Let's Go to Wicked

A Problem in Counting and Probability

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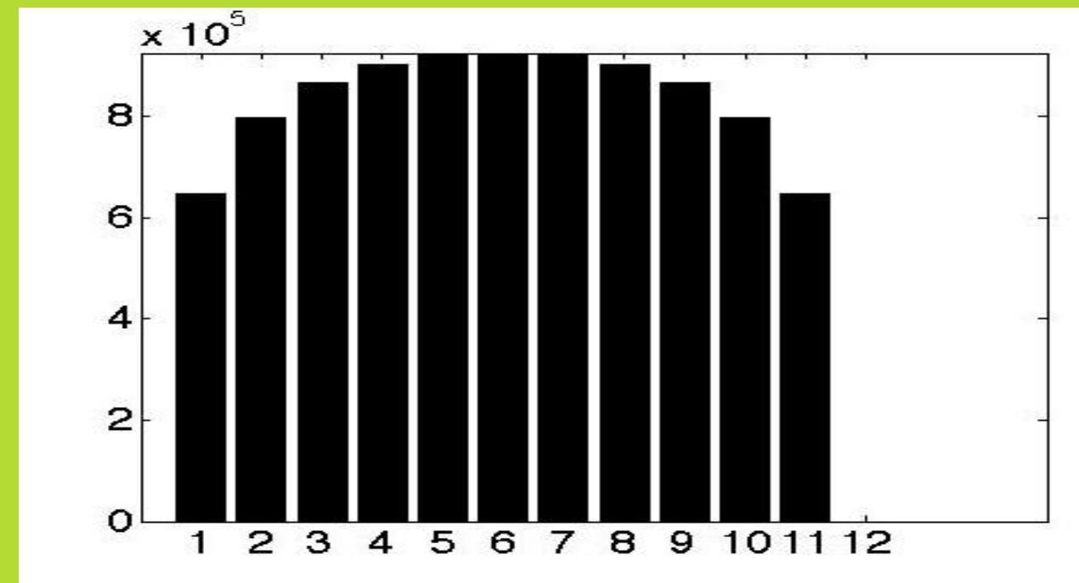
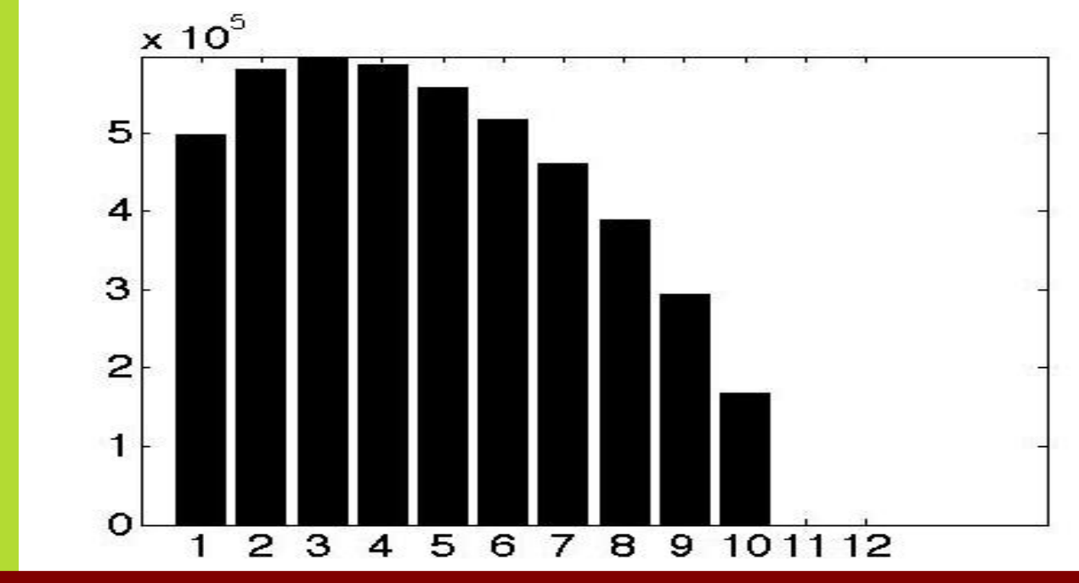
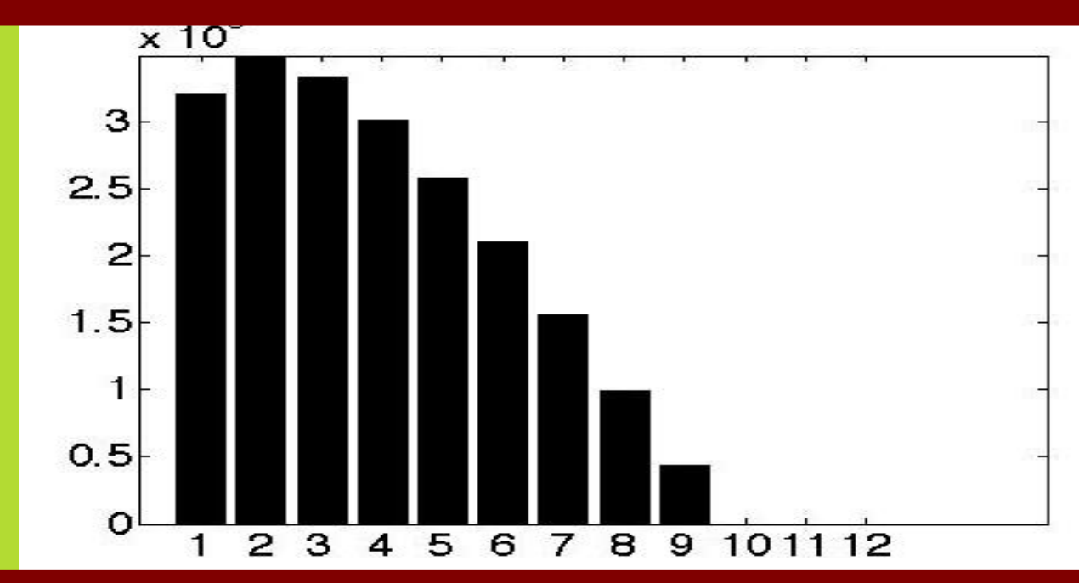
Graphs: Horizontal - Size of group joined
Vertical - Number of possible Secondary Wins



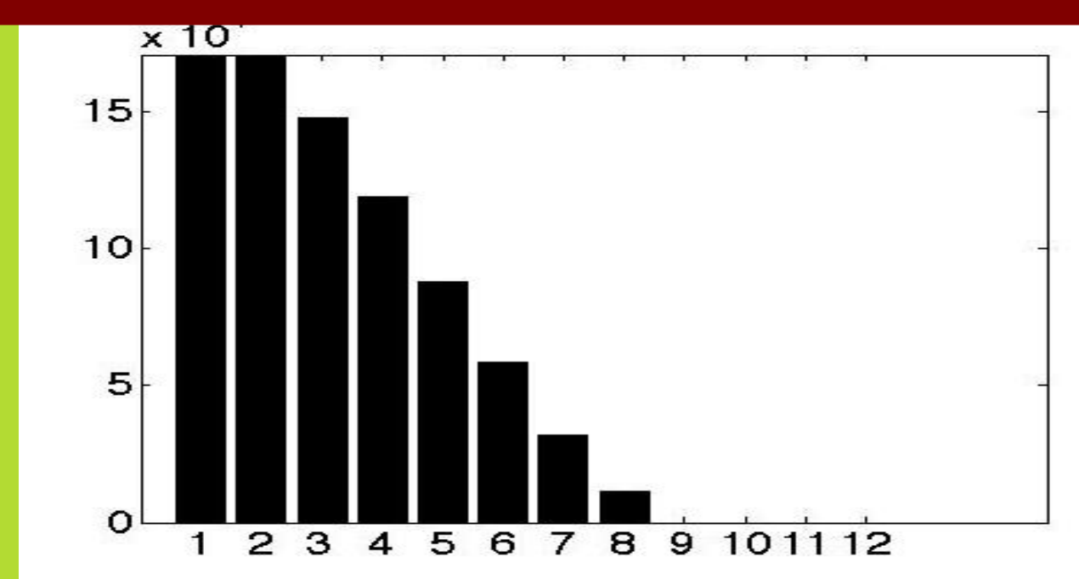
n = 12
t = 12
g = 6



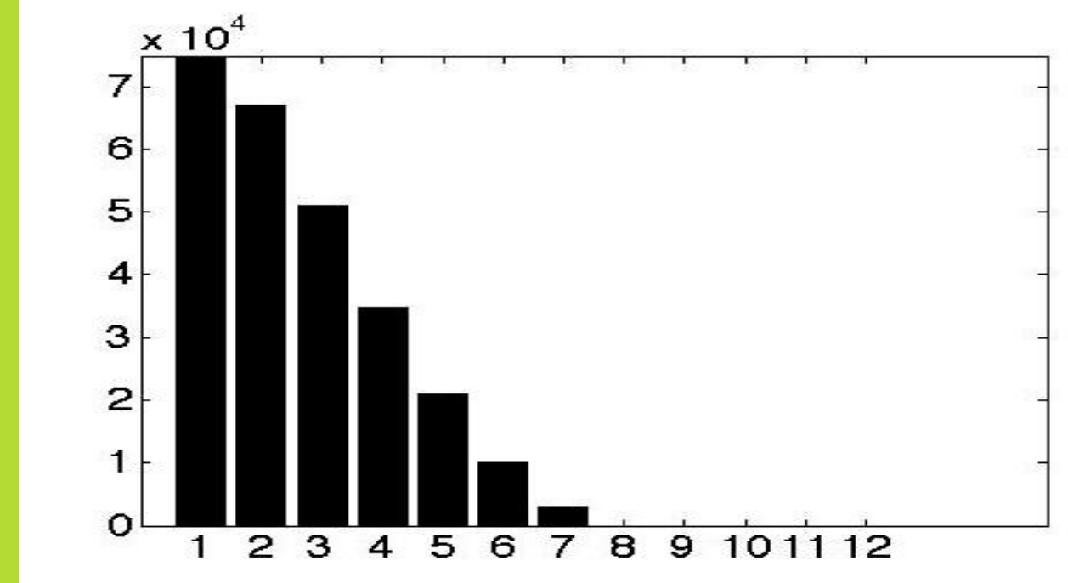
n = 12
t = 11
$g = \frac{1}{2} \cdot 6 = 3$



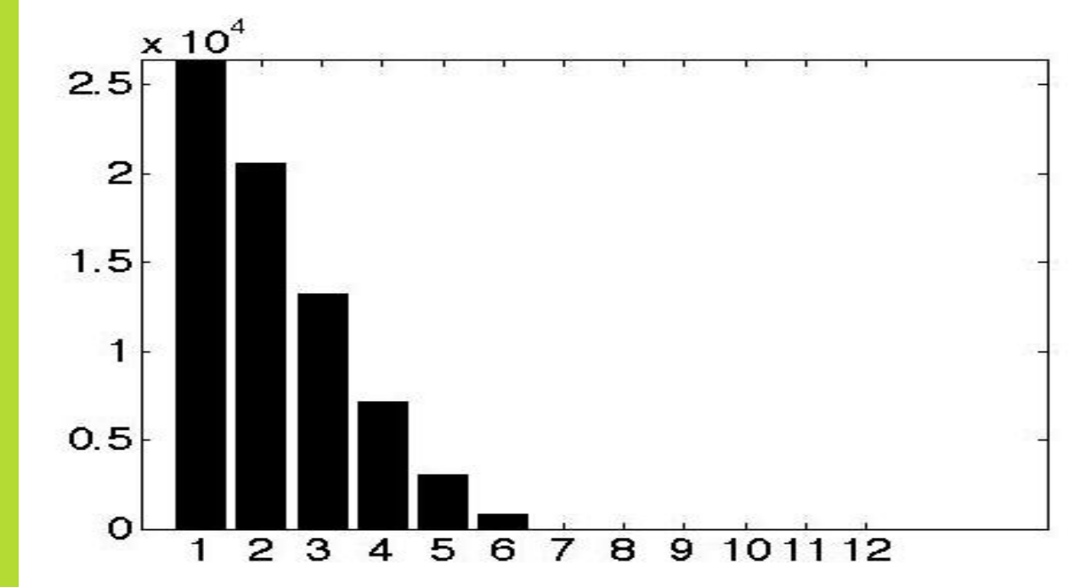
n = 12
t = 10
$g = \frac{1}{3} \cdot 6 = 2$



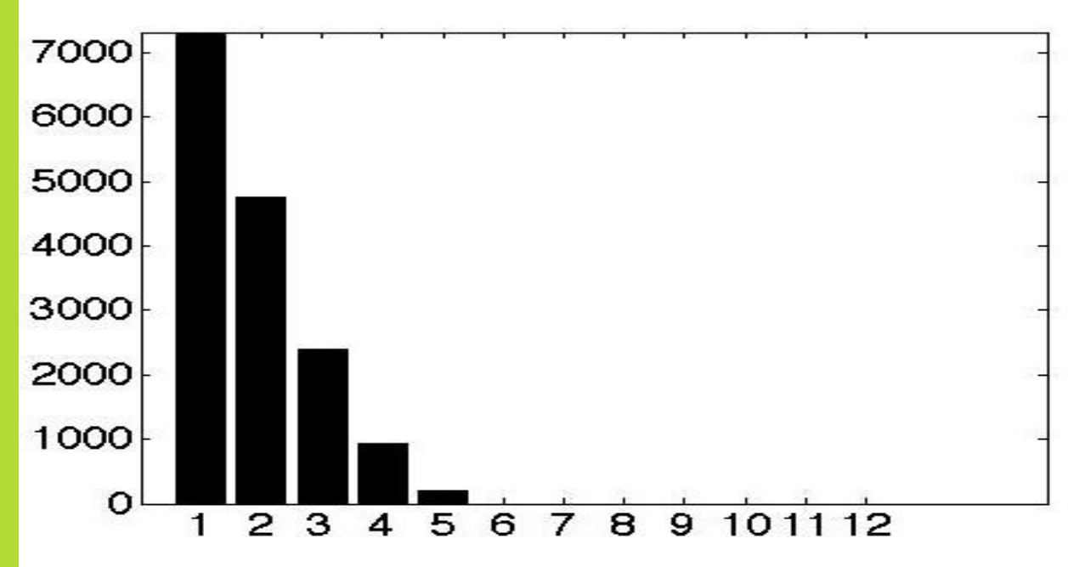
n = 12
t = 9
$g = \frac{1}{4} \cdot 6 = 1.5 \Rightarrow 1, 2$



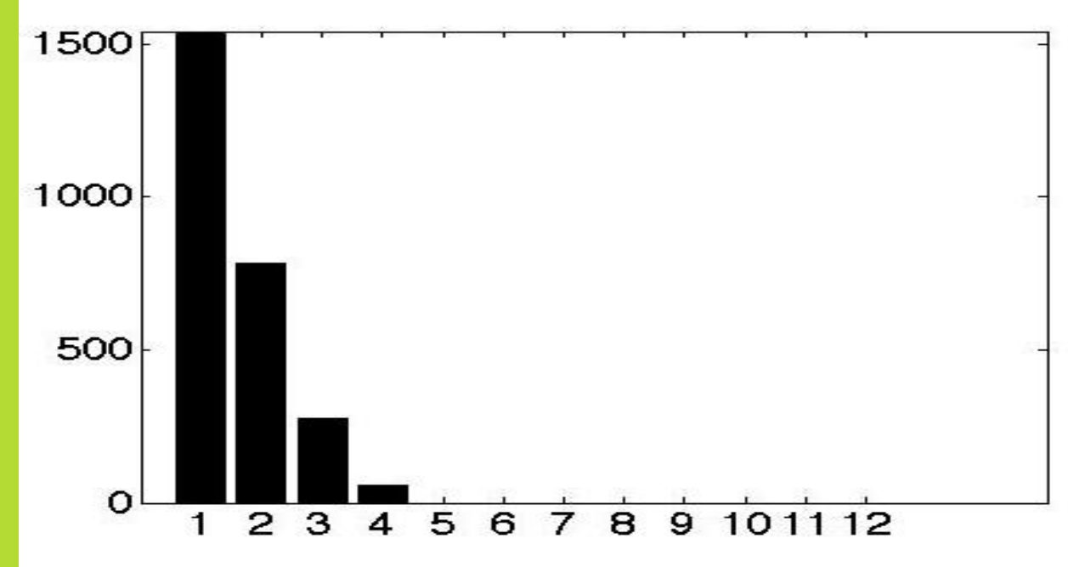
n = 12
t = 8
$g = \frac{1}{5} \cdot 6 = 1.2 \Rightarrow 1$



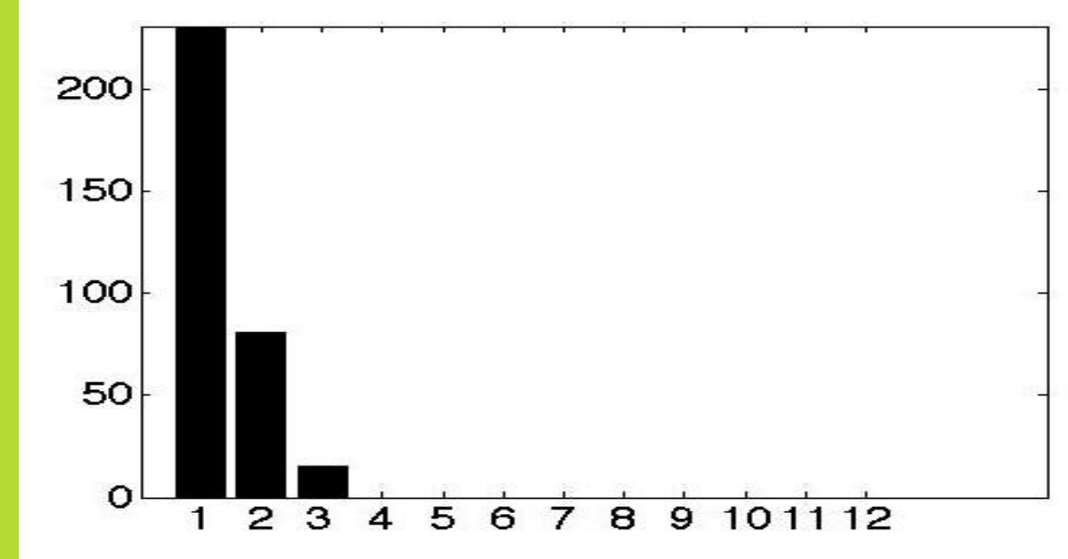
n = 12
t = 7
$g = \frac{1}{6} \cdot 6 = 1$



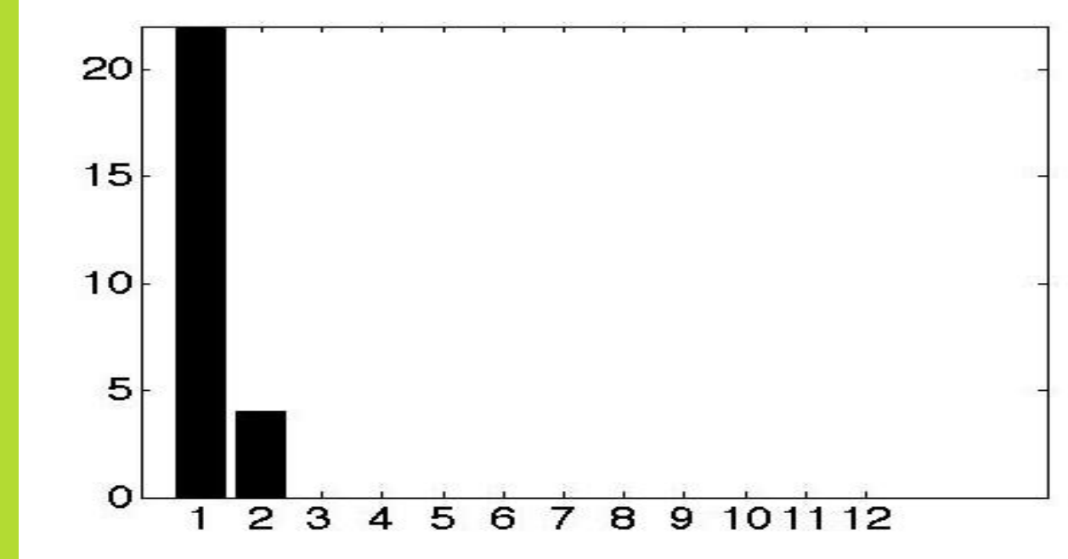
n = 12
t = 6
$g = \frac{1}{7} \cdot 6 = 0.857 \Rightarrow 1$



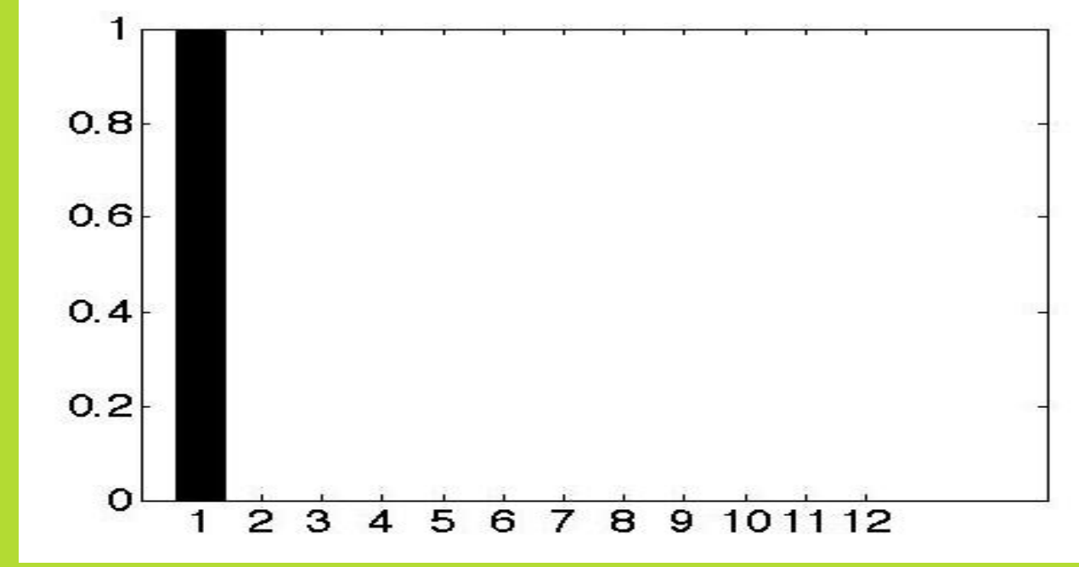
n = 12
t = 5
$g = \frac{1}{8} \cdot 6 = 0.75 \Rightarrow 1$



n = 12
t = 4
$g = \frac{1}{9} \cdot 6 = 0.667 \Rightarrow 1$



n = 12
t = 3
$g = \frac{1}{10} \cdot 6 = 0.6 \Rightarrow 1$



n = 12
t = 2
$g = \frac{1}{11} \cdot 6 = 0.545 \Rightarrow 1$

Introduction

Scenario: Every night before "Wicked" is performed in Hollywood, a nightly pre-show drawing is held for an opportunity to buy pairs of discounted front row seat tickets. Each person whose name is drawn receives a pair of tickets. Since it is possible for any group to win extra tickets (for example, 2 people could win 4 tickets if both names are drawn), it is advantageous for one group to team up with another group in order to potentially receive their extra tickets. So the question is, what is the optimal group size for us to team up with in order to maximize our chances of receiving extra tickets from that group?

Notation:

- n - the total number of pairs of people participating in the drawing (in addition to Amy and Anandi)
- t - the number of pairs of tickets given out
- g - the number of pairs of people in the group that Amy and Anandi join with

Observations and Comments:

- We consider the number of ways we could get tickets from the group we teamed with (Secondary Wins)
- We only consider the case that people come in pairs since tickets are given out in pairs
- We wrote a program in Matlab to do computations
- We found a pattern and a corresponding formula for predicting the pattern

Example

n = 12 pairs of other people (24 total)
t = 10 pairs of tickets given out
g = 1, 2, 3, 4, 5, 6, 7, 8, 9 pairs of people to team with

Group Size	Number of Possible Secondary Wins
1	$\binom{2}{2}\binom{22}{8} = 319,770$
2	$\binom{4}{3}\binom{20}{7} + \binom{4}{4}\binom{20}{6} = 348,840$
3	$\binom{6}{4}\binom{18}{6} + \binom{6}{5}\binom{18}{5} + \binom{6}{6}\binom{18}{4} = 332,928$
4	$\binom{8}{5}\binom{16}{5} + \binom{8}{6}\binom{16}{4} + \binom{8}{7}\binom{16}{3} + \binom{8}{8}\binom{16}{2} = 300,168$
5	$\binom{10}{6}\binom{14}{4} + \binom{10}{7}\binom{14}{3} + \binom{10}{8}\binom{14}{2} + \binom{10}{9}\binom{14}{1} + \binom{10}{10}\binom{14}{0} = 258,126$
6	$\binom{12}{7}\binom{12}{3} + \binom{12}{8}\binom{12}{2} + \binom{12}{9}\binom{12}{1} + \binom{12}{10}\binom{12}{0} = 209,616$
7	$\binom{14}{8}\binom{10}{2} + \binom{14}{9}\binom{10}{1} + \binom{14}{10}\binom{10}{0} = 156,156$
8	$\binom{16}{9}\binom{8}{1} + \binom{16}{10}\binom{8}{0} = 99,528$
9	$\binom{18}{10}\binom{6}{0} = 43,758$

General Equation and Conjecture

n = pairs of other people (2n total)
t = pairs of tickets being given out
g = 1, 2, 3, … , t pairs of people to team with

For a given n, t, and g, the number of possible secondary wins is:

$$\sum_{i=1}^{\min(t-g,\,g)} \binom{2g}{g+i}\binom{2n-2g}{t-(g+i)}$$

Maximizing the above sum for a given n and t, the optimal group size (in pairs) is:

$$g_{optimal} = \frac{1}{(n-t+1)} \bullet \frac{n}{2}$$

where g rounds to the nearest whole number unless the decimal part is exactly .5, in which case the optimal g is either one of the whole numbers below or above the fraction.

Pattern?

n = number of total pairs of people
t = number of pairs of tickets
(n,t) = optimal group size to join with

n

t

	1	2	3	4	5	6	7	8	9	10	11	12
1	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5
2		1	1	1	1	1	1	1	1	1	1	1
3			1.5	1	1	1	1	1	1	1	1	1
4				2	1	1	1	1	1	1	1	1
5					2.5	1.5	1	1	1	1	1	1
6						3	2	1	1	1	1	1
7							3.5	2	1.5	1	1	1
8								4	2	2	1	1
9									4.5	2.5	2	1.5
10										5	3	2
11											5.5	3
12												6

n

t

	1	2	3	4	5	6	7	8	9	10	11	12
1	.5	$\frac{1}{2}\bullet 1$	$\frac{1}{3}\bullet 1.5$	$\frac{1}{4}\bullet 2$	$\frac{1}{5}\bullet 2.5$	$\frac{1}{6}\bullet 3$	$\frac{1}{7}\bullet 3.5$	$\frac{1}{8}\bullet 4$	$\frac{1}{9}\bullet 4.5$	$\frac{1}{10}\bullet 5$	$\frac{1}{11}\bullet 5.5$	$\frac{1}{12}\bullet 6$
2		1	$\frac{1}{2}\bullet 1.5$	$\frac{1}{3}\bullet 2$	$\frac{1}{4}\bullet 2.5$	$\frac{1}{5}\bullet 3$	$\frac{1}{6}\bullet 3.5$	$\frac{1}{7}\bullet 4$	$\frac{1}{8}\bullet 4.5$	$\frac{1}{9}\bullet 5$	$\frac{1}{10}\bullet 5.5$	$\frac{1}{11}\bullet 6$
3			1.5	$\frac{1}{2}\bullet 2$	$\frac{1}{3}\bullet 2.5$	$\frac{1}{4}\bullet 3$	$\frac{1}{5}\bullet 3.5$	$\frac{1}{6}\bullet 4$	$\frac{1}{7}\bullet 4.5$	$\frac{1}{8}\bullet 5$	$\frac{1}{9}\bullet 5.5$	$\frac{1}{10}\bullet 6$
4				2	$\frac{1}{2}\bullet 2.5$	$\frac{1}{3}\bullet 3$	$\frac{1}{4}\bullet 3.5$	$\frac{1}{5}\bullet 4$	$\frac{1}{6}\bullet 4.5$	$\frac{1}{7}\bullet 5$	$\frac{1}{8}\bullet 5.5$	$\frac{1}{9}\bullet 6$
5					2.5	$\frac{1}{2}\bullet 3$	$\frac{1}{3}\bullet 3.5$	$\frac{1}{4}\bullet 4$	$\frac{1}{5}\bullet 4.5$	$\frac{1}{6}\bullet 5$	$\frac{1}{7}\bullet 5.5$	$\frac{1}{8}\bullet 6$
6						3	$\frac{1}{2}\bullet 3.5$	$\frac{1}{3}\bullet 4$	$\frac{1}{4}\bullet 4.5$	$\frac{1}{5}\bullet 5$	$\frac{1}{6}\bullet 5.5$	$\frac{1}{7}\bullet 6$
7							3.5	$\frac{1}{2}\bullet 4$	$\frac{1}{3}\bullet 4.5$	$\frac{1}{4}\bullet 5$	$\frac{1}{5}\bullet 5.5$	$\frac{1}{6}\bullet 6$
8								4	$\frac{1}{2}\bullet 4.5$	$\frac{1}{3}\bullet 5$	$\frac{1}{4}\bullet 5.5$	$\frac{1}{5}\bullet 6$
9									4.5	$\frac{1}{2}\bullet 5$	$\frac{1}{3}\bullet 5.5$	$\frac{1}{4}\bullet 6$
10										5	$\frac{1}{2}\bullet 5.5$	$\frac{1}{3}\bullet 6$
11											5.5	$\frac{1}{2}\bullet 6$
12												6

n

t

	1	2	3	4	5	6	7	8	9	10	11	12
1	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5
2		1	.75	.667	.625	.6	.583	.571	.563	.556	.55	.545
3			1.5	1	.833	.75	.7	.667	.643	.625	.611	.6
4				2	1.25	1	.875	.8	.75	.714	.688	.667
5					2.5	1.5	1.17	1	.9	.833	.786	.75
6						3	1.75	1.33	1.13	1	.917	.857
7							3.5	2	1.5	1.25	1.1	1
8								4	2.25	1.67	1.38	1.2
9									4.5	2.5	1.83	1.5
10										5	2.75	2
11											5.5	3
12												6