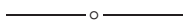


inequalities translate to  $(1 + \frac{1}{n})^n \leq e \leq (1 + \frac{1}{n})^{n+1}$  which is enough to conclude:

$$\bullet e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n.$$

This approach leaves out some details, but so do most treatments in modern introductory calculus texts. It is an honest approach in the sense that a rigorous treatment is available at an introductory analysis level, making it a reasonable alternative to typical presentations of  $e^x$ .



## Why It Might Seem That Christmas Is Coming Early This Year

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“Daddy, how long until Christmas?” That is the question that my kids seem to ask all through the year. When I was younger, it sometimes felt that time moved very slowly, especially when I was looking forward to something such as Christmas. Now that I am an adult, time has speeded up and it seems that each year passes more quickly than the last. Now asking myself, “Is it already Christmas again?” is a regular occurrence each winter. Using the fact that at age  $n$  the current year is  $1/n$  of one’s life, this note presents a model of passing time that could be used as source of classroom exercises.

Let us define  $D(a_1, a_2)$ , the perceived duration of life from age  $a_1$  to age  $a_2$  by

$$D(a_1, a_2) = \sum_{n=a_1+1}^{a_2} \frac{1}{n}.$$

Take the precarious teenage years (from the 13th birthday to the 20th) as an example. People’s seven teenage years may seemed to have lasted longer than their first seven years of adulthood. This can be quantified using the definition:

$$\frac{D(13, 20)}{D(20, 27)} = \frac{\sum_{n=14}^{20} 1/n}{\sum_{n=21}^{27} 1/n} = 1.42,$$

so the teenage years might seem to last 42% longer than the early adult years, even though both periods are of course the same length.

Since our lives are continuous and not discrete, the definition should be modified to

$$L(a_1, a_2) = \int_{a_1}^{a_2} \frac{1}{t} dt = \ln \left( \frac{a_2}{a_1} \right)$$

( $L$  for length). Again comparing the teenage to early adult years we have

$$\frac{L(13, 20)}{L(20, 27)} = \frac{\ln(20/13)}{\ln(27/20)} = 1.44$$

which, as expected, is approximately the same as in the discrete case.

Of course, the perception of time will obviously be different for each person, depending on a number of factors. Rather than using  $L$  as a strict prediction for one person, we can look at it as giving the average perception of time for all people, similar to how the normal high temperature for a given day is an average of previous years’ measurements of the high temperature on that day.

Here are some examples of the use of the definition. Suppose that for a current period of my life (from  $a_1$  to  $a_2$ ) I want to compute the age which is its perceptual midpoint. That is, I want to find  $a_m$  which satisfies  $L(a_1, a_m) = L(a_m, a_2)$ . We find that  $a_m = \sqrt{a_1 a_2}$ . If I use for  $a_1$  the age of my earliest memory (for me about 5), then I can find the age at which my life from then to my present age, 31.33 (31 years and 121 days) was, relatively speaking, at its halfway point:  $a_m = \sqrt{5 \cdot 31.33} = 12.52$ .

Suppose that I want to know when my life will reach its perceptual halfway point. If I assume that I'll live to age 80, and again using age 5 as my starting age, the halfway point in my life is exactly age 20. So, according to the model, my life is already more than half over. In fact,

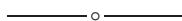
$$\frac{L(31.33, 80)}{L(5, 80)} = \frac{\ln(80/31.33)}{\ln(80/5)} = 0.34$$

so it might seem that I have only about a third of my life yet to live.

For a person of a certain age, the perceived length of the coming year relative to the perceived length of the previous year is

$$N(a) = \frac{L(a, a+1)}{L(a-1, a)} = \frac{\ln((a+1)/a)}{\ln(a/(a-1))}.$$

So, the day after Christmas, when my daughter poses the "How long until next Christmas?" question I can do more than tell her "One year." I can use our ages—she will be 5.21 and I will be 31.48—to find the answer: since  $N(5.21) = 0.82$  and  $N(31.48) = 0.97$ , I could tell her, "For you it will seem to take about 82% as long until next Christmas as it took to get to this Christmas and for me it will seem to take about 97% as long." Then she'll ask, "Daddy, what does 'percent' mean?"



## Sum Rearrangements

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Although it is known that the terms of a conditionally convergent series can be rearranged to form a series that converges to any given real number (see Rudin [3, p. 76]), in general it is difficult to find the sum of a given rearrangement. We will show that, for an entire class of series and type of rearrangement, the sum of the rearrangement is easy to find.

Let  $a$  and  $b$  be real numbers such that both  $a$  and  $a + b$  are positive, and consider the conditionally convergent series

$$\sum_{i=1}^{\infty} \frac{(-1)^{i+1}}{ai + b}.$$

Let  $\{s_n\}$  be its sequence of partial sums and let  $S$  be its sum. Let  $p$  and  $q$  be positive integers with  $p > q$ . Rearrange the series by taking the first  $p$  positive terms of the original series followed by the first  $q$  negative terms, then the next  $p$  positive terms, the next  $q$  negative terms, and so on. For example, with  $a = 2$ ,  $b = -1$ ,  $p = 3$ , and  $q = 2$ , the original series is