Lesson Topics

**Own Price Elasticity** measures the responsiveness of the demand for a good to its own price. Demand is more inelastic when you distinguish your product from competitors.

**Elasticity and Revenue (2)** are inversely related: If demand is elastic, an increase in price decreases revenue; if inelastic, an increase in price increases revenue. — So, inelastic demand is more profitable.

**Cross Elasticity (1)** measures the responsiveness of the demand for one good to the price of another good. The sign indicates whether goods are gross substitutes or gross complements.

**Demand Functions (3)** are typically linear or log-linear. The coefficients of linear functions determine equilibrium quantity and surplus. The coefficients of log-linear functions equal elasticities.
Elasticity and Revenue

Question. You are a manager of Toyota. If your marketing department estimates that the semiannual demand for the Toyota Prius is \( Q = 75,000 - 1.5 P \)

What price should you charge in order to maximize revenues from sales of the Prius? What is the own price elasticity of demand at that price?
Tip: A full answer to each question on this review includes all highlighted conclusions below and an explanation of how you reached those conclusions.

Answer to Question: The own price elasticity of demand at any price $P$ is $E = \frac{dQ}{dP}(P/Q) = -1.5\left(P/(75,000-1.5P)\right)$. To maximize revenue, set that elasticity to -1, making demand unit elastic,

$$-1 = E = -1.5\left(P/(75,000-1.5P)\right),$$
so $75,000-1.5P = 1.5P$,
so $3P=75,000$, so $P = 25,000$.

An alternative way to find that price is to solve for inverse demand $P = 50,000-Q/1.5$, and $MR = 50,000-2Q/1.5$, and set $0=MR=50,000-2Q/1.5$,
so $Q = 50,000(1.5/2) = 37,500$,
so $P = 50,000-Q/1.5 = 50,000-25,000 = 25,000$. 

Elasticity and Revenue

Question. You are managing ticket sales for Delta Airlines. For each flight, no matter how many passengers are on the flight, Delta spends $5,000 for fuel, and $4,000 in airport fees. Each flight can hold up to 400 passengers, and your marketing department estimates that the demand for passenger seats on each flight is $Q = 500 - 5\ P$, with $P$ measured in $\$. 

What price should you charge? What profit should you expect?
**Answer to Question:** The own price elasticity of demand at any price P is 
\[ E = \frac{dQ}{dP} \frac{P}{Q} = -5 \left( \frac{P}{500-5P} \right). \] 
To maximize profit with all costs fixed, maximize revenue. And to maximize revenue, set own price elasticity to -1, making demand unit elastic, 
-1 = E = -5 \left( \frac{P}{500-5P} \right),
so 500-5P = 5P,
so 10P = 500, so \( P = $50. \)

An alternative way to find that price is to solve for inverse demand 
P = 100 - Q/5, and MR = 100-2Q/5, and set 
0 = MR = 100-2Q/5, 
so Q = 100(5/2) = 250, 
so P = 100-Q/5 = 100-250/5 = $50

In either case, quantity is \( Q = 500 - 5 \times 50 = 250, \) revenue \( PQ = 50 \times 250 = $12,500, \) and profit on each flight is \( \Pi = 12,500 - 9,000 = $3,500 \)
Cross Elasticity

Question. Suppose the demand for sunscreen (X) has been estimated to be $\ln Q_x = 5 - 1.7P_x + 3 \ln S - 4 \ln A_y$, where $S$ denotes the average hours of sunshine per day and $A_y$ represents the level of advertising for good Y.

a. What would be the effect on demand of a 5 percent increase in the daily amount of sunshine?

b. What would be the effect of a 10 percent reduction in the amount of advertising toward good Y?

c. What might be good Y in this example?

Explain your answers.
Answer to Question:

a. A five percent increase in the daily amount of sunshine leads to a
   3x5 percent = **15 percent increase in the demand for sunscreen (X)**.

b. A 10 percent reduction in the amount of advertising toward good Y
   results in a 4x10 percent = **40 percent increase in demand for X**.

c. Beach umbrellas (or any other **gross substitute for sunscreen**).
A.3 Elasticity  

**Demand Functions**

**Question.** You are a manager in charge of monitoring cash flow at the Coca-Cola Company. You must determine how to change the price of Coke in 2009 to increase by $10.2 million the Coca-Cola Company’s overall revenues from both Coke and Powerade (which is another one of your drink products).

So, you collect monthly data on the prices you charged in the past on Coke and Powerade, and on the demand for Coke and Powerade, and determine the following regression lines:

\[
\ln(CQ) = 9 - 2 \ln(CP) - 0.1 \ln(AP) \\
\ln(AQ) = 4 - 0.5\ln(CP) - 0.2 \ln(AP)
\]

for variables  
CQ = demand quantity for Coke,  
CP = price for Coke,  
AQ = demand quantity for Powerade,  
AP = price for Powerade.

You also collect data on revenue, and find out that in 2007, the Coca-Cola Company earned about $1 billion from sales of Coke, and about $100 million from sales of Powerade.

How should you change the price of Coke in 2009 to increase by $10.2 million the Coca-Cola Company’s overall revenues from both Coke and Powerade sales?
A.3 Elasticity

Answer to Question: The regression lines

\[ \ln(CQ) = 9 - 2 \ln(CP) - 0.1 \ln(AP) \]
\[ \ln(AQ) = 4 - 0.5 \ln(CP) - 0.2 \ln(AP) \]

imply

- the own price elasticity of demand for Coke is -2
- the cross-price elasticity between demand for Powerade and price of Coke is -0.5.

Using the change in revenue formula for two products, the estimate of how any change in the price of coke in 2009 will affect the Coca-Cola Company’s overall revenues from both Coke and Powerade sales is

\[ \Delta R = \left[ 1000M(1-2) + 100M(-0.5) \right] \times \% \Delta \text{Price} \]
\[ = [-1050M] \times \% \Delta \text{Price} \]

So, setting \( \Delta R = 10.2M \) yields
\[ \% \Delta \text{Price} = \frac{10.2M}{-1050M} = -0.097. \]
That is, the price of Coke must decrease 0.97%.
A.3 Elasticity

Demand Functions

Question. You are a manager in charge of monitoring cash flow at the Coca-Cola Company. You must determine how to change the price of Coke in 2009 to increase by $2.04 million the Coca-Cola Company’s overall revenues from both Coke and Powerade (which is another one of your drink products).

So, you collect monthly data on the prices you charged in the past on Coke and Powerade, and on the demand for Coke and Powerade, and determine the following regression lines:

\[
\ln(CQ) = 9 - 2 \ln(CP) - 0.1 \ln(AP) \\
\ln(AQ) = 4 - 0.5 \ln(CP) - 0.2 \ln(AP)
\]

for variables
CQ = demand quantity for Coke,
CP = price for Coke,
AQ = demand quantity for Powerade,
AP = price for Powerade.

You also collect data on revenue, and find out that in 2007, the Coca-Cola Company earned about $1 billion from sales of Coke, and about $100 million from sales of Powerade.

How should you change the price of Coke in 2009 to increase by $2.04 million the Coca-Cola Company’s overall revenues from both Coke and Powerade sales?
A.3 Elasticity

Answer to Question: The regression lines

\[
\begin{align*}
\ln(CQ) &= 9 - 2 \ln(CP) - 0.1 \ln(AP) \\
\ln(AQ) &= 4 - 0.5 \ln(CP) - 0.2 \ln(AP)
\end{align*}
\]

imply
- the own price elasticity of demand for Coke is -2
- the cross-price elasticity between demand Powerade and price of Coke is -0.5.

Using the change in revenue formula for two products, the estimate of how any change in the price of coke in 2009 will affect the Coca-Cola Company’s overall revenues from both Coke and Powerade sales is

\[
\Delta R = \left[ \$1000M(1-2) + \$100M(-0.5) \right] \times \%\Delta \text{Price}
\]
\[
= [-\$1050M] \times \%\Delta \text{Price}
\]

So, setting \(\Delta R = \$2.04M\) yields
\[
\%\Delta \text{Price} = \frac{$2.04M}{-$1050M} = -0.0019.
\]
That is, the price of Coke must decrease 0.19%.
Demand Functions

Question. You are a manager in charge of monitoring cash flow at Kodak. You must determine how a planned 2 percent increase in the price of disposable cameras in 2009 will affect Kodak’s overall revenues from both disposable and digital camera sales.

So, you collect monthly data on the prices you charged in the past on digital cameras and disposable cameras, and on the demand for digital cameras and disposable cameras and determine the following regression lines:

\[
\ln(DQ) = 5 - 3\ln(DP) - 0.1\ln(XP)
\]
\[
\ln(XQ) = 3 - 0.5\ln(XP) - 0.3\ln(DP)
\]

for variables
DQ = demand quantity for digital cameras,
DP = price for digital cameras,
XQ = demand quantity for disposable cameras,
XP = price for disposable cameras,

You also collect data on revenue, and find out that in 2007, Kodak earned about $600 million from sales of digital cameras about $100 million from sales of disposable cameras.

What is your estimate of how the planned 2 percent increase in the price of disposable cameras in 2009 will affect Kodak’s overall revenues from both disposable and digital camera sales?
A.3 Elasticity

**Answer to Question:**
The regression lines imply
- the own price elasticity of demand for disposable cameras is -.5
- the cross-price elasticity of demand between digital and disposable cameras is -0.1.

Using the change in revenue formula for two products, the estimate of how the planned 2 percent increase in the price of disposable cameras in 2009 will affect Kodak’s overall revenues from both disposable and digital camera sales is

\[ \Delta R = [100(1-.5) + 600(-0.1)] \times 0.02 \text{ million} = [-10] \times 0.02 \text{ million} = -$200,000, \text{ so revenues will decrease by $200,000.} \]

Comment: The cross-price elasticity of demand between digital and disposable cameras is negative, which means digital and disposable cameras are gross complements. How can that be? Remember, gross complements includes an income effect as well as a substitution effect. In particular, if the price of disposable cameras increases, purchasing power decreases, so the demand for digital cameras decreases if it is a normal good.