Lesson Topics

Production Scheduling (1) Problems are Resource Allocation Problems when outputs are fixed and when outputs and inputs occur at different periods in time. The simplest problems consider only two time periods.

Production Scheduling Problems with Dynamic Inventory (4) help managers find an efficient low-cost production schedule for one or more products over several time periods (weeks or months).

Workforce Assignment (1) Problems are Resource Allocation Problems when labor is a resource with a flexible allocation; some labor can be assigned to more than one work center.

Make or Buy (2) Problems are Linear Programming Profit Maximization problems when outputs are fixed and when inputs can be either made or bought. Make or Buy Problems help minimize cost.

Product Mix (1) Problems are Resource Allocation Problems when outputs have different physical characteristics. Product Mix Problems thus determine production levels that meet demand requirements.

Blending Problems with Weight Constraints (1) help production managers blend resources to produce goods of a specific weight at minimum cost.
Production Scheduling

Question. Greenville Cabinets received a contract to produce speaker cabinets for a major speaker manufacturer. The contract calls for the production of 3300 bookshelf speakers and 4100 floor speakers over the next two months, with the following delivery schedule:

<table>
<thead>
<tr>
<th>Model</th>
<th>Month 1</th>
<th>Month 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bookshelf</td>
<td>2200</td>
<td>1100</td>
</tr>
<tr>
<td>Floor</td>
<td>1000</td>
<td>3100</td>
</tr>
</tbody>
</table>

Greenville estimates that the production time for each bookshelf model is 0.7 hour and the production time for each floor model is 1 hour. The raw material costs are $10 for each bookshelf model and $12 for each floor model. Labor costs are $22 per hour using regular production time and $33 using overtime. Greenville has up to 1900 hours of regular production time available each month and up to 500 additional hours of overtime available each month. If production for either cabinet exceeds demand in month 1, the cabinets can be stored at a cost of $5 per cabinet. For each product, Greenville Cabinets must determine the number of units that should be manufactured each month on regular time and on overtime to minimize total production and storage costs.

Formulate a linear program for finding the optimal production schedule for Greenville Cabinets. Formulate the problem, but you need not solve for the optimum.
Answer to Question:

Decision variables: Regular time labor

<table>
<thead>
<tr>
<th>Model</th>
<th>Month 1</th>
<th>Month 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bookshelf</td>
<td>B1R</td>
<td>B2R</td>
</tr>
<tr>
<td>Floor</td>
<td>F1R</td>
<td>F2R</td>
</tr>
</tbody>
</table>

Decision variables: Overtime labor

<table>
<thead>
<tr>
<th>Model</th>
<th>Month 1</th>
<th>Month 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bookshelf</td>
<td>B1O</td>
<td>B2O</td>
</tr>
<tr>
<td>Floor</td>
<td>F1O</td>
<td>F2O</td>
</tr>
</tbody>
</table>

Labor costs per unit

<table>
<thead>
<tr>
<th>Model</th>
<th>Regular</th>
<th>Overtime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bookshelf</td>
<td>.7 (22) = 15.40</td>
<td>.7 (33) = 23.10</td>
</tr>
<tr>
<td>Floor</td>
<td>1 (22) = 22</td>
<td>1 (33) = 33</td>
</tr>
</tbody>
</table>

IB = Month 1 ending inventory for bookshelf units
IF = Month 1 ending inventory for floor model

Objective function
A.9 Operations Management Applications         Review Questions

Min $ 15.40 \text{B1R} + 15.40 \text{B2R} + 22 \text{F1R} + 22 \text{F2R} $

+ $ 23.10 \text{B1O} + 23.10 \text{B2O} + 33 \text{F1O} + 33 \text{F2O} $

+ $ 10 \text{B1R} + 10 \text{B2R} + 12 \text{F1R} + 12 \text{F2R} $

+ $ 10 \text{B1O} + 10 \text{B2O} + 12 \text{F1O} + 12 \text{F2O} $

+ $ 5 \text{IB} + 5 \text{IF} $

or

Min $ 25.40 \text{B1R} + 25.40 \text{B2R} + 34 \text{F1R} + 34 \text{F2R} $

+ $ 33.10 \text{B1O} + 33.10 \text{B2O} + 45 \text{F1O} + 45 \text{F2O} $

+ $ 5 \text{IB} + 5 \text{IF} $

s.t.

$ .7 \text{B1R} + 1 \text{F1R} \leq 1900 \quad \text{Regular time: month 1} $

$ .7 \text{B2R} + 1 \text{F2R} \leq 1900 \quad \text{Regular time: month 2} $

$ .7 \text{B1O} + 1 \text{F1O} \leq 500 \quad \text{Overtime: month 1} $

$ .7 \text{B2O} + 1 \text{F2O} \leq 500 \quad \text{Overtime: month 2} $

$ \text{B1R} + \text{B1O} - \text{IB} = 2200 \quad \text{Bookshelf: month 1} $

$ \text{IB} + \text{B2R} + \text{B2O} = 1100 \quad \text{Bookshelf: month 2} $

$ \text{F1R} + \text{F1O} - \text{IF} = 1000 \quad \text{Floor: month 1} $

$ \text{IF} + \text{F2R} + \text{F2O} = 3100 \quad \text{Floor: month 2} $
**Production Scheduling with Dynamic Inventory**

**Question.** The Silver Star Bicycle Company will be manufacturing both men’s and women’s models for its Easy-Pedal 10-speed bicycles during the next two months. Management wants to develop a production schedule indicating how many bicycles of each model should be produced in each month. Current demand forecasts call for 150 men’s and 125 women’s models to be shipped during the first month and 200 men’s and 150 women’s models to be shipped in the second month. Additional data are shown:

<table>
<thead>
<tr>
<th>Model</th>
<th>Production Costs</th>
<th>Labor Requirement (hours)</th>
<th>Assembly</th>
<th>Current Inventory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men’s</td>
<td>$120</td>
<td>2.0</td>
<td>1.5</td>
<td>20</td>
</tr>
<tr>
<td>Women’s</td>
<td>$90</td>
<td>1.6</td>
<td>1.0</td>
<td>30</td>
</tr>
</tbody>
</table>

Last month the company used a total of 1000 hours of labor. The company’s labor relations policy will not allow the combined total hours of labor (manufacturing plus assembly) to increase or decrease by more than 100 hours from month to month. In addition, the company charges monthly inventory at a rate of 2% of the production cost based on the inventory levels at the end of the month. The company would like to have at least 25 units of each model in inventory at the end of the two months.

a) Establish a production schedule that minimizes production and inventory costs and satisfies the labor-smoothing, demand, and inventory requirements. What inventories will be maintained and what are the monthly labor requirements?

b) If the company changed the constraints so that the monthly labor increases and decreases could not exceed 50 hours, what would happen to the production schedule? How much will the cost increase? What would you recommend?
Answer to Question:

a. Let

\[ x_{11} = \text{amount of men's model in month 1} \]
\[ x_{21} = \text{amount of women's model in month 1} \]
\[ x_{12} = \text{amount of men's model in month 2} \]
\[ x_{22} = \text{amount of women's model in month 2} \]
\[ s_{11} = \text{inventory of men's model at end of month 1} \]
\[ s_{21} = \text{inventory of women's model at end of month 1} \]
\[ s_{12} = \text{inventory of men's model at end of month 2} \]
\[ s_{22} = \text{inventory of women's model at end of month 2} \]

The model formulation for part (a) is given.

\[
\text{Min } 120x_{11} + 90x_{21} + 120x_{12} + 90x_{22} + 2.4s_{11} + 1.8s_{21} + 2.4s_{12} + 1.8s_{22}
\]
\[
\text{s.t.} \\
20 + x_{11} - s_{11} = 150 \quad \text{or} \quad x_{11} - s_{11} = 130 \quad \text{Satisfy Demand} \\
30 + x_{21} - s_{21} = 125 \quad \text{or} \quad x_{21} - s_{21} = 95 \quad \text{Satisfy Demand} \\
s_{11} + x_{12} - s_{12} = 200 \quad \text{Satisfy Demand} \\
s_{21} + x_{22} - s_{22} = 150 \quad \text{Satisfy Demand} \\
s_{12} \geq 25 \quad \text{Ending Inventory} \\
s_{22} \geq 25 \quad \text{Ending Inventory} \\
\]

Labor Hours: Men's = 2.0 + 1.5 = 3.5
Women's = 1.6 + 1.0 = 2.6

\[
3.5x_{11} + 2.6x_{21} \geq 900 \quad \text{Labor Smoothing for Month 1} \\
3.5x_{11} + 2.6x_{21} \leq 1100 \\
3.5x_{11} + 2.6x_{21} - 3.5x_{12} - 2.6x_{22} \leq 100 \quad \text{Labor Smoothing for} 
\]
-3.5 \, x_{11} - 2.6 \, x_{21} + 3.5 \, x_{12} + 2.6 \, x_{22} \leq 100 \quad \text{Month 2} \tag{10}

x_{11}, \ x_{12}, \ x_{21}, \ x_{22}, \ s_{11}, \ s_{12}, \ s_{21}, \ s_{22} \geq 0

The optimal solution is to produce 193 of the men's model in month 1, 162 of the men's model in month 2, 95 units of the women's model in month 1, and 175 of the women's model in month 2. Total Cost = $67,156

<table>
<thead>
<tr>
<th>Inventory Schedule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Month 1</td>
</tr>
<tr>
<td>Month 2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Labor Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Previous month</td>
</tr>
<tr>
<td>Month 1</td>
</tr>
<tr>
<td>Month 2</td>
</tr>
</tbody>
</table>

b. To accommodate this new policy the right-hand sides of constraints [7] to [10] must be changed to 950, 1050, 50, and 50 respectively. The revised optimal solution is given.

\begin{align*}
  x_{11} &= 201 \\
  x_{21} &= 95 \\
  x_{12} &= 154 \\
  x_{22} &= 175 \quad \text{Total Cost = $67,175}
\end{align*}
Production Scheduling with Dynamic Inventory Question.

Wilson Sporting Goods produces baseballs. Wilson must decide how many baseballs to produce each month. It has decided to use a 4-month planning horizon. The forecasted demands for the next 4 months are 12,000; 13,000; 3,000; and 13,000.

Wilson wants to meet these demands on time, knowing that it currently has 2,000 baseballs in inventory and that it can use a given month’s production to help meet the demand for that month. During each month there is enough production capacity to produce up to 12,000 baseballs, and there is enough storage capacity to store up to 2,000 baseballs at the end of the month, after demand has occurred.

The forecasted production costs per baseball for the next 4 months are $12.50, $12.55, $12.70, $12.80.

The holding cost per baseball held in inventory at the end of the month is figured at 5% of the production cost for that month: $0.625, $0.6275, $0.635, $0.64.

The selling price for baseballs is not considered relevant to the production decision because Wilson will satisfy all customer demand exactly when it occurs – at whatever the selling price.

Therefore, Wilson wants to determine the production schedule that minimizes the total production and holding costs.

Formulate and solve a linear program for finding the optimal production schedule for Wilson Sporting Goods.

Tip: Your written answer should define the decision variables, formulate the objective and constraints, and solve for the optimum. --- You will not earn full credit if you just solve for the optimum; you must also define the decision variables, and formulate the objective and constraints.
Answer to Question: The decision variables are the production quantities for the 4 months, labeled P1 through P4. To keep quantities small, all quantities are in hundreds of baseballs.

The objective function and constraints are easier to understand if we add variables I1 through I4 to be the corresponding end-of-month inventories (after meeting demand). For example, I3 is the number of baseballs left over at the end of month 3.

Minimize the total production and holding costs:
Min 12.50 P1 + 12.55 P2 + 12.70 P3 + 12.80 P4
+ 0.625 I1 + 0.6275 I2 + 0.635 I3 + 0.64 I4.

The following constraints define inventories:

- P1 – I1 = 120-20 (production–inventory = net demand)
- P2 + I1–I2 = 130 (production-net inventory = demand)
- P3 + I2–I3 = 30 (production-net inventory = demand)
- P4 + I3–I4 = 130 (production-net inventory = demand)

There are obvious constraints are on production and inventory storage capacities: Pj ≤ 120 and Ij ≤ 20 for each month j (j = 1, …, 4).

Finally, production and inventory storage are assumed non-negative.
A.9 Operations Management Applications

Review Questions

- Objective Function Coefficients:
  - P1: 12.5
  - P2: 12.5
  - P3: 12.7
  - P4: 12.8
  - I1: 0.025
  - I2: 0.0275
  - I3: 0.025
  - I4: 0.54

- Constraints:
  - Constraint 1: P1 + P2 + P3 + P4 = 100
  - Constraint 2: 0 + 0 + 0 + 0 = 100
  - Constraint 3: 0 + 0 + 0 + 0 = 100
  - Constraint 4: 0 + 0 + 0 + 0 = 100
  - Constraint 5: 0 + 0 + 0 + 0 = 100
  - Constraint 6: 0 + 0 + 0 + 0 = 100
  - Constraint 7: 0 + 0 + 0 + 0 = 100
  - Constraint 8: 0 + 0 + 0 + 0 = 100
  - Constraint 9: 0 + 0 + 0 + 0 = 100
  - Constraint 10: 0 + 0 + 0 + 0 = 100
  - Constraint 11: 0 + 0 + 0 + 0 = 100
  - Constraint 12: 0 + 0 + 0 + 0 = 100

- Optimal Solution:
  - Objective Function Value = 4937.6000

- Variable Values:
  - P1: 110.0000
  - P2: 120.0000
  - P3: 40.0000
  - P4: 120.0000
  - I1: 10.0000
  - I2: 0.0000
  - I3: 10.0000
  - I4: 0.0000
Production Scheduling with Dynamic Inventory

Question. Wilson Sporting Goods produces baseballs. Wilson must decide how many baseballs to produce each month. It has decided to use a 3-month planning horizon. The forecasted demands for the next 3 months are 12,000; 13,000; and 3,000. Wilson wants to meet these demands on time, knowing that it currently has 2,000 baseballs in inventory and that it can use a given month’s production to help meet the demand for that month.

During each month there is enough production capacity to produce up to 12,000 baseballs, and there is enough storage capacity to store up to 2,000 baseballs at the end of the month, after demand has occurred.

The forecasted production costs per baseball for the next 3 months are $12, $13, $14. The holding cost per baseball held in inventory at the end of each of the 3 months are $2, $3, $4.

The selling price for baseballs is not considered relevant to the production decision because Wilson will satisfy all customer demand exactly when it occurs – at whatever the selling price. Therefore, Wilson wants to determine the production schedule that minimizes the total production and holding costs.

Formulate and solve a linear program for finding the optimal production schedule for Wilson Sporting Goods.

What is the net cost if 2 more baseballs were demanded in month 2 and 1 less baseball were demanded in month 3?

Tip: Your written answer should define the decision variables, formulate the objective and constraints, solve for the optimum, and write down your solution. --- You will not earn full credit if you just solve for the optimum; you must also define the decision variables, and formulate the objective and constraints.
Answer to Question:
The decision variables are the production quantities for the 3 months, labeled P1 through P3. To keep quantities small, all quantities are in hundreds of baseballs.

The objective function and constraints are easier to understand if we add variables I1 through I3 to be the corresponding end-of-month inventories (after meeting demand). For example, I3 is the number of baseballs left over at the end of month 3.

Minimize the total production and holding costs:
Min $12P_1 + 13P_2 + 14P_3 + 2I_1 + 3I_2 + 4I_3$.

The following constraints define inventories:
- $P_1 - I_1 = 120-20$ (production–inventory = net demand)
- $P_2 + I_1 - I_2 = 130$ (production-net inventory = demand)
- $P_3 + I_2 - I_3 = 30$ (production-net inventory = demand)

There are obvious constraints are on production and inventory storage capacities: $P_j \leq 120$ and $I_j \leq 20$ for each month $j (j = 1, \ldots, 3)$.

Finally, production and inventory storage are assumed non-negative.
### A.9 Operations Management Applications

#### Review Questions

**Objective Function**

<table>
<thead>
<tr>
<th>Variable Names</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>I1</th>
<th>I2</th>
<th>I3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficients</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

**Constraints**

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Subject to</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>I1</th>
<th>I2</th>
<th>I3</th>
<th>Relation (&lt;=, &gt;=)</th>
<th>Right-Hand-Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraint 1</td>
<td></td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
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<td>=</td>
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</tr>
<tr>
<td>Constraint 2</td>
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<td>1</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>=</td>
<td>130</td>
</tr>
<tr>
<td>Constraint 3</td>
<td></td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>=</td>
<td>30</td>
</tr>
<tr>
<td>Constraint 4</td>
<td></td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>&lt;</td>
<td>120</td>
</tr>
<tr>
<td>Constraint 5</td>
<td></td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>&lt;</td>
<td>120</td>
</tr>
<tr>
<td>Constraint 6</td>
<td></td>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>&lt;</td>
<td>120</td>
</tr>
<tr>
<td>Constraint 7</td>
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<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>&lt;</td>
<td>20</td>
</tr>
<tr>
<td>Constraint 8</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>&lt;</td>
<td>20</td>
</tr>
<tr>
<td>Constraint 9</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>&lt;</td>
<td>20</td>
</tr>
</tbody>
</table>

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**Linear Programming Module**
A.9 Operations Management Applications

Review Questions

What is the net cost of producing 2 more baseballs in month 2 and 1 less baseball in month 3? Using the 100% rule, those changes are within the range of feasibility, so the dual prices for constraints 2 and 3 apply.

Net cost = $14 \times 2 + $14 \times (-1) = $14.
### RIGHT HAND SIDE RANGES

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Lower Limit</th>
<th>Current Value</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-10.000</td>
<td>100.000</td>
<td>110.000</td>
</tr>
<tr>
<td>2</td>
<td>120.000</td>
<td>130.000</td>
<td>140.000</td>
</tr>
<tr>
<td>3</td>
<td>0.000</td>
<td>30.000</td>
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</tr>
<tr>
<td>4</td>
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<td>No Upper Limit</td>
</tr>
<tr>
<td>5</td>
<td>110.000</td>
<td>120.000</td>
<td>130.000</td>
</tr>
<tr>
<td>6</td>
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<td>7</td>
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<tr>
<td>8</td>
<td>0.000</td>
<td>20.000</td>
<td>No Upper Limit</td>
</tr>
<tr>
<td>9</td>
<td>0.000</td>
<td>20.000</td>
<td>No Upper Limit</td>
</tr>
</tbody>
</table>
Production Scheduling with Dynamic Inventory

Question. Wilson Sporting Goods produces baseballs. Wilson must decide how many baseballs to produce each month. It has decided to use a 3-month planning horizon. The forecasted demands for the next 3 months are 13,000; 3,000; and 13,000.

Wilson wants to meet these demands on time, knowing that it currently has 2,000 baseballs in inventory and that it can use a given month’s production to help meet the demand for that month.

During each month there is enough production capacity to produce up to 12,000 baseballs, and there is enough storage capacity to store up to 2,000 baseballs at the end of the month, after demand has occurred.

The forecasted production costs per baseball for the next 3 months are $12.55, $12.70, $12.80.

The holding cost per baseball held in inventory at the end of the month is figured at 5% of the production cost for that month: $0.6275, $0.635, $0.64.

The selling price for baseballs is not considered relevant to the production decision because Wilson will satisfy all customer demand exactly when it occurs – at whatever the selling price.

Therefore, Wilson wants to determine the production schedule that minimizes the total production and holding costs.

Formulate the Wilson Sporting Goods problem as a linear program, but you need not solve for the optimum.

Tip: Your written answer should define the decision variables, and formulate the objective and constraints.
Answer to Question:
The decision variables are the production quantities for the 3 months, labeled P1 through P3.

The objective function and constraints are easier to understand if variables I1 through I3 are end-of-month inventories (after meeting demand). For example, I2 is the number of baseballs left over at the end of month 2.

Minimize the total production and holding costs:
Min 12.55 P1 + 12.70 P2 + 12.80 P3
   + 0.6275 I1 + 0.635 I2 + 0.64 I3.

The following constraints define inventories:
• P1 − I1 = 13000−2000 (production−inventory = net demand)
• P2 + I1−I2 = 3000 (production-net inventory = demand)
• P3 + I2−I3 = 13000 (production-net inventory = demand)

There are constraints are on production and inventory storage capacities:
Pj ≤ 12000 and Ij ≤ 2000 for each month j (j = 1, 2, 3).

Finally, assume production and inventory storage are non-negative.
Workforce Assignment

Question.
National Wing Company (NWC) is gearing up for the new B-48 contract. NWC has agreed to supply 20 wings in April, 24 in May, and 30 in June. Wings can be freely stored from one month to the next.

Currently, NWC has 90 fully-qualified workers. A fully qualified worker can either be placed in production or can train new recruits. A new recruit can be trained to be an apprentice in one month. After another month, the apprentice becomes a fully-qualified worker. Each trainer can train three recruits. At the end of June, NWC wishes to have at least 140 fully-qualified workers.

The production rate and salary per employee type is listed below.

<table>
<thead>
<tr>
<th>Type of Employee</th>
<th>Production Rate (Wings/Month)</th>
<th>Wage per Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production</td>
<td>.4</td>
<td>$3,000</td>
</tr>
<tr>
<td>Trainer</td>
<td>.3</td>
<td>$3,300</td>
</tr>
<tr>
<td>Apprentice</td>
<td>.2</td>
<td>$2,600</td>
</tr>
<tr>
<td>Recruit</td>
<td>.1</td>
<td>$2,200</td>
</tr>
</tbody>
</table>

Formulate a linear programming model to minimize cost. Formulate the problem, but you need not solve for the optimum.

Tip: Your written answer should define the decision variables, and formulate the objective and constraints.
Answer to Question:

\( P_i \) = number of producers in month \( i \) (where \( i = 1, 2, 3 \) for April, May, June)
\( T_i \) = number of trainers in month \( i \) (where \( i = 1, 2 \) for April, May)
\( A_i \) = number of apprentices in month \( i \) (where \( i = 2, 3 \) for May, June)
\( R_i \) = number of recruits in month \( i \) (where \( i = 1, 2 \) for April, May)

Define the objective function to minimize total wage cost for producers, trainers, apprentices, and recruits for April, May, and June:

\[
\text{Min } 3000P_1 + 3300T_1 + 2200R_1 + 3000P_2 + 3300T_2
+ 2600A_2 + 2200R_2 + 3000P_3 + 2600A_3
\]

Define the constraint that total production in Month 1 (April) must equal or exceed contract for Month 1:

\[
(1) \quad .4P_1 + .3T_1 + .1R_1 \geq 20
\]

Define the constraint that total production in Months 1-2 (April, May) must equal or exceed total contracts for Months 1-2:

\[
(2) \quad .4P_1 + .3T_1 + .1R_1 + .4P_2 + .3T_2 + .2A_2 + .1R_2 \geq 44
\]

Define the constraint that total production in Months 1-3 (April, May, June) must equal or exceed total contracts for Months 1-3:

\[
(3) \quad .4P_1 + .3T_1 + .1R_1 + .4P_2 + .3T_2 + .2A_2 + .1R_2 + .4P_3 + .2A_3 \geq 74
\]

Define the constraint that the number of producers and trainers in a month (fully qualified workers) must not exceed the initial supply of 100, plus any apprentices employed in a previous month:

\[
(4) \quad P_1 + T_1 \leq 90
\]
\[
(5) \quad P_2 + T_2 \leq 90
\]
\[
(6) \quad P_3 + T_3 \leq 90 + A_2
\]

Define the constraint that the number of apprentices in a month must not exceed the number of recruits in the previous month:

\[
(7) \quad A_2 - R_1 \leq 0; \quad (8) \quad A_3 - R_2 \leq 0
\]
A.9 Operations Management Applications          Review Questions

Define the constraint that each trainer can train three recruits:

(9) \(3T_1 - R_1 \geq 0\); (10) \(3T_2 - R_2 \geq 0\)

Define the constraint that at the end of June, there are to be at least 140 fully-qualified workers (50 more than you started with):

(11) \(90 + A_2 + A_3 \geq 140\)
Make or Buy

Question. Danchuk Manufacturing produces a variety of classic automobiles, including a 1955 Chevy and a 1955 Thunderbird. Each car consists of three components that can be manufactured by Danchuk: a body, an interior, and an engine. Both cars use the same interior and engine, but different bodies.

Danchuk’s sales forecast indicates that 400 Chevys and 600 Thunderbirds will be needed to satisfy demand during the next year. Because only 2000 hours of in-house manufacturing time is available, Danchuk is considering purchasing some, or all, of the components from outside suppliers. If Danchuk manufactures a component in-house, it incurs a variable manufacturing cost. The following table shows the manufacturing time per component, the manufacturing cost per component, and the cost to purchase each of the components from an outside supplier:

<table>
<thead>
<tr>
<th>Component</th>
<th>Manufacturing Time per Unit (hours)</th>
<th>Manufacturing Cost per Unit (thousands of dollars)</th>
<th>Purchase Cost per Unit (thousands of dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chevy Body</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Thunderbird Body</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Interior</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Engine</td>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Formulate a linear program for Danchuk to minimize total cost to meet the sales forecasts. Formulate the program, but you need not solve for the optimum.

Tip: Your written answer should define the decision variables and formulate the objective and constraints.
Answer to Question:
Continuous (non-integer) variables are used for the number of units made:
CB = the number of Chevy Bodies to make
TB = the number of Thunderbird Bodies to make
IN = the number of Interiors to make
E = the number of Engines to make

Continuous variables are also used for the number of units purchased:
CBP = the number of Chevy Bodies to purchase
TBP = the number of Thunderbird Bodies to purchase
INP = the number of Interiors to purchase
EP = the number of Engines to purchase

Objective: Minimize Total Cost
4CB + 2TB + 1IN + 4E (production costs)
+5CBP + 3TBP + 2INP + 5EP (purchase costs)

Input Constraints:
3CB + 2TB + 1IN + 2E ≤ 2000 (In-house hours)

Sales Constraints:
CB + CBP = 400 (manufactured or purchased bodies used for Chevys)
TB + TBP = 600 (manufactured or purchased bodies used for Thunderbirds)
IN + INP = 1000 (manufactured or purchased interiors used for both Chevys and Thunderbirds)
E + EP = 1000 (manufactured or purchased engines used for both Chevys and Thunderbirds)
Make or Buy

Question. Danchuk Manufacturing produces a variety of classic automobiles, including a 1955 Chevy and a 1955 Thunderbird. Each car consists of three components that can be manufactured by Danchuk: a body, an interior, and an engine. Both cars use the same interior and engine, but different bodies.

Danchuk’s sales forecast indicates that 500 Chevys and 300 Thunderbirds will be needed to satisfy demand during the next year. Because only 3000 hours of in-house manufacturing time is available, Danchuk is considering purchasing some, or all, of the components from outside suppliers. If Denchuk manufactures a component in-house, it incurs a variable manufacturing cost. The following table shows the manufacturing time per component, the manufacturing cost per component, and the cost to purchase each of the components from an outside supplier:

<table>
<thead>
<tr>
<th>Component</th>
<th>Manufacturing Time per Unit (hours)</th>
<th>Manufacturing Cost per Unit (thousands of dollars)</th>
<th>Purchase Cost per Unit (thousands of dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chevy Body</td>
<td>4</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>Thunderbird Body</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Interior</td>
<td>5</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Engine</td>
<td>9</td>
<td>9</td>
<td>11</td>
</tr>
</tbody>
</table>

Formulate a linear program for Denchuk to minimize total cost to meet the sales forecasts. Formulate the program, but you need not solve for the optimum.

Tip: Your written answer should define the decision variables and formulate the objective and constraints.
Answer to Question:
Continuous (non-integer) variables are used for the number of units made:
CB = the number of Chevy Bodies to make
TB = the number of Thunderbird Bodies to make
IN = the number of Interiors to make
E = the number of Engines to make

Continuous variables are also used for the number of units purchased:
CBP = the number of Chevy Bodies to purchase
TBP = the number of Thunderbird Bodies to purchase
INP = the number of Interiors to purchase
EP = the number of Engines to purchase

Objective: Minimize Total Cost
7CB + 4TB + 3IN + 9E (production costs)
+9CBP + 5TBP + 4INP + 11EP (purchase costs)

Input Constraints:
4CB + 3TB + 5IN + 9E ≤ 3000 (In-house hours)

Sales Constraints:
CB + CBP = 500 (manufactured or purchased bodies used for Chevys)
TB + TBP = 300 (manufactured or purchased bodies used for Thunderbirds)
IN + INP = 800 (manufactured or purchased interiors used for both Chevys and Thunderbirds)
E + EP = 800 (manufactured or purchased engines used for both Chevys and Thunderbirds)
**Product Mix**

**Question.** Island Water Sports is a business that provides rental equipment and instruction for a variety of water sports in a resort town. On one particular morning, a decision must be made of how many Wildlife Raft Trips and how many Group Sailing Lessons should be scheduled. Each Wildlife Raft Trip requires one captain and one crew person, and can accommodate six passengers. The revenue per raft trip is $120. Ten rafts are available, and 30 people are on the list for reservations this morning. Each Group Sailing Lesson requires one captain and two crew people for instruction. Two boats are needed for each group. Four students form each group. There are 12 sailboats available, and 20 people are on the list for sailing instruction this morning. The revenue per group sailing lesson is $160. The company has 12 captains and 18 crew available this morning. The company would like to maximize the number of customers served while generating at least $180 in revenue and honoring all reservations. Formulate and use the *Management Scientist* to solve the appropriate linear programming problem.
Answer to Question:
The following description of constraints also mentions assumptions that fill in information left out of the verbal description of the problem. Let

\[ R = \text{the number of Wildlife Raft Trips to schedule} \]
\[ S = \text{the number of Group Sailing Lessons to schedule} \]

\[
\begin{align*}
\text{Max} & \quad 6R + 4S \\
\text{s.t.} & \quad R + S \leq 12 \quad \text{(captain constraint, assuming captains not used do not affect the objective function or other constraints. Idle captains do no alternative work)} \\
& \quad R + 2S \leq 18 \quad \text{(crew constraint, assuming crew not used do not affect the objective function or other constraints. Idle crew do no alternative work)} \\
& \quad 6R \geq 30 \quad \text{(6R is the number of raft customers assuming you have all the paying customers you have room for)} \\
& \quad 4S \geq 20 \quad \text{(4S is the number of sailing customers assuming you have all the paying customers you have room for)} \\
& \quad 120R + 160S \geq 180 \\
& \quad R \leq 10 \quad \text{(raft constraint, assuming 1 raft per trip (you cannot do two trips per day) and rafts not used do not affect the objective function or other constraints. Idle rafts have no alternative purpose, and depreciate as much as rafts used)} \\
& \quad 2S \leq 12 \quad \text{(sailboat constraint, assuming boats not used do not affect the objective function or other constraints. Idle boats have no alternative purpose, and depreciate as much as boats used)} \\
& \quad R, S \geq 0
\end{align*}
\]
### A.9 Operations Management Applications

#### Review Questions

**The Management Scientist Version 6.0**

**Note:** Decision variable names can be changed if desired. Enter Objective constrains section, enter constraint coefficients, constraint relationship, and right-hand-side values.

**Optimization Type:** Max

**Objective Function**

<table>
<thead>
<tr>
<th>Variable Names</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>6</td>
</tr>
<tr>
<td>S</td>
<td>4</td>
</tr>
</tbody>
</table>

**Constraints**

<table>
<thead>
<tr>
<th>Subject To</th>
<th>R</th>
<th>S</th>
<th>Relation</th>
<th>Right-Hand-Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraint 1</td>
<td>1</td>
<td>1</td>
<td>&lt;</td>
<td>12</td>
</tr>
<tr>
<td>Constraint 2</td>
<td>1</td>
<td>2</td>
<td>&lt;</td>
<td>18</td>
</tr>
<tr>
<td>Constraint 3</td>
<td>6</td>
<td></td>
<td>&gt;</td>
<td>30</td>
</tr>
<tr>
<td>Constraint 4</td>
<td></td>
<td>4</td>
<td>&gt;</td>
<td>20</td>
</tr>
<tr>
<td>Constraint 5</td>
<td>120</td>
<td>160</td>
<td>&gt;</td>
<td>180</td>
</tr>
<tr>
<td>Constraint 6</td>
<td>1</td>
<td></td>
<td>&lt;</td>
<td>10</td>
</tr>
<tr>
<td>Constraint 7</td>
<td>2</td>
<td></td>
<td>&lt;</td>
<td>12</td>
</tr>
</tbody>
</table>

**Optimal Solution**

**Objective Function Value = 62**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>7.000</td>
</tr>
<tr>
<td>S</td>
<td>5.000</td>
</tr>
</tbody>
</table>
Blending with Weight Constraints

Question.
The Maruchan Corporation receives wheat, monosodium glutamate, dehydrated soy sauce, and Maltodextrin, which it blends into Maruchan Ramen Noodle Soup. The Maruchan Corporation advertises that each 3-ounce packet meets the minimum daily requirements for Sodium, Dietary Fiber, and Iron. The following is the cost per pound of wheat, monosodium glutamate, dehydrated soy sauce, and Maltodextrin. And the Sodium, Dietary Fiber, and Iron per pound of the 4 ingredients.

<table>
<thead>
<tr>
<th>Ingredient</th>
<th>Sodium Units/lb</th>
<th>Dietary Fiber Units/lb</th>
<th>Iron Units/lb</th>
<th>Cost/lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheat</td>
<td>9</td>
<td>12</td>
<td>0</td>
<td>.75</td>
</tr>
<tr>
<td>Monosodium glutamate</td>
<td>16</td>
<td>10</td>
<td>14</td>
<td>.90</td>
</tr>
<tr>
<td>Dehydrated soy sauce</td>
<td>8</td>
<td>10</td>
<td>15</td>
<td>.80</td>
</tr>
<tr>
<td>Maltodextrin</td>
<td>10</td>
<td>8</td>
<td>7</td>
<td>.70</td>
</tr>
</tbody>
</table>

The Maruchan Corporation is interested in producing the 3-ounce mixture at minimum cost while meeting the minimum daily requirements of 6 units of Sodium, 5 units of Dietary Fiber, and 5 units of Iron.

Formulate The Maruchan Corporation’s problem as a linear program, but you need not solve for the optimum.

Tip: Your written answer should define the decision variables, and formulate the objective and constraints.
Define the decision variables:

\[ x_j = \text{the pounds of Ingredient } j \ (j = 1,2,3,4) \text{ used in 3-ounce mixture} \]

Define the objective: Minimize the total cost for a 3-ounce mixture:

\[
\text{Min } .75x_1 + .90x_2 + .80x_3 + .70x_4
\]

Constrain the total weight of the mix to 3-ounces (3/16 pounds):

\[ (1) \ x_1 + x_2 + x_3 + x_4 = 3/16 \]

Constrain the total amount of Sodium in the mix to be at least 6 units:

\[ (2) \ 9x_1 + 16x_2 + 8x_3 + 10x_4 \geq 6 \]

Constrain the total amount of Dietary Fiber in the mix to be at least 5 units:

\[ (3) \ 12x_1 + 10x_2 + 10x_3 + 8x_4 \geq 5 \]

Constrain the total amount of Iron in the mix to be at least 5 units:

\[ (4) \ 14x_2 + 15x_3 + 7x_4 \geq 5 \]