Can calculus carry nerve signals?

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First I would like to thank President Samuelson for his kind introduction. I am deeply honored and privileged to give this lecture. As I thought of this recognition, I realized that this recognition belongs to a large group of people, my colleagues at the department of mathematics and students, whom I have worked with and learned from. It has been a privilege for me to be a member of this great institution and to share joyful learning experiences with students here at BYU.

I thought about several topics for this talk. But I liked this one the most. Not only does it reach deep into my professional life, but more importantly it has touched everyone's life to a varying degree. This may sound shocking to those of you who have not taken another mathematics course beyond college algebra, but I hope you will be convinced after my talk. Alternative titles of this talk are: Can calculus predict weather? Can calculus send a satellite to its orbit? Can calculus broadcast radio? Can calculus make money for you? Why calculus?

I have taught mathematics for the last seventeen years here at BYU. There is one course that many of you either have taken or are going to take from the mathematics department. The course of the subject is called calculus, it goes by the names of Math 112, 113, and 214 or Math 119 at BYU. My research area is dynamical systems and differential equations which are direct descendents of calculus. One can say that it is the pursuit of knowledge that has brought us together today on this campus, but personally, it is my pursuit of the subject of calculus that has brought me to you at this moment.

In ancient times, the word “calculus” was a pebble used in counting. Centuries later, it meant “to compute”. Today calculus is a language and tool of science used by engineers, scientists, and economists to describe the complex inter-relationships of physical quantities. And their work has a huge impact on our day to day life.

The development of calculus has a long and rich history. It involves both numbers and geometry.

Originally mathematics arose as a part of everyday life. The notion of numbers first appeared. It was born directly from observations of real physical phenomena exhibited in nature. The concept of number was a consequence of observing both the similarities and contrasts found within nature. Number five is the abstract notion of five fingers of a hand, five vertices of a star figure, and so on.

The extension of operations among numbers to letters gives rise to algebra. By 2000 BC the Babylonians had a well-developed algebra. They were capable of solving quadratic equations by using the quadratic formula as well as completing the square. These are basically the same procedures we use today in middle school.

The word geometry is a combination of earth and measure in Greek. Geometry arose from practical needs in measuring the size of lands and laying out accurate right angles for the corners of buildings. In ancient Egypt, geometry was used to reestablish boundary lines on the land affected by the periodic flooding of the Nile River.

The first important geometer mentioned in history is Thales, a Greek who lived about 600 BC. One important theorem he proved is that the angle inscribed in a semicircle is a right angle. He was also known for producing the most famous student, Pythagoras. We all learned the Pythagorean Theorem. Pythagoras also discovered irrational numbers such as the square root of 2. The existence of irrational numbers was against their beliefs at that time.

Geometry was further developed in the third century B.C. and was put into axiomatic form by Euclid in his publication **Elements**, which has served as a basic textbook in geometry that we still use today.

In the third century B.C. another great Greek scientist, Archimedes, also made a number of important contributions to geometry. He devised ways to measure the areas and volumes of many curved figures including the volume and surface area of a sphere. He also found the sum of a geometric series.

A way to compute the area of a circle is to divide the circle into a number of sectors and to use rectangles or triangles to compute the area. People believe that ancient Egyptian and Chinese knew this approach, which is a primitive form of integral calculus.

One important observation is that for all circles the ratio between the circumference and the diameter is the same constant, which we call it pi today. It is an irrational number.

Calculus was not developed in a rigorous or systematic way until the 17^{th} century.

The foundation for modern calculus was set by Descartes, a French mathematician, in the first half of the 17^{th} century. He introduced the Cartesian coordinate system and founded analytic geometry that bridges between algebra and geometry.

Important contributions to the early development of calculus were made by many people including Fermat and Pascal.

The true turning point in the development of calculus occurred in the 17^{th} century when Newton studied planetary motion. The question was to find the relationships between the position, the velocity, and the acceleration of moving bodies. To borrow the catchiest phrase of this year's presidential race, it is all about `change', about how fast a change can be made. The answer to the rate of change leads to the development of the differential calculus, while computing the total distance a particle has traveled leads to the development of integral calculus.

Leibniz, a German mathematician and contemporary of Newton, independently developed calculus. The main reason that Leibniz's contribution is overshadowed by Newton's is because Newton used his new discovery to usher in the age of modern science and enlightenment.

Newton used his newly founded tool to establish the modern theory of celestial mechanics. For the first time, a century's worth of empirical observation on the celestial bodies and a mathematical theory fit to each other like a glove to a hand, establishing in the modern methodology for scientific investigations. The idea that natural phenomena can be understood by mathematics was absolutely shocking at that time and still surprises many people today.

Some profound impacts of Newton's calculus are direct, some are indirect, and some others are of great philosophical importance. Let me just briefly comment on a few extraordinary examples.

As I said, a main concern of calculus is on the rate of changes of things. This idea can be found in a hugely influential theory of population in the 18^{th} century by the British economist Thomas Malthus. Malthus' theory is based on the idea that the number of an unchecked population grows at an exponential or geometrical rate while its food grows only at an arithmetic rate.

Maxwell's electromagnetic theory was directly built on calculus, which gave rise to mankind's mastering of electricity and all means of modern communications, including fiber optics.

Under the primary assumption that the speed of light is constant for all observers, stationary or in constant motion, Einstein used calculus to derive his theory of special relativity and later his theory of general relativity, which in turn gave birth to the big bang theory, the black hole theory, and modern cosmology in general, which in fact originated from Newton's celestial mechanics.

Quantum mechanics is another theory formulated in terms of calculus. Without quantum mechanics, the world would be a less colorful place. Let's just imagine a list of inconveniencies in a quantum-theory-free world. How about to start the list by taking away your TVs, or your computers, or your iPods, or your cell phones? In fact, all useful physics theories are built on the language of calculus.

Right at this moment, billions of neurons in our brain are firing electrical pulses run our body and enable whatever mental ponderings are ours. Certainly it is a long way to go before we will fully understand the brain, but the electrical dynamics of neurons can be modeled and understood by differential equations. That was done by two British physiologists, Alan Hodgkin and Andrew Huxley. They were awarded the 1963's Nobel Prize in Physiology or Medicine for both their experimental and theoretical works on the problem of neuronal pulse propagation.

In finance, calculus was used to derive the Black-Scholes equation for option pricing, which was further developed by Robert Merton. Again differential equations proved to be indispensable. For their work, Merton and Scholes were awarded the 1997 Nobel Prize in Economics, not to mention how much richer many stock traders have become because of their theory. Of course, the bigger picture lies in their contribution to the management of stock volatilities, and to the creation of wealth in general.

To end my enumeration of applications of calculus, let me just mention that calculus is absolutely vital to mathematical modeling of food webs in ecology, weather forecasting in meteorology, global warming simulation in climatology, and infectious disease simulation, prediction, and intervention in epidemiology. All have important implications to mankind's immediate and future well-being.

On a lighter side, the derivative that is the rate of change, a fundamental concept in calculus, even found its way into presidential speeches when President Nixon announced in the fall 1972 "the rate of increase of inflation was decreasing''. As Professor Hugo Rossi, the former vice president of American Mathematical Society, pointed out that "(that) was the first time a sitting president used the third derivative to advance his case for reelection."

As I mentioned before, my research area is dynamical systems and differential equations which are direct descendents of calculus. Currently, my research concerns with systems when uncertainty or a random noise taken into account. The issues I study are stability, long term dynamics, and chaotic behavior of states. A fundamental tool is calculus and its stochastic version.

The takeaway message, if any and perhaps a small one, is this: the field of human inquiry that is opened up by calculus is huge and the applications of calculus are endless. If you are going into a profession that is several degrees in separation from calculus, stay tuned to the most profound and new scientific discoveries. If history is a reliable guide, such discoveries will be made with calculus being used directly or indirectly as one of the most fundamental tools.

Throughout my many years of teaching calculus, I have polled, unscientifically, the question why do we study calculus? In moving toward the end of my talk, let me share with you the top 5 answers to the question. My remarks above may have provided some insights to some of the answers. Here they are.

Reason number one, "It is required for my major''--- meaning it is not my choice but I have to get it over with. I hope I have made the case why it is required, and perhaps, you and I can all approach it with a greater excitement and enthusiasm.

Reason number two, "I may help my children with their homework on calculus in twenty years.'' This one is surprising in one aspect, to say the least. I thought that many young people don't plan something so far into the future.

But it does bring up a point that is unique about the subject. Just ask ourselves what other subjects will be taught the same way in twenty, in one hundred, or in one thousand years from now? We used calculus to help Neil Armstrong to land on the moon, to send the Exploration Rover to Mars. A few decades from now, we will use it again to land man on Mars. Hundred and thousand years into the future, if mankind is to embark on a journey to another star system, which no one can predict with the absolute certainty. But suppose it is so. I can predict, with absolute certainty, that future astronautic engineers will use the same calculus that we teach and learn today to plan their journey. Calculus is truly a body of knowledge that will last as long as human civilization will.

Reason number three why study calculus: “It can help me to get rich since finance and economics use it a lot.’’ A good practical and motivating reason and it is true. Newton became fabulously rich after he attained his reputation due in no small a part to calculus. But for ordinary people like us, I don’t know quantitatively how much more you can make with calculus credits on your resume than without. It is an interesting statistical question that someone may have done the research on it for the answer. I hope your calculus experience will make you feel richer well before your first paycheck after college.

Number four, “I prefer it over a foreign language class for my GE requirement.” One point is important here. Calculus is a language and a tool. A language that many more foreigners speak well before they pick up a second foreign language. I was one of them, obviously --- the only time as a mathematician that I can safely use the word ‘obviously’ in a statement. My comment to this answer is why not take both?

Number five, “I like mathematics.” I like this one the best, but I wish it were the first choice for most students. I’m biased, as you can tell.

Finally, for those of you who will take calculus some day, my advice, in addition to that you must have heard from your grade school teachers, is that never stop asking the question “why?” The following point has been made many times by many people, and I want to pass it on to you if you have not heard it before. That is, millions saw the apple fall but Newton asked why, and the rest has been an unprecedented scientific and technological revolution in the history of mankind.

I would like to end my talk by quoting Brigham Young:

> *Every accomplishment, every polished grace, every useful attainment in mathematics, music, and in all sciences and art belong to the saints.*

Thank you.