(r_1k^2_1+k_1(M^2_{11}+M^2_{21}))}+\frac{T(\delta_{12}+\delta_{22})^2(M^2_{12}+M^2_{22})}{2(r_2k^2_2+k_2(M^2_{12}+M^2_{22}))}$$

Corollary 7.4 *(i) If both agents exhibit envy/jealousy, that is $0<\alpha_i<1$, then*

$$V^{SB}_P(\alpha_1,\alpha_2)<V^{SB}_P(0,0)$$

(ii) If both agents exhibit admiration towards each other's pay, that is, if $-1<\alpha_i<0$, then

$$V^{SB}_P(\alpha_1,\alpha_2)>V^{SB}_P(0,0)$$

This means the principal is better off with agents who don't exhibit jealousy/envy, but admire each other. Thus, Envy/Jealousy is costly for the principal.

Remark 7.5 *The problem is complicated when the agents' attitudes towards each other are opposite, that is, one is envious of his coworker, but the coworker admires him. We do not address that problem here.*

Using the findings in the previous proposition, we now can compute the pay-for-performance sensitivities: $\frac{\partial C_T^i}{\partial S_T^i}, \frac{\partial C_T^i}{\partial S_T^{3-i}}$.

Lemma 7.3 *For the case of hidden action and in the presence of JE or AD, the compensation sensitivities to the performance are given by the following formulas:*

$$\begin{aligned}\frac{\partial C_T^1}{\partial S_T^1} = & \left(M_{11} - \frac{\rho}{\sqrt{1-\rho^2}}M_{12}\right)\frac{k_1(\delta_{11}+\delta_{21})\sigma_1\sigma_2^2\left(1-\rho^2\right)}{(1+\alpha_2)(r_1k_1^2\sigma_1^2\sigma_2^2(1-\rho^2)+\sigma_1^2\delta_{11}^2-2\rho\sigma_1\sigma_2\delta_{11}\delta_{21}+\sigma_2^2\delta_{21}^2)}\\ &+\left(M_{21} - \frac{\rho}{\sqrt{1-\rho^2}}M_{22}\right)\frac{\alpha_1k_2(\delta_{12}+\delta_{22})\sigma_1\sigma_2^2\left(1-\rho^2\right)}{(1+\alpha_1)(r_2k_2^2\sigma_1^2\sigma_2^2(1-\rho^2)+\sigma_1^2\delta_{22}^2-2\rho\sigma_1\sigma_2\delta_{12}\delta_{22}+\sigma_2^2\delta_{12}^2)}\\ \frac{\partial C_T^1}{\partial S_T^2} = & \frac{M_{12}k_1(\delta_{11}+\delta_{21})\sigma_1^2\sigma_2\left(1-\rho^2\right)}{(1+\alpha_2)\sqrt{1-\rho^2}(r_1k_1^2\sigma_1^2\sigma_2^2(1-\rho^2)+\sigma_1^2\delta_{11}^2-2\rho\sigma_1\sigma_2\delta_{11}\delta_{21}+\sigma_2^2\delta_{21}^2)}\\ &+\frac{\alpha_1M_{22}k_2(\delta_{12}+\delta_{22})\sigma_1^2\sigma_2\left(1-\rho^2\right)}{(1+\alpha_1)\sqrt{1-\rho^2}(r_2k_2^2\sigma_1^2\sigma_2^2(1-\rho^2)+\sigma_1^2\delta_{22}^2-2\rho\sigma_1\sigma_2\delta_{12}\delta_{22}+\sigma_2^2\delta_{12}^2)}\end{aligned}$$

$$\begin{aligned}\frac{\partial C_T^2}{\partial S_T^1} = & \frac{M_{21}k_2(\delta_{12}+\delta_{22})\sigma_1\sigma_2^2\left(1-\rho^2\right)}{(1+\alpha_1)\sqrt{1-\rho^2}(r_2k_2^2\sigma_1^2\sigma_2^2(1-\rho^2)+\sigma_1^2\delta_{22}^2-2\rho\sigma_1\sigma_2\delta_{12}\delta_{22}+\sigma_2^2\delta_{12}^2)}\\ &+\frac{\alpha_2M_{11}k_1(\delta_{11}+\delta_{21})\sigma_1\sigma_2^2\left(1-\rho^2\right)}{(1+\alpha_2)\sqrt{1-\rho^2}(r_1k_1^2\sigma_1^2\sigma_2^2(1-\rho^2)+\sigma_1^2\delta_{11}^2-2\rho\sigma_1\sigma_2\delta_{11}\delta_{21}+\sigma_2^2\delta_{21}^2)}\\ \frac{\partial C_T^2}{\partial S_T^2} = & \left(M_{22} - \frac{\rho}{\sqrt{1-\rho^2}}M_{21}\right)\frac{k_2(\delta_{22}+\delta_{12})\sigma_1^2\sigma_2\left(1-\rho^2\right)}{(1+\alpha_1)(r_2k_2^2\sigma_1^2\sigma_2^2(1-\rho^2)+\sigma_1^2\delta_{22}^2-2\rho\sigma_1\sigma_2\delta_{12}\delta_{22}+\sigma_2^2\delta_{12}^2)}\\ &+\left(M_{12} - \frac{\rho}{\sqrt{1-\rho^2}}M_{11}\right)\frac{\alpha_2k_1(\delta_{21}+\delta_{11})\sigma_1^2\sigma_2\left(1-\rho^2\right)}{(1+\alpha_2)(r_1k_1^2\sigma_1^2\sigma_2^2(1-\rho^2)+\sigma_1^2\delta_{11}^2-2\rho\sigma_1\sigma_2\delta_{11}\delta_{21}+\sigma_2^2\delta_{21}^2)}\end{aligned}$$

Proof. Follows directly from the formulas in the above proposition. ∎

Next we will find the impact of the envy and jealousy on the pay-for-performance sensitivities (PPS from now on).

Corollary 7.5 *(i) If* $\frac{\delta_{11}}{\sigma_1} > \rho\frac{\delta_{21}}{\sigma_2}$ *then* $\frac{\partial C_T^1}{\partial S_T^1}(\alpha_1, \alpha_2)$ *is decreasing in* α_2*, and, if* $\frac{\delta_{11}}{\sigma_1} < \rho\frac{\delta_{21}}{\sigma_2}$ *then* $\frac{\partial C_T^1}{\partial S_T^1}(\alpha_1, \alpha_2)$ *is increasing in* α_2*.*

(ii)If $\frac{\delta_{12}}{\sigma_1} > \rho\frac{\delta_{22}}{\sigma_2}$ *then* $\frac{\partial C_T^1}{\partial S_T^1}(\alpha_1, \alpha_2)$ *is increasing in* α_1*, and, if* $\frac{\delta_{12}}{\sigma_1} < \rho\frac{\delta_{22}}{\sigma_2}$ *then* $\frac{\partial C_T^1}{\partial S_T^1}(\alpha_1, \alpha_2)$ *is decreasing in* α_1*.*

(iii)$\frac{\partial C_T^1}{\partial S_T^2}(\alpha_1, \alpha_2)$ *is decreasing in* α_2 *and is increasing in* α_1*.*

Proof. Follows directly from the formulas in the above lemma. ■

From the last corollary it is easy to infer some results on the pay-for performance sensitivities:

The PPS of a worker is negatively associated with the level of envy or jealousy of his coworker if: (a) the productivity (δ_{11}) of the worker is high, or (b) the uncertainty (σ_1) of the project assigned to the worker is low, or (c) the correlation (ρ) between the projects is low, or (d) the contribution (δ_{21})of the worker to the project of the coworker is low, r (e) the uncertainty (σ_2) of the project assigned to the coworker is high.

On the other hand, the PPS of a worker is positively correlated with the level of envy or jealousy of his coworker in the opposite of the (a)-(e) situations.

Also, the pay-to-a-worker-for-the-performance-of-the-other-coworker $\frac{\partial C_T^1}{\partial S_T^2}(\alpha_1, \alpha_2)$ is always increasing in one's own level of envy and is always decreasing in the level of jealousy of the other worker.

Thus, PPS is related to productivity of the workers, the uncertainty of the outcomes of their projects, and the correlation between the outcomes of the projects and the level of envy or jealousy of workers. Also, for an envious worker, the pay must be sensitive to the performance of the coworker; however, if the coworker is not envious, then the sensitivity is lower.

The results here are consistent with the suggestions of Tirole (2001) that in the presence of agency problems (induced by JE), the optimal compensation should exhibit high pay-for-performance sensitivity.

Now, combining all the results, we can state the following result:[25]

Corollary 7.6 *(i) In the first-best case, the level of the optimal effort is less than the one we would get in the absence of envy or jealousy, if the other coworker is envious. That is, it is costly for workers to have envious coworkers: for* $0 < \alpha_{3-i} < 1$

$$u^{i,FB}(\alpha_1, \alpha_2) = \frac{(\delta_{1i} + \delta_{2i})(1 - \alpha_1\alpha_2)}{k_i(1 + \alpha_{3-i})} < \frac{(\delta_{1i} + \delta_{2i})}{k_i} := \hat{u}^i$$

(ii) In the second-best case with hidden actions, the level of the optimal effort is less than the one we would get in the absence of envy or jealousy, if the other coworker is envious. That is, workers would simply exert less effort knowing that their coworkers are envious/jealous of their pay: for $0 < \alpha_{3-i} < 1$

$$\bar{u}^{i,SB}(\alpha_1, \alpha_2) = \bar{u}^{i,SB}(0,0)\frac{1 - \alpha_1\alpha_2}{1 + \alpha_{3-i}} < \bar{u}^{i,SB}(0,0)$$

[25] we only state the results for the case of envy and jealousy, not for the case of admiration.

(iii)In the first-best case, if both agents exhibit envy or jealousy, that is, $0 < \alpha_1, \alpha_2 < 1$, *then the principal is worse off by having envious workers:*

$$V_P^{FB}(\alpha_1, \alpha_2) < V_P^{FB}(0, 0)$$

(iv) In the second-best case with hidden actions, if both agents exhibit envy/jealousy, that is $0 < \alpha_i < 1$, *then*

$$V_P^{SB}(\alpha_1, \alpha_2) < V_P^{SB}(0, 0)$$

That is, the principal is worse off if the workers are envious of the others.

(v) The optimal effort levels are decreasing in the level of envy/jealousy: for $0 < \alpha_1, \alpha_2 < 1$

$$\frac{\partial \bar{u}^1}{\partial \alpha_1} = -\frac{\alpha_2}{(1+\alpha_2)} \frac{(M_{11}^2 + M_{12}^2)(\delta_{11} + \delta_{21})}{(r_1 k_1^2 + M_{11}^2 + M_{12}^2)} < 0$$

and

$$\frac{\partial \bar{u}^1}{\partial \alpha_2} = -\frac{1+\alpha_1}{(1+\alpha_2)^2} \frac{(M_{11}^2 + M_{12}^2)(\delta_{11} + \delta_{21})}{(1+\alpha_2)^2 (r_1 k_1^2 + M_{11}^2 + M_{12}^2)} < 0$$

(vi) The sensitivity of the pay for a worker to his own performance is increasing in the envy/jealousy level of the other coworker for a less productive worker or if the outcome of the assigned project is very uncertain. That is, if $\frac{\delta_{11}}{\sigma_1} < \rho \frac{\delta_{21}}{\sigma_2}$ *then* $\frac{\partial C_T^1}{\partial S_T^1}(\alpha_1, \alpha_2)$ *is increasing in* α_2.

(vii) If the contribution of a worker (say worker1) to his own project (which is $\frac{\delta_{11}}{\sigma_1}$*) is less than his contribution to the other project (which is* $\frac{\delta_{21}}{\sigma_2}$*), then his compensation is more sensitive to his own performance with an increasing level of coworker envy. That is, if* $\frac{\delta_{11}}{\sigma_1} < \rho \frac{\delta_{21}}{\sigma_2}$*, then* $\frac{\partial C_T^1}{\partial S_T^1}(\alpha_1, \alpha_2)$ *is increasing in* α_2.

Also, a worker's (say worker 1) compensation is sensitive to his own performance if the coworker's contribution to worker 1's project is more than the one for worker 2's project. That is, if $\frac{\delta_{12}}{\sigma_1} > \rho \frac{\delta_{22}}{\sigma_2}$*, then* $\frac{\partial C_T^1}{\partial S_T^1}(\alpha_1, \alpha_2)$ *is increasing in* α_1.

Proof. Follows directly from the results obtained earlier in the paper. ■

The last corollary has certain empirical implications:

(1)The value of a group/organization where JE is present (small in size or compensation information is revealed) is less than the one of an otherwise similar group, in which there is no JE.

(2) The pay of a jealous worker (for example, workers in small groups or in groups in which the compensation info is publicly known) to own performance is higher in case if the variability of own project is lower relative to the variability of others' projects.

(3) The pay of a jealous worker to own performance is higher if the own project is lower or no-relationship with other's project.

(4) The pay of a worker (not necessary with JE, but in a group where coworkers exhibit JE) is increasing in the JE of other coworkers, if the variability of own project is higher relative to the variability of others' projects. That is, in an enviroment where JE exists,the pay of the workers responsible for uncertain projects is increasing in own performance.

(5) The pay-for performance in divisions of a large diversified firm would be lower than the one in otherwise stand alone firms, as they are less volatile. If the divisions are related and have joint projects then the PPS would be lower, however, if the groups are independent, then the PPS would be higher. This may be related to the literature on the undervaluation of diversified firms, compared to otherwise similar stand alone ones.[26]

Some other implications can be drawn from the last corollary, as it offers some results, that relate the value of the firm, the optimal effort, and the pay-for performance ratios to the uncertainty of and the correlation between projects, and the productivity of workers.

8 Conclusion

This paper extends the standard incentive contracting problem in continuous time, by modeling the interaction of multiple agents that exhibit envy or jealousy towards the compensation of their coworkers. Rather than assuming workers act only out of concern for their own self interest, there is now a sizeable literature that extends economic theory to recognize that people may also be motivated by relative income concerns. We model here the joint production of workers; i.e., each worker's effort has impact not only on the outcome of his own project, but also on the outcome of the coworker's project. Many papers show that individuals' self-reported happiness is negatively affected by the earnings of others in their area. In modeling utility functions, we take into account this psychological externality, i.e., people having utility functions that depend on relative consumption in addition to absolute consumption. In this paper, we use compensation as a proxy for consumption and derive the optimal contracts in explicit forms in many situations. We study the first-best case (symmetric and complete information) and the second-best case with hidden actions, assuming a principal is contracting multiple risk-averse agents who exhibit envy/jealousy or admiration towards their coworkers' pay, using the continuous-time stochastic modeling approach.

[26]We did an analysis of variation of CEO pay for 5522 firms in Compustat for the period 1992-2001. Our regression results show that the variation of CEO pay (The standard deviation as a percantage of mean pay) is decreasing in the number of divisions of firms. We find that Variability in CEO Pay = -0.0459* (Number of segments)+0.678, with $R^2 = 76\%$. That is, in large firms (possibly with lower JE) the PPS is lower than in smaller ones. This issue should be studied further in firms where there are many business segments, especially the breakdown between related-segments vs. unrelated-segments in the firm. Our conjecture is that in large diversified firms with unrelated-segments, the PPS would be lower that in otherwise similar firms, in which business operations in different segments are more correlated.

Our model provides us with a rich framework to analyze the interplay between workers and the relationship with their optimal efforts,the uncertainties of their projects, their skills, and their personalities. We apply the methods of backward stochastic differential equations to obtain the results. Having the incentive pay, optimal efforts, and the value function in closed forms allows us to compute and analyze the sensitivities of the optimal compensation, effort, and total utility of the principal with respect to some parameters of interest, such as the measure of envy/jealousy, the productivity of the workers, the contribution of the worker to the outcome of his own and of the coworker's project, the uncertainty of the outcomes of the projects, and the correlation between the projects.

Our findings suggest that incorporating psychological externalities (envy, jealousy, or admiration) to Contract Theory can be useful to study the interplay between workers in organizations. We show that the design of the optimal contract should reflect the relative concerns of workers.

We show that in the first-best case (symmetric and complete information), the level of the optimal effort is less than the one we would get in the absence of any envy or jealousy if the other coworker is envious. In this case we show that the principal is worse off by having envious workers.

In the second-best case with hidden actions, the level of the optimal effort is less than the one we would get in the absence of envy or jealousy if the other coworker is envious. That is, workers would simply exert less effort knowing that their coworkers are envious/jealous of their pay. In this case, we show that the principal is worse off if the workers are envious of the others. We show that the sensitivity of the pay for a worker to his own performance is increasing in the envy/jealousy level of the other coworker for a less productive worker or if the outcome of the assigned project is very uncertain. If the contribution of a worker to his own project is less than his contribution to the other project, then his compensation is more sensitive to his own performance with increasing level of coworker envy.

Since we are modeling a risk-neutral principal here, then the findings suggest that the value of the firm is reduced by having envy or jealousy in the workplace. We also show that the optimal effort levels are decreasing in the level of envy/jealousy. That is, envious workers exert less effort. Thus, envy/jealousy is destructive to the organizations. Since envy or jealousy is destructive, there is a need for ways to eliminate it. One way of reducing it is not disclosing the compensation details (see Bebchuk and Fried, 2003 on the subject).[27]

Our study and results offer the opportunity for further research in Behavioral Contract Theory and in Asset Pricing Theory, when agents' utility functions depend not only on their consumption levels but also on those of the "others". The model we study in this paper

[27] Jorgensen and Herby (2004) find that higher comparison income is related with lower and that this is significant for people who socialize more frequently. We do not suggest that not socializing would be a solution to the envious behavior towards the pay of the other coworkers.

can be generalized into one with more general recursive utility functions. The latter may be particulary useful in modeling utility functions in asset pricing. We also derive expressions for pay-for performance sensitivities in a rich environment that can be empirically tested for stand-alone and large diversified firms. We leave these for future research.

9 Appendix

9.1 Proof of Theorem 4.1

The problem is equivalent to the following:

$$\max_{u_t^1, u_t^2, C_T^1, C_T^2} E^{u^1,u^2} V(u, C, \lambda)$$

where

$$\begin{aligned}
V(u,C,\lambda) \quad &= \left[S_t^1 + S_t^2 - \frac{(1+\alpha_2)G_T^1 + (1+\alpha_1)G_T^2}{1-\alpha_1\alpha_2} - (C_T^1 - \frac{G_T^1 + \alpha_1 G_T^2}{1-\alpha_1\alpha_2}) - (C_T^2 - \frac{G_T^2 + \alpha_2 G_T^1}{1-\alpha_1\alpha_2}) \right] \\
&- \left[\frac{\lambda_1}{r^1} \exp\left\{ -r_1 \left((C_T^1 - \frac{G_T^1 + \alpha_1 G_T^2}{1-\alpha_1\alpha_2}) - \alpha_1 (C_T^2 - \frac{G_T^2 + \alpha^2 G_T^1}{1-\alpha_1\alpha_2}) \right) \right\} \right] \\
&- \left[\frac{\lambda_2}{r^2} \exp\left\{ -r_2 \left((C_T^2 - \frac{G_T^2 + \alpha_2 G_T^1}{1-\alpha_1\alpha_2}) - \alpha_2 (C_T^1 - \frac{G_T^1 + \alpha_1 G_T^2}{1-\alpha_1\alpha_2}) \right) \right) \right]
\end{aligned}$$

where (λ_1, λ_2) are lagrange multipliers.

Denote $X_1 = C_T^1 - \frac{G_T^1 + \alpha_1 G_T^2}{1-\alpha_1\alpha_2}$, $X_2 = C_T^2 - \frac{G_T^2 + \alpha_2 G_T^1}{1-\alpha_1\alpha_2}$. Then the problem is to solve

$$\begin{aligned}
\max_{X_1, X_2, u} E^{u_1,u_2} \quad & \left\{ (S_T^1 + S_T^2 - \frac{(1+\alpha_2)G_T^1 + (1+\alpha_1)G_T^2}{1-\alpha_1\alpha_2} - X_1 - X_2) \right. \\
& \left. - \frac{\lambda_1}{r_1} exp(-r_1(X_1 - \alpha_1 X_2)) - \frac{\lambda_2}{r_2} exp(-r_2(X_2 - \alpha_2 X_1)) \right\}
\end{aligned}$$

This, in turn, is equivalent to solving

$$\begin{aligned}
& \max_{u^1,u^2} E^{u^1,u^2} \left\{ (S_T^1 + S_T^2 - \frac{(1+\alpha_2)G_T^1 + (1+\alpha_1)G_T^2}{1-\alpha_1\alpha_2} \right\} \\
& + \max_{X_1,X_2} E^{u^1,u^2} \left\{ (-X_1 - X_2) - \frac{\lambda_1}{r_1} exp(-r_1(X_1 - \alpha_1 X_2)) - \frac{\lambda_2}{r_2} exp(-r_2(X_2 - \alpha_2 X_1)) \right\}
\end{aligned}$$

The rest is straightforward, by noticing that the objective function here is jointly concave in X_1, X_2.

9.2 Theorem on NEE

Theorem 9.3 *Given the efforts and contracts of agents $(u^1, u^2, C_T^1, C_T^2) \in \mathcal{A}$, there exist unique processes $(Y_t^i, Z_t^i) \in \mathbf{S}^2 \times \mathbf{H}^2 : (t,\omega) \to \mathbf{R} \times \mathbf{R^2}$, $i = 1, 2$, which are the solutions of the following backward stochatic differential equations(BSDE):*

$$Y_t^{i,u^1,u^2} = C_T^i - \alpha_i C_T^{3-i} + \int_t^T \left(H_i\left(Z_s^i, u_s^1, u_s^2\right) - \frac{1}{2r_i} \left\| Z_s^{i,u^1,u^2} \right\|^2 \right) ds - \frac{1}{r_i} \int_t^T Z_s^{i,u^1,u^2} dB_s \quad (9.1)$$

where

$$H_i\left(Z_s^i, u_s^1, u_s^2\right) \quad = \tfrac{i}{r_i} Z_s^{i,u^1,u^2}\Sigma^{-1}Mu_s - \tfrac{k_i}{2}\left|u_s^i\right|^2$$

and $Z^{i,u^i,u^j} = [\, Z^{i,1,u^i,u^j}, \quad Z^{i,2,u^i,u^j}\,]$.

In addition, the value process of each agent i

$$V_t^i = E^{u^1,u^2}\left\{-\frac{1}{r_i}\exp\left\{-r_i\left(C_T^i - \int_t^T \frac{k_i}{2}\left(u_s^i\right)ds - \alpha_i C_T^{3-i}\right)\right\}|\mathcal{F}_t\right\}$$

is given by the following explicit expression, in terms of Y_t^{i,u^1,u^2} *and effort* $\{u_t^i\}_{t\geq 0}^T$ *:*

$$V_t^i = -\frac{1}{r_i}\exp\left\{-r_i\left\{Y_t^i - \int_0^t \frac{k_i}{2}\left|u_s^i\right|^2 ds\right\}\right\}$$

First consider the existence and uniqueness of the following BSDE

$$Y_t^{i,u^1,u^2} \quad = C_T^i - \alpha_i C_T^{3-i} - \int_t^T\left(\tfrac{1}{2}k_i\left|u_s^i\right|^2 + \tfrac{1}{2r_i}\left\|Z_s^{i,u^1,u^2}\right\|^2\right)ds - \tfrac{1}{r_i}\int_t^T Z_s^{i,u^1,u^2}dB_s^{u^1,u^2}$$

It is a quadratic BSDE. In general, if the terminal condition $C_T^i - \alpha_i C_T^{3-i}$ is bounded, there exists a unique solution for this BSDE in $\mathbf{S}^2(\mathbf{R}) \times \mathbf{H}^2(\mathbf{R}^2)$ sense, which is not applicable in our case. Notice that

$$\begin{aligned} -r_i\left(Y_t^{i,u^1,u^2} - \int_0^t \frac{1}{2}k_i\left|u_s^i\right|^2 ds\right) \quad &= -r_i\left(C_T^i - \alpha_i C_T^{3-i} - \int_0^T \frac{1}{2}k_i\left|u_s^i\right|^2 ds\right) \\ &+ \int_t^T \frac{1}{2}\left\|Z_s^{i,u^1,u^2}\right\|^2 ds + \int_t^T Z_s^{i,u^1,u^2}dB_s^{u^1,u^2} \end{aligned}$$

To show the existence and uniqueness of the solutions, we need the following lemma:

Lemma 9.4 *Assume* $\xi \in \mathbf{L}^2$ *and consider a BSDE given by*

$$Y_t = \xi + \int_t^T \frac{\gamma}{2}|Z_s|^2 ds - \int_t^T Z_s dB_s$$

Then, there exists a solution $(Y, Z) \in \mathbf{H}^2(\mathbf{R}^{d+1})$*, if and only if*

$$E(exp(\gamma\xi)) < \infty$$

Proof. Let (Y, Z) be the solution of the problem. By Ito's formula, we have

$$exp(\gamma Y_t) = exp(\gamma Y_0) + \gamma\int_0^t exp(\gamma Y_s)Z_s dB_s$$

Where $X_t = \gamma\int_0^t exp(\gamma Y_s)Z_s dB_s$ is a local martingale.

Consider $\tau_n = \inf\{t > 0 : Y_t \geq n\} \wedge T$. So $\tau_n \to T$ as $n \to \infty$. Then $X_{t\wedge\tau_n}$ is a bounded martingale and by Fatou's lemma, we have

$$E(liminf_{n\to\infty} exp(\gamma Y_{\tau_n})) = E(exp(\gamma\xi)) \leq E(exp(\gamma Y_0)) < \infty$$

Next, we show the sufficiency. We have $E(exp(\gamma\xi)) < \infty$. Define

$$X_t =: E(exp(\gamma\xi)|\mathcal{F}_t) = X_0 + \int_0^t Z_s \cdot dB_s$$

Applying Ito's formula, we have

$$\frac{1}{\gamma} logX_t = \frac{1}{\gamma} logX_0 - \frac{1}{2\gamma}\int_t^T |\frac{Z_s}{X_s}|^2 ds + \frac{1}{\gamma}\int_t^T \frac{Z_s}{M_s} dB_s; \quad \frac{1}{\gamma} logX_T = \xi$$

Uniqueness of $(\frac{1}{\gamma}logX_t, \frac{1}{\gamma}\frac{Z_t}{X_t})$ follows from the unique martingale representation of X_t. So to prove that $(\frac{1}{\gamma}logX_t, \frac{1}{\gamma}\frac{Z_t}{X_t})$ is the solution of the BSDE, it is enough to show that :

$$\frac{Z_t}{M_t} \in H^2(\mathbf{R}^d), \quad \frac{1}{\gamma} logX_t \in S^2$$

Since the exponential function is concave, we have

$$\frac{1}{\gamma} logX_t = \frac{1}{\gamma} logE(exp(\gamma\xi)|\mathcal{F}_t) \geq E(\xi|\mathcal{F}_t) \geq -E(\xi^-|\mathcal{F}_t) =: N_t$$

For $m > 0$, define the following stopping time

$$T_m = inf\left\{t > 0 : |N_t| > m, \int_0^t |\frac{Z_s}{X_s}|^2 ds > m, |\int_0^t Z_s \cdot dB_s| > a\right\} \wedge T$$

Since

$$\begin{aligned} \int_0^{T_m} |\frac{Z_s}{X_s}|^2 ds &= 2logX_0 - 2logX_{T_m} + 2\int_t^{T_m} \frac{Z_s}{M_s} dB_s \\ &\leq 2logX_0 - 2\gamma N_{T_m} + 2\int_t^{T_m} \frac{Z_s}{M_s} dB_s \end{aligned}$$

then

$$\begin{aligned} (\int_0^{T_m} |\frac{Z_s}{X_s}|^2 ds)^2 &\leq (2logX_0 - 2\gamma N_{T_m} + 2\int_t^{T_m} \frac{Z_s}{M_s} dB_s)^2 \\ &\leq 12(logX_0)^2 + 12\gamma^2 N_{T_m}^2 + 12(\int_0^{T_m} \frac{Z_s}{M_s} dB_s)^2 \end{aligned}$$

Also, noticing that N_t^2 is submartingale, we have $EN_{T_m}^2 \leq E(\xi^-)^2$. Then

$$E\left(\int_0^{T_m} |\frac{Z_s}{X_s}|^2 ds\right)^2 \leq 12(logX_0)^2 + 12\gamma^2 E(\xi^-)^2 + 12E(\int_0^{T_m} |\frac{Z_s}{M_s}|^2 ds)$$

We also have

$$12(\int_0^{T_m} |\frac{Z_s}{M_s}|^2 ds) \leq \frac{1}{2}\left((\int_0^{T_m} |\frac{Z_s}{M_s}|^2 ds) \right)^2 + 72$$

so

$$E\left(\int_0^{T_m} |\frac{Z_s}{X_s}|^2 ds \right)^2 \leq 24(logX_0)^2 + 24\gamma^2 E(\xi^-)^2 + 72) = C_1; \qquad C_1 < \infty$$

Finally, we have

$$E\left(\int_0^{T_m} |\frac{Z_s}{X_s}|^2 ds \right) \leq \sqrt{E\left(\int_0^{T_m} |\frac{Z_s}{X_s}|^2 ds \right)^2} < \sqrt{C_1} < \infty$$

Since $T_m \to T$ as $m \to \infty$, by monotone convergence theorem, we have

$$E\left(\int_0^{T} |\frac{Z_s}{X_s}|^2 ds \right) \leq \sqrt{C_1} < \infty$$

That is $\frac{Z}{X} \in H^2(\mathbf{R}^d)$

Also, we have

$$(\frac{1}{\gamma}logX_t)^2 = 2\left[\left(\frac{1}{\gamma}logX_0 \right)^2 + \left(\frac{1}{2\gamma}\int_t^T |\frac{Z_s}{X_s}|^2 ds \right)^2 + \left(\frac{1}{\gamma}\int_t^T \frac{Z_s}{M_s} dB_s \right)^2 \right]$$

By Burkholder-Davis-Gundy inequality, we have

$$E\left[\sup_{0\leq t\leq T} \left(\frac{1}{\gamma}logX_t \right)^2 \right] < \infty$$

So there exists a solution $(Y, Z) \in \mathbf{S}^2(\mathbf{R}) \times \mathbf{H}^2(\mathbf{R}^d)$ of the problem

$$Y_t = \xi + \int_t^T \frac{\gamma}{2}|Z_s|^2 ds - \int_t^T Z_s dB_s$$

■

Remark 9.6 *The previous lemma is a generalization of Theorem 3.1 in Briand, Lepeltier and Martin(2005).*

Remark 9.7 *The contracts C_T^i, in general, are bounded by some constant, so the usual conditions for uniqueness and existence of solutions of BSDEs with quadratic growth don't apply in our problem.*

Our BSDE is

$$-r_i\left(Y_t^{i,u^1,u^2}-\int_0^t \frac{1}{2}k_i\left|u_s^i\right|^2 ds\right) \qquad = -r_i\left(C_T^i-\alpha_i C_T^{3-i}-\int_0^T \frac{1}{2}k_i\left|u_s^i\right|^2 ds\right)$$
$$+\int_t^T \frac{1}{2}\left\|Z_s^{i,u^1,u^2}\right\|^2 ds+\int_t^T Z_s^{i,u^1,u^2}dB_s^{u^1,u^2}$$

By conditions of set $\mathcal{A}$, we have

$$E^u\left(C_T^i-\alpha_i C_T^{3-i}-\int_0^T \frac{1}{2}k_i\left|u_s^i\right|^2 ds\right)^2<\infty$$

and

$$E^u\left\{exp\left[-r_i\left(C_T^i-\alpha_i C_T^{3-i}-\int_0^T \frac{1}{2}k_i\left|u_s^i\right|^2 ds\right)\right]\right\}<\infty$$

So it follows from the lemma that there exists a unique (Y_t^i, Z_t^i), such that

$$E^u\int_0^T |Z_s^i|^2 ds<\infty$$

$$E^u\left\{\sup_{0\le t\le T}\left(Y_t^{i,u^1,u^2}-\int_0^t \frac{1}{2}k_i\left|u_s^i\right|^2 ds\right)^2\right\}<\infty$$

Since

$$\left(Y_t^{i,u^1,u^2}\right)^2\le 2\left[\left(Y_t^{i,u^1,u^2}-\int_0^t \frac{1}{2}k_i\left|u_s^i\right|^2 ds\right)^2+\left(\int_0^t \frac{1}{2}k_i\left|u_s^i\right|^2 ds\right)^2\right]$$

we also have

$$E^u\left\{\sup_{0\le t\le T}\left(Y_t^{i,u^1,u^2}\right)^2\right\}<\infty$$

Define

$$P_t^i=-\frac{1}{r_i}\exp\left\{-r_i\left(Y_t^{i,u^1,u^2}-\int_0^t \frac{1}{2}k_i\left|u_s^i\right|^2 ds\right)\right\}$$
$$Q_t^i=-P_t^i Z_t^{i,u^1,u^2}$$

leading to this equation:

$$P_t^i=-\frac{1}{r_i}exp\left(-r_i\left(C_T^i-\alpha_i C_T^{3-i}-\int_0^T \frac{1}{2}k_i\left|u_s^i\right|^2 ds\right)\right)-\int_t^T Q_s^i dB_s^{u^1,u^2}$$

So,

$$Y_t^i=-\frac{1}{r_i}ln(-r_iP_t^i)=-\frac{1}{r_i}ln\left[E^{u^1,u^2}\left[-\frac{1}{r_i}\exp\left\{-r_i\left(C_T^i-\int_t^T \frac{1}{2k_i}|u_s^i|^2 ds-\alpha_1 C_T^2|\mathcal{F}_t\right)\right\}\right]\right]$$

This proves the theorem.

Remark 9.8 *BSDEs with quadratic growth and unbounded terminal condition was studied in detail by P. Briand and Y. Hu (2005).*

Remark 9.9 *We notice that the value process Y_t^i is represented by a backward stochastic differential equation with respect to a random process B_t, which is Brownian motion under measure Q^0. Considering that we are going to use this representation in the principal's problem, and that the principal's objective expected utility is measured under probability Q^u, we need to represent the value process with respect to B_t^u for later use, which is the Brownian motion under measure Q^u. Following Theorem 9.3,*

$$
\begin{aligned}
Y_t^{1,u^1,u^2} &= C_T^1 - \alpha_1 C_T^2 - \int_t^T \left(\frac{k_1}{2}|u_s^1|^2 + \frac{1}{2r_1} \left\| Z_s^{1,u^1,u^2} \right\|^2 \right) ds - \frac{1}{r_1} \int_t^T Z_s^{1,u^1,u^2} dB_s^u \\
Y_t^{2,u^1,u^2} &= C_T^2 - \alpha_2 C_T^1 - \int_t^T \left(\frac{k_2}{2}|u_s^2|^2 + \frac{1}{2r_2} \left\| Z_s^{2,u^1,u^2} \right\|^2 \right) ds - \frac{1}{r_2} \int_t^T Z_s^{2,u^1,u^2} dB_s^u
\end{aligned}
$$

Remark 9.10 *To simplify the notations, we have the following*

$$
\Sigma^{-1} M = \begin{bmatrix} \frac{\delta_{11}}{\sigma_1} & \frac{\delta_{12}}{\sigma_1} \\ -\frac{\rho\delta_{11}}{\sigma_1\sqrt{1-\rho^2}} + \frac{\delta_{21}}{\sigma_2\sqrt{1-\rho^2}} & -\frac{\rho\delta_{12}}{\sigma_1\sqrt{1-\rho^2}} + \frac{\delta_{22}}{\sigma_2\sqrt{1-\rho^2}} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{21} \\ M_{12} & M_{22} \end{bmatrix}
$$

So

$$
\begin{aligned}
H_1\left(Z_s^1, u_s^1, u_s^2\right) &= \frac{1}{r_1}\left[(M_{11}Z_s^{1,1} + M_{12}Z_s^{1,2})u_s^1 + (M_{21}Z_s^{1,1} + M_{22}Z_s^{1,2})u_s^2\right] - \frac{k_1}{2}|u_s^1|^2 \\
H_2\left(Z_s^2, u_s^1, u_s^2\right) &= \frac{1}{r_2}\left[(M_{11}Z_s^{2,1} + M_{12}Z_s^{2,2})u_s^1 + (M_{21}Z_s^{2,1} + M_{22}Z_s^{2,2})u_s^2\right] - \frac{k_2}{2}|u_s^2|^2
\end{aligned}
$$

Now, finding Nash-Equilibrium-Efforts is equivalent to finding $(\overline{u}^1, \overline{u}^2)$, such that $\overline{u}^1 \in \arg\max_{u^1} Y_0^{1,u^1,\overline{u}^2}$ and $\overline{u}^2 \in \arg\max_{u^2} Y_0^{2,\overline{u}^1,u^2}$. Applying the comparison theorem from the theory of BSDE[28], we have the following result:

Proposition 9.3 *There exists at most one solution, $(Y_t^1, Y_t^2, Z_t^1 Z_t^2) \in \mathbf{S}^2 \times \mathbf{S}^2 \times \mathbf{H}^2 \times \mathbf{H}^2$ for the multidimensional BSDEs (9.1)*

such that

$$
Eexp\{\int_0^T |\Sigma^{-1}M[f_1(Z_s^1), f_2(Z_s^2)]'|^2 ds\} < \infty \tag{9.2}
$$

where

[28]Given backward SDE $Y_t = \zeta + \int_t^T f(s, Y_s, Z_s, u_s)ds - \int_t^T Z_s dB_s$, under certain technical conditions, the comparison theorem gives the optimal solutions for $\max_u Y_t^u$, which is equivalent to $\max_u f(t, Y_t, Z_t, u_t)$. So, there is a certain function $h(.)$, such that $\bar{u}_t = h(t, Y_t, Z_t)$, and the optimal process Y is given by BSDE $Y_t = \zeta + \int_t^T f(s, Y_s, Z_s, h(t, Y_t, Z_t))ds - \int_t^T Z_s dB_s$.

$$f_1(Z_t^1) = \frac{1}{k_1 r_1}(M_{11}Z_t^{1,1} + M_{12}Z_t^{1,2}), f_2(Z_t^2) = \frac{1}{r_2 k_2}(M_{21}Z_t^{2,1} + M_{22}Z_t^{2,2}).$$

If the solution exists, the equilibrium efforts for the agents $(\overline{u}^1, \overline{u}^2)$ *are given by*[29]

$$\bar{u}_t^1 = f_1(Z_t^1), \bar{u}_t^2 = f_2(Z_t^2)$$

Proof.

Assume there exist two solutions $(Y_t^1, Y_t^2, Z_t^1, Z_t^2)$ and $(\tilde{Y}_t^1, \tilde{Y}_t^2, \tilde{Z}_t^1, \tilde{Z}_t^2)$, such that

$$\begin{aligned} Y_t^1 &= C_T^1 - \alpha_1 C_T^2 + \int_t^T \left(H_1 \left(Z_s^1, f_1(Z_s^1), f_2(Z_s^2) \right) - \tfrac{1}{2r_1} \left\| Z_s^1 \right\|^2 \right) ds - \tfrac{1}{r_1} \int_t^T Z_s^1 dB_s \\ Y_t^2 &= C_T^2 - \alpha_2 C_T^1 + \int_t^T \left(H_2 \left(Z_s^2, f_1(Z_s^1), f_2(Z_s^2) \right) - \tfrac{1}{2r_2} \left\| Z_s^2 \right\|^2 \right) ds - \tfrac{1}{r_2} \int_t^T Z_s^2 dB_s \end{aligned}$$

$$\begin{aligned} \tilde{Y}_t^1 &= C_T^1 - \alpha_1 C_T^2 + \int_t^T \left(H_1 \left(\tilde{Z}_s^1, f_1(\tilde{Z}_s^1), f_2(\tilde{Z}_s^2) \right) - \tfrac{1}{2r_1} \left\| \tilde{Z}_s^1 \right\|^2 \right) ds - \tfrac{1}{r_1} \int_t^T \tilde{Z}_s^1 dB_s \\ \tilde{Y}_t^2 &= C_T^2 - \alpha_2 C_T^1 + \int_t^T \left(H_2 \left(\tilde{Z}_s^2, f_1(\tilde{Z}_s^1), f_2(\tilde{Z}_s^2) \right) - \tfrac{1}{2r_2} \left\| \tilde{Z}_s^2 \right\|^2 \right) ds - \tfrac{1}{r_2} \int_t^T \tilde{Z}_s^2 dB_s \end{aligned}$$

Define $\nabla Y_t^i = Y_t^i - \tilde{Y}_t^i, \nabla Z_t = Z_t^i - \hat{Z}_t^i$.

Then

$$\nabla Y_t^i = \int_t^T a(Z_t^i + \tilde{Z}_s^i)\nabla Z_s^i ds - \int_t^T \nabla Z_s^i dB_s^{f_1(Z^1), f^2(Z^2)}$$

for some constant a which depends on parameters $\delta_{i,j} r_i, \rho, \sigma_i$. Furthermore, we have

$$\nabla Y_t^i = \int_t^T \nabla Z_s^i dB_s^a$$

where $dB_t^a = dB_t - \Sigma^{-1}\{M[f_1(Z_t^1), f_2(Z_t^2)]' - a(Z_t^1 + Z_t^2)\}^2 dt = dB_t - \kappa_t dt$.

Since we assume $Eexp\{N \int_0^T (\Sigma^{-1} M[f^1(Z_s^1), f^2(Z_s^2)]')^2 ds < \infty\}$, we have $Eexp\{\int_0^T |\kappa_s|^2 ds\} < \infty$, then $K_t = exp\{-\frac{1}{2}\int_0^T |\kappa_s|^2 ds + \int_0^T \kappa_s dB_s\}$ is a martingale. Thus, $\nabla Y_t^i = \int_t^T \nabla Z_s^i dB_s^a$ has a unique solution $\nabla Y_t^i = \nabla Z_t^i = 0$.

If multidimensional BSDEs above have a solution, by comparison method, we know that solving $\max_{u^1} Y_0^{1,u^1,\overline{u}^2}$ is equivalent to solving

$$\max_{u_1} H_1 \left(Z^1, u^1, \overline{u}^2 \right),$$

and that

[29]From this proposition, equilibrium efforts are functions of the intensity process (Z^1, Z^2). So, instead of finding optimal effort levels (u^1, u^2), it suffices to discuss the optimality conditions of (Z^1, Z^2) and $u_t^i = f_i(Z_t^i)$. So we make the following changes in the notations: $E^u \to E^Z, Q^u \to Q^Z, B_t^u \to B_t^Z$

$$H_i\left(Z_s^i, u^i, \overline{u}^{3-i}\right) = \frac{1}{r_i} Z_s^{i,u^1,u^2} \Sigma^{-1} M u_s - \frac{k_i}{2}\left|u_s^i\right|^2$$

Since $H_i\left(Z_s^i, u^i, \overline{u}^{3-i}\right)$ is quadratic on u^i, the first order condition will yield the Nash Equilibrium-Efforts.

■

Remark 9.11 *The condition (9.2) is sufficient for K_t^u to be a martingale, given $u_t^i = f_i(Z_t^i)$. This assumption can further be relaxed.*

Remark 9.12 *Proposition 9.3 shows that the equilibrium efforts can be represented by intensity process (Z_t^1, Z_t^2) of the value process (Y_t^1, Y_t^2), and $(Y_t^1, Z_t^1), (Y_t^2, Z_t^2)$ are uniquely determined by $C_T^1 - \alpha_1 C_T^2$ and $C_T^2 - \alpha_2 C_T^1$. In other words, there exists a unique pair of equilibrium efforts for every pair of contracts (C_T^1, C_T^2).*

Remark 9.13 *Based on the relation between B_t and B_t^Z, the optimal value processes can be written as follows:*

$$\begin{aligned} Y_t^1 &= C_T^1 - \alpha_1 C_T^2 - \int_t^T \left(\tfrac{k_1}{2}|f_1(Z_s^1)|^2 + \tfrac{1}{2r_1}\left\|Z_s^1\right\|^2\right) ds - \tfrac{1}{r_1}\int_t^T Z_s^1 dB_s^Z \\ Y_t^2 &= C_T^2 - \alpha_2 C_T^1 - \int_t^T \left(\tfrac{k_2}{2}|f_2(Z_s^2)|^2 + \tfrac{1}{2r_2}\left\|Z_s^2\right\|^2\right) ds - \tfrac{1}{r_2}\int_t^T Z_s^2 dB_s^Z \end{aligned}$$

9.3 Proof of Theorem 6.2

Proof.

Given $f_1(Z^1), f_2(Z^2)$, we can represent (Y_0^1, Y_0^2) forwardly, that is

$$\begin{aligned} C_T^1 - \alpha_1 C_T^2 &= Y_0^1 - \int_t^T \left(H_1\left(Z_s^1, f_1(Z_s^1), f_2(Z_s^2)\right) + \tfrac{1}{2r_1}\left\|Z_s^1\right\|^2\right) ds + \tfrac{1}{r_1}\int_t^T Z_s^1 dB_s \\ C_T^2 - \alpha_2 C_T^1 &= Y_0^2 - \int_t^T \left(H_2\left(Z_s^2, f_1(Z_s^1), f_2(Z_s^2)\right) + \tfrac{1}{2r_2}\left\|Z_s^2\right\|^2\right) ds + \tfrac{1}{r_2}\int_t^T Z_s^2 dB_s \end{aligned}$$

Denote $Y_0^i = \tilde{L}_i$, then we can solve for (C_T^1, C_T^2). Proposition 9.3 says there exists at most one solution $(\tilde{Y}_t^1, \tilde{Y}_t^2, \tilde{Z}_t^1, \tilde{Z}_t^2)$ for these equations:

$$\begin{aligned} \tilde{Y}_t^1 &= C_T^1 - \alpha_1 C_T^2 + \int_t^T \left(H_1\left(\tilde{Z}_s^1, f_1(\tilde{Z}_s^1), f_2(\tilde{Z}_s^2)\right) - \tfrac{1}{2r_1}\left\|\tilde{Z}_s^1\right\|^2\right) ds - \tfrac{1}{r_1}\int_t^T \tilde{Z}_s^1 dB_s \\ \tilde{Y}_t^2 &= C_T^2 - \alpha_2 C_T^1 + \int_t^T \left(H_2\left(\tilde{Z}_s^2, f_1(\tilde{Z}_s^1), f_2(\tilde{Z}_s^2)\right) - \tfrac{1}{2r_2}\left\|\tilde{Z}_s^2\right\|^2\right) ds - \tfrac{1}{r_2}\int_t^T \tilde{Z}_s^2 dB_s \end{aligned}$$

satisfying $Eexp\{N\int_0^T |\Sigma^{-1}M[f_1(\tilde{Z}_s^1), f_2(\tilde{Z}_s^2)]'|^2\} < \infty$. Thus, $(Y_t^1, Y_t^2, Z_t^1, Z_t^2)$ is the solution to the problem and the theorem is proved.

■

9.4 Proof of Proposition 7.1

Proof.

First, we consider one part of of $V(Z^1, Z^2)$:

$$\begin{gathered} K\left(Z^{1,1}, Z^{1,2}\right) \\ = \left(\delta_{11}+\delta_{21}\right) h_1\left(Z_s^1\right) - \tfrac{1+\alpha_2}{2(1-\alpha_1\alpha_2)}\left(k_1\left|h_1\left(Z_s^1\right)\right| + \tfrac{1}{r_1}\left\|Z_s^1\right\|^2\right) \\ = \tfrac{(\delta_{11}+\delta_{21})}{k_1 r_1}\left(Z_s^{1,1} M_{11} + Z_s^{1,2} M_{21}\right) \\ -\tfrac{(1+\alpha_2)}{2(1-\alpha_1\alpha_2) r_1}\left(2 Z_s^{1,1} Z_s^{1,2} M_{11} M_{21} + \left(1+M_{11}^2\right)\left|Z_s^{1,1}\right|^2 + \left(1+M_{21}^2\right)\left|Z_s^{1,2}\right|^2\right) \end{gathered}$$

The second order derivative of function $K\left(Z^{1,1}, Z^{1,2}\right)$

$$\begin{aligned} K_{11} & = -\tfrac{1}{1-\alpha_1\alpha_2} \tfrac{(1+\alpha_2)\left(1+M_{11}^2\right)}{r_1} \\ K_{22} & = -\tfrac{1}{1-\alpha_1\alpha_2} \tfrac{(1+\alpha_2)\left(1+M_{21}^2\right)}{r_1} \\ K_{12} & = K_{21} = -\tfrac{1}{1-\alpha_1\alpha_2}\left(\tfrac{(1+\alpha_2) M_{11} M_{21}}{r_1}\right) \end{aligned}$$

It is easy to compute the Hessian matrix of $K\left(Z^{1,1}, Z^{1,2}\right)$. That matrix is negatively definite, so the F.O.C. yields the global maximum. That is

$$\begin{aligned} \frac{\left(\delta_{11}+\delta_{21}\right)\left(1-\alpha_1\alpha_2\right) M_{11}}{k_1\left(1+\alpha_2\right)} & = \left(1+M_{11}^2\right) Z_s^{1,1} + M_{11} M_{21} Z_s^{1,2} \\ \frac{\left(\delta_{11}+\delta_{21}\right)\left(1-\alpha_1\alpha_2\right) M_{21}}{k_1\left(1+\alpha_2\right)} & = M_{11} M_{21} Z_s^{1,1} + \left(1+M_{21}^2\right) Z_s^{1,2} \end{aligned}$$

so

$$Z_s^{1,1} = \tfrac{(\delta_{11}+\delta_{21})(1-\alpha_1\alpha_2) M_{11}}{k_1(1+\alpha_2)\left(1+M_{11}^2+M_{21}^2\right)}, Z_s^{1,2} = \frac{\left(\delta_{11}+\delta_{21}\right)\left(1-\alpha_1\alpha_2\right) M_{21}}{k_1\left(1+\alpha_2\right)\left(1+M_{11}^2+M_{21}^2\right)}$$

Similarly,

$$Z_s^{2,1} = \tfrac{(\delta_{12}+\delta_{22})(1-\alpha_1\alpha_2) M_{12}}{k_2(1+\alpha_1)\left(1+M_{12}^2+M_{22}^2\right)}, Z_s^{2,2} = \frac{\left(\delta_{12}+\delta_{22}\right)\left(1-\alpha_1\alpha_2\right) M_{22}}{k_2\left(1+\alpha_1\right)\left(1+M_{12}^2+M_{22}^2\right)}$$

Now we can get the optimal efforts:

$$\begin{aligned} \overline{u}_t^1 & = h_1\left(Z^1\right) = \tfrac{(\delta_{11}+\delta_{21})(1-\alpha_1\alpha_2)}{r_1 k_1^2(1+\alpha_2)\left(1+M_{11}^2+M_{21}^2\right)}\left(M_{11}^2+M_{21}^2\right) \\ \overline{u}_t^2 & = h_2\left(Z^2\right) = \tfrac{(\delta_{12}+\delta_{22})(1-\alpha_1\alpha_2)}{r_2 k_2^2(1+\alpha_1)\left(1+M_{12}^2+M_{22}^2\right)}\left(M_{12}^2+M_{22}^2\right) \end{aligned}$$

■

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