Lesson Topics

Proportion Constraints restrict the size of one decision variable as a proportion of another. For example, \( B \geq 0.1J \) or \( B - 0.1J \geq 0 \) says that variable B is at least 10% of variable J.

Portfolio Selection with Risk Indices use one specific measure of the risk of a portfolio. When the risk index is part of a linear risk function, risk minimization is a linear-programming problem.

100% Rules expand the range of optimality and the range of feasibility to simultaneous changes in two or more objective function coefficients, or in two or more right-hand-side constants.

Negative Dual Prices (1) measure the rate of deterioration (negative improvement) in the objective function value per unit increase in a right-hand side constant.

Blending Problems with Sensitivity Analysis helps production managers decide how cost is affected by changing product characteristics (shade tolerance, calories, vitamin content, …).
Negative Dual Prices

**Question.** Benson Electronics manufactures three components used to produce cell telephones and other communication devices. In a given production period, demand for the three components may exceed Benson’s manufacturing capacity. In this case, the company meets demand by purchasing the components from another manufacturer at an increased cost per unit. Benson’s manufacturing cost per unit and purchasing cost per unit for the three components are as follows:

<table>
<thead>
<tr>
<th>Source</th>
<th>Component 1</th>
<th>Component 2</th>
<th>Component 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacture</td>
<td>$4.50</td>
<td>$5.00</td>
<td>$2.75</td>
</tr>
<tr>
<td>Purchase</td>
<td>$6.50</td>
<td>$8.80</td>
<td>$7.00</td>
</tr>
</tbody>
</table>

Manufacturing times in minutes per unit for Benson’s three departments are as follows:

<table>
<thead>
<tr>
<th>Department</th>
<th>Component 1</th>
<th>Component 2</th>
<th>Component 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Assembly</td>
<td>1</td>
<td>1.5</td>
<td>3</td>
</tr>
<tr>
<td>Testing &amp;</td>
<td>1.5</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Packaging</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For instance, each unit for component 1 that Benson manufactures requires 2 minutes of production time, 1 minute of assembly time, and 1.5 minutes of testing and packaging time. For the next production period, Benson has capacities of 360 hours in the production department, 250 hours in the assembly department, and 300 hours in the testing and packaging department.
A.7 Sensitivity Analysis Applications

Review Questions

a. Formulate a linear programming model that can be used to determine how many units of each component to manufacture and how many units of each component to purchase. Assume that component demands that must be satisfied are 6000 units for component 1, 4000 units for component 2, and 3500 units for component 3. The objective is to minimize the total manufacturing and purchasing costs.

b. Use the *Management Scientist* to determine the optimal solution? How many units of each component should be manufactured and how many units of each component should be purchased?

c. Which departments are limiting Benson’s manufacturing quantities? Use the dual price to determine the value of an *extra hour* in each of these departments.

d. Suppose that Benson had to obtain one additional unit of component 2. Discuss what the dual price for the component 2 constraints tells us about the cost to obtain the additional unit.
A.7 Sensitivity Analysis Applications

Answer to Question:

a. Let  \( M_1 = \) units of component 1 manufactured
    \( M_2 = \) units of component 2 manufactured
    \( M_3 = \) units of component 3 manufactured
    \( P_1 = \) units of component 1 purchased
    \( P_2 = \) units of component 2 purchased
    \( P_3 = \) units of component 3 purchased

To proceed convert Benson’s capacity of 360 hours in the production department into 360x60 = 21600 minutes, and so on.

\[
\begin{align*}
\text{Min} & \quad 4.50M_1 + 5.00M_2 + 2.75M_3 + 6.50P_1 + 8.80P_2 + 7.00P_3 \\
\text{s.t.} & \quad 2M_1 + 3M_2 + 4M_3 \leq 21,600 \quad \text{Production} \\
& \quad 1M_1 + 1.5M_2 + 3M_3 \leq 15,000 \quad \text{Assembly} \\
& \quad 1.5M_1 + 2M_2 + 5M_3 \leq 18,000 \quad \text{Testing/Packaging} \\
& \quad M_1 + 1P_1 = 6,000 \quad \text{Component 1} \\
& \quad 1M_2 + 1P_2 = 4,000 \quad \text{Component 2} \\
& \quad 1M_3 + 1P_3 = 3,500 \quad \text{Component 3} \\
& \quad M_1, M_2, M_3, P_1, P_2, P_3 \geq 0
\end{align*}
\]

b.

<table>
<thead>
<tr>
<th>Source</th>
<th>Component 1</th>
<th>Component 2</th>
<th>Component 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacture</td>
<td>2000</td>
<td>4000</td>
<td>1400</td>
</tr>
<tr>
<td>Purchase</td>
<td>4000</td>
<td>0</td>
<td>2100</td>
</tr>
</tbody>
</table>

Total Cost: $73,550

c. Since the slack is 0 in the production and the testing & packaging departments, these department are limiting Benson’s manufacturing quantities.

Dual price information:

Production \( \quad \$0.906/\text{minute} \times 60 \text{ minutes} = \$54.36 \text{ per hour} \)

Testing/Packaging \( \quad \$0.125/\text{minute} \times 60 \text{ minutes} = \$7.50 \text{ per hour} \)
d. The dual price is -$7.969. This tells us that the value of the optimal solution will worsen (the cost will increase) by $7.969 for an additional unit of component 2. Note that although component 2 has a purchase cost per unit of $8.80, it would only cost Benson $7.969 to obtain an additional unit of component 2.