Lesson Topics

Decision Formulation lists alternatives, uncertain states of nature (hot, cold, ...), and resulting consequences. Decision Formulation is especially important when a decision is unprecedented.

Decision Making without Probabilities assigned to the states of nature is possible for optimists (who assume the best happens) and for pessimists (who assume the worst happens).

Expected Value is the average consequence of a sequence of decisions, according to the central limit theorem. Hence, people facing repeated decisions should maximize expected value.

Backward Induction (4) finds your optimal sequence decisions by making your last decision first. — So, before your first cigarette, think about your last.

Decision Tree (1) Formulation pictures a decision with nodes and branches that lists alternatives, uncertain states of nature, and consequences. Decision Trees are useful for a sequence of decisions.
Backward Induction

Question. Use backward induction (and draw a decision tree, if needed) to help make the following sequential decisions, where later decisions depend on earlier decisions:

1. Should Dell Computer invest $3 million in research to create a faster external hard drive for computers?
   - The probability of a successful project is 0.8.
   - If the project were successful, it requires a new $10 million production facility to make the new products.
   - If the new products were made, demand and revenues are uncertain:
     - With probability 0.5, demand is high and revenue is $20 million.
     - With probability 0.3, demand is med. and revenue is $10 million.
     - With probability 0.2, demand is low and revenue is $5 million.

2. If the project were successful, should Dell Computer sell its rights in the project for $5 million?
C.3 Statistical Decision Making  

Review Questions

**Answer to Question:**

Step 1: Replace uncertain payoffs with expected value:

1. Should Dell Computer invest $3 million in research to create a faster external hard drive for computers?
   - The probability of a successful project is 0.8.
   - If the project were successful, it requires a new $10 million production facility to make the new products.
   - If the new products were made, demand and revenues are uncertain.
     - Expected revenue = 0.5(20) + 0.3(10) + 0.2(5) = $14 million
       - With probability 0.5, demand is high and revenue is $20 million.
       - With probability 0.3, demand is med. and revenue is $10 million.
       - With probability 0.2, demand is low and revenue is $5 million.

2. If the project were successful, should Dell Computer sell its rights in the project for $5 million?
Step 2: Make the second decision:

1. Should Dell Computer invest $3 million in research to create a faster external hard drive for computers?

   - The probability of a successful project is 0.8.
   - If the project were successful, it requires a new $10 million production facility to make the new products.
   - If the new products were made, demand and revenues are uncertain. Expected revenue = 0.5(20)+0.3(10)+0.2(5) = $14 million
     - With probability 0.5, demand is high and revenue is $20 million.
     - With probability 0.3, demand is med. and revenue is $10 million.
     - With probability 0.2, demand is low and revenue is $5 million.

2. If the project were successful, should Dell Computer sell its rights in the project for $5 million? Answer: Sell rights in the project and earn $(5-3) = $2 million profit, since keeping the rights is worth $(14-10-3) = $1 million profit.

Step 3: Replace uncertain payoffs with expected value, and make the first decision:

Should Dell Computer invest $3 million in research to create a faster external hard drive for computers? If the research is done, there is a 0.8 probability of earning $2 million profit, and a 0.2 probability of losing the $3 million investment. So expected value = 0.8(2) + 0.2(-3) = $1 million, which being positive indicates that, yes, Dell Computer should invest $3 million in research to create a faster external hard drive for computers and, if the project is successful, they should sell their rights for $5 million.
Backward Induction

Question. Use backward induction (and draw a decision tree, if needed) to help make the following sequential decisions, where later decisions depend on earlier decisions. The objective is to maximize expected profit:

Seneca Hill Winery recently purchased land for the purpose of establishing a new vineyard. Management is considering two varieties of white grapes for the new vineyard: Chardonnay and Riesling. The Chardonnay grapes would be used to produce a dry Chardonnay wine, and the Riesling grapes would be used to produce a semidry Riesling wine. It takes approximately four years from the time of planting before new grapes can be harvested. This length of time creates a great deal of uncertainty about future demand and makes the decision concerning the type of grapes to plant difficult. Three possibilities are being considered: Chardonnay grapes only; Riesling grapes only, and both Chardonnay and Riesling grapes.

Seneca management decided that for planning purposes it would be adequate to consider only two demand possibilities for each type of wine: Strong or weak. With two possibilities for each type of wine it was necessary to assess four probabilities. With the help of some forecasts in industry publications management made the following probability assessments:

<table>
<thead>
<tr>
<th>Demand</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chardonnay Demand Weak &amp; Riesling Demand Weak</td>
<td>0.05</td>
</tr>
<tr>
<td>Chardonnay Demand Weak &amp; Riesling Demand Strong</td>
<td>0.50</td>
</tr>
<tr>
<td>Chardonnay Demand Strong &amp; Riesling Demand Weak</td>
<td>0.25</td>
</tr>
<tr>
<td>Chardonnay Demand Strong &amp; Riesling Demand Strong</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Profit projections show the following.
1) If Seneca Hill only plants Chardonnay grapes, profit is $20,000 if demand is weak for Chardonnay wine, and $70,000 if demand is strong for Chardonnay wine.
2) If they only plant Riesling grapes, profit is $25,000 if demand is weak for Riesling grapes, and $45,000 if demand is strong for Riesling grapes.

3) If they plant both types of grapes, then profits are as follows:

<table>
<thead>
<tr>
<th>Demand</th>
<th>Total Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chardonnay Demand Weak &amp; Riesling Demand Weak</td>
<td>$22,000</td>
</tr>
<tr>
<td>Chardonnay Demand Weak &amp; Riesling Demand Strong</td>
<td>$40,000</td>
</tr>
<tr>
<td>Chardonnay Demand Strong &amp; Riesling Demand Weak</td>
<td>$26,000</td>
</tr>
<tr>
<td>Chardonnay Demand Strong &amp; Riesling Demand Strong</td>
<td>$60,000</td>
</tr>
</tbody>
</table>

Should Seneca Hill plant Chardonnay grapes only; Riesling grapes only, or both Chardonnay and Riesling grapes.
Answer to Question:
Step 1: Replace uncertain payoffs with expected value:
Profit projections show the following.

1) If Seneca Hill only plants Chardonnay grapes, profit is $20,000 if demand is weak for Chardonnay wine, and $70,000 if demand is strong for Chardonnay wine.
   Expected profit = (0.05+0.50)$20,000 + (0.25+0.20)$70,000 = $42,500
2) If they only plant Riesling grapes, profit is $25,000 if demand is weak for Riesling grapes, and $45,000 if demand is strong for Riesling grapes.
   Expected profit = (0.05+0.25)$25,000 + (0.50+0.20)$45,000 = $39,600
3) If they plant both types of grapes, then profits are as follows:
   Expected profit = 0.05$22,000 + 0.50$40,000 + 0.25$26,000 + 0.20$60,000 = $39,000

Step 2: Make the decision: Seneca Hill should only plant Chardonnay grapes, and expect profit $42,500.
C.3 Statistical Decision Making

**Backward Induction**

**Question.** Use backward induction (and draw a decision tree, if needed) to help make the following sequential decisions, where later decisions depend on earlier decisions. The objective is to maximize expected profit:

Activision is considering marketing one of two new video games for the coming holiday season: Call of Duty Black Ops, or Halo.

On the one hand, Call of Duty Black Ops is a unique game and appears to have no competition. Estimated profits if demand is Strong, or Medium, or Weak are as follows:

<table>
<thead>
<tr>
<th>Call of Duty Black Ops Demand</th>
<th>Strong</th>
<th>Medium</th>
<th>Weak</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit</td>
<td>$1000</td>
<td>$700</td>
<td>$300</td>
</tr>
<tr>
<td>Probability</td>
<td>0.2</td>
<td>0.5</td>
<td>0.3</td>
</tr>
</tbody>
</table>

On the other hand, the profitability of Halo could be affected by a competitor's introduction of Zombie Smash, which is viewed as similar to Halo. Estimated profits from Halo if demand is Strong, or Medium, or Weak are as follows:

<table>
<thead>
<tr>
<th>Halo Demand with competition from Zombie Smash</th>
<th>Strong</th>
<th>Medium</th>
<th>Weak</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit</td>
<td>$800</td>
<td>$400</td>
<td>$200</td>
</tr>
<tr>
<td>Probability</td>
<td>0.3</td>
<td>0.4</td>
<td>0.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Halo Demand without competition from Zombie Smash</th>
<th>Strong</th>
<th>Medium</th>
<th>Weak</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit</td>
<td>$1600</td>
<td>$800</td>
<td>$400</td>
</tr>
<tr>
<td>Probability</td>
<td>0.5</td>
<td>0.3</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Suppose Activision believes the probability of a competitor introducing Zombie Smash is 0.6. Which of the two games should Activision market --- Call of Duty Black Ops or Halo? What profit should they expect?
C.3 Statistical Decision Making

Review Questions

Answer to Question:
Step 1: Replace uncertain payoffs with expected value:

$640 = Expected profit from Call of Duty Black Ops

$460 = Expected profit Halo with competition from Zombie Smash

$1120 = Expected profit Halo without competition from Zombie Smash

Given the probability of a competitor introducing Zombie Smash is 0.6,

$724 = Expected profit Halo

Step 2: Make the decision: Activision should market Halo, and expect profit $724.
C.3 Statistical Decision Making

**Backward Induction**

**Question.** Use backward induction (and draw a decision tree, if needed) to help make the following sequential decisions, where later decisions depend on earlier decisions:

1. Should the Bayer Biological Products Group spend $20 million to begin preclinical development of a new blood-clot-busting drug?
   a. The probability of successful testing on humans is 0.7
   b. If the testing is not successful, the project is terminated
   c. If the testing is successful, the project requires $2 million to file a license with to the FDA. The probability of successful filing is 0.9
   d. If the filing is successful, demand and revenues are uncertain:
      i. With probability 0.4, demand is high and revenue is $60 million.
      ii. With probability 0.1, demand is medium and revenue is $80 million.
      iii. With probability 0.5, demand is low and revenue is $100 million.

2. If the testing on humans were successful, should the Bayer Biological Products Group sell its rights in the project for $70 million?

Compute expected profit for the Bayer Biological Products Group.
C.3 Statistical Decision Making

Review Questions

Answer to Question:
Step 1: Replace uncertain payoffs with expected value:

1. Should the Bayer Biological Products Group spend $20 million to begin preclinical development of a new blood-clot-busting drug?
   a. The probability of successful testing on humans is 0.7
   b. If the testing is not successful, the project is terminated
   c. If the testing is successful, the project requires $2 million to file a license with to the FDA. The probability of successful filing 0.9
   d. If the filing is successful, expected revenue =
      
      \[ 0.4(60)+0.1(80)+0.5(100)=82\text{million} \]

   demand and revenues are uncertain:
      i. With probability 0.4, demand is high and revenue is $60 million.
      ii. With probability 0.1, demand is medium and revenue is $80 million.
      iii. With probability 0.5, demand is low and revenue is $100 million.

2. If the testing on humans were successful, should the Bayer Biological Products Group sell its rights in the project for $70 million?
C.3 Statistical Decision Making

Step 2: Replace uncertain payoffs with expected value:

1. Should the Bayer Biological Products Group spend $20 million to begin preclinical development of a new blood-clot-busting drug?
   a. The probability of successful testing on humans is 0.7
   b. If the testing is not successful, the project is terminated
   c. If the testing is successful, the project requires $2 million to file a license with the FDA, and earn expected revenue = 0.9(82) + 0(0) = $73.8 million. The probability of successful filing 0.9
   d. If the filing is successful, expected revenue = 0.4(60)+0.1(80)+0.5(100)=$82 million demand and revenues are uncertain:
      i. With probability 0.4, demand is high and revenue is $60 million.
      ii. With probability 0.1, demand is medium and revenue is $80 million.
      iii. With probability 0.5, demand is low and revenue is $100 million.

2. If the testing on humans were successful, should the Bayer Biological Products Group sell its rights in the project for $70 million?
C.3 Statistical Decision Making

Review Questions

Step 3: Make the second decision:

1. Should the Bayer Biological Products Group spend $20 million to begin preclinical development of a new blood-clot-busting drug?
   a. The probability of successful testing on humans is 0.7
   b. If the testing is not successful, the project is terminated
   c. If the testing is successful, the project requires $2 million to file a license with to the FDA, and earn expected revenue = 0.9(82) + 0(0) = $73.8 million. The probability of successful filing 0.9
   d. If the filing is successful, expected revenue = 0.4(60)+0.1(80)+0.5(100)=$82million demand and revenues are uncertain:
      i. With probability 0.4, demand is high and revenue is $60 million.
      ii. With probability 0.1, demand is medium and revenue is $80 million.
      iii. With probability 0.5, demand is low and revenue is $100 million.

2. If the testing on humans were successful, should the Bayer Biological Products Group sell its rights in the project for $70 million?

Answer: Assume testing on humans were successful. Selling rights in the project earns $(70-20) = $50 million profit. Keeping rights earns $(73.8-2-20) = $51.8 million profit. So keep the rights.
C.3 Statistical Decision Making

Step 4: Replace uncertain payoffs with expected value, and make the first decision:

1. Should the Bayer Biological Products Group spend $20 million to begin preclinical development of a new blood-clot-busting drug?
   a. The probability of successful testing on humans is 0.7
   b. If the testing is not successful, profit = -$20 million. If the testing is successful, expected profit = $51.8 million.

So, preclinical development yields expected profit = 0.3(-20) + 0.7(51.8) = $30.26 million, which being positive indicates that, yes, the Bayer Biological Products Group should begin preclinical development and, if the project is successful, they should not sell their rights.
Decision Tree

Question. You may use a hand-held calculator on this problem, but not a computer. Consider the following decision tree, where square nodes are decision nodes and round nodes are chance node. Each chance node has branches marked with their probabilities. For example, branch D occurs with probability .42. And at the end of each branch is its payoff. For example, branch I ends in payoff 100, and branch E ends in payoff -50. Use backward induction (that is, fold back the decision tree) and expected-payoff maximization to determine the optimal initial choice of A or B or C. Be sure to explain your answer.
Answer to Question: Choose A. The full strategy is: Choose A. If F happens, choose K.