The Mathematics of Field Goal Kicking

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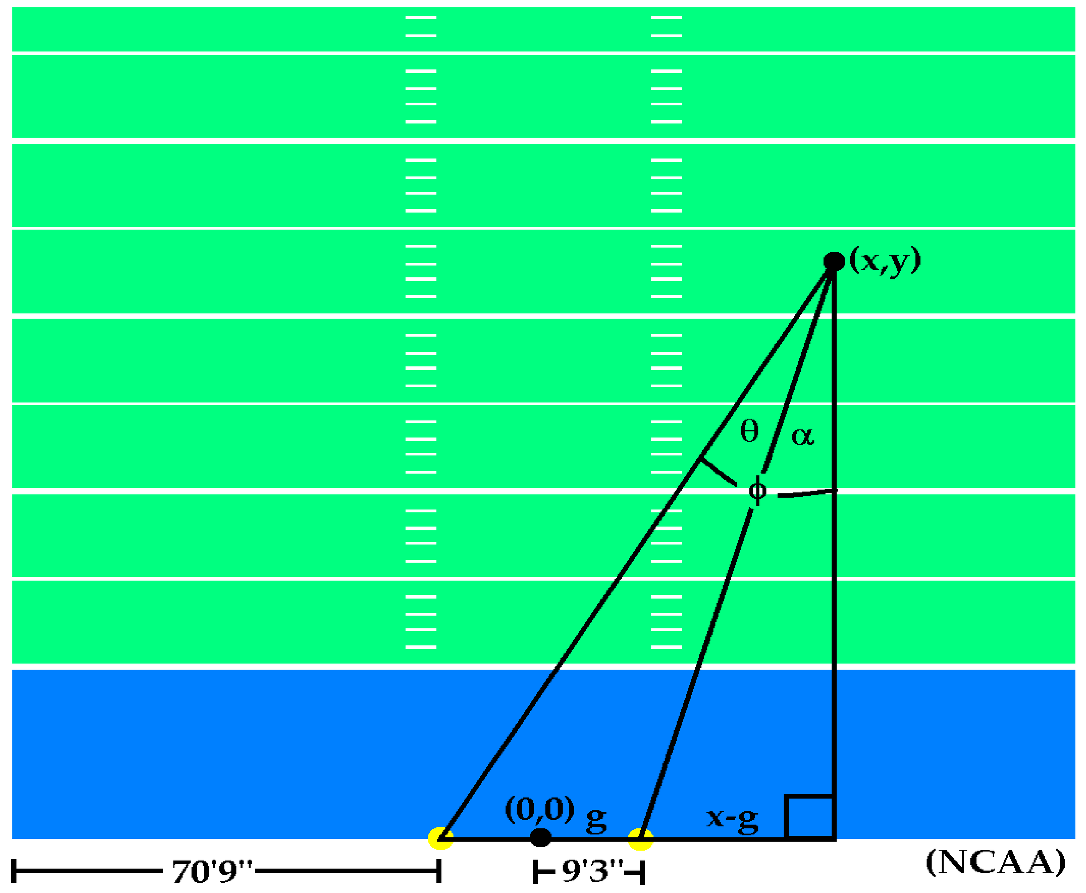


We find a simple formula for the kicking angle θ using the above diagram, where 2*g* is the width of the field goal posts, and *x* and *y* are (positive) horizontal and vertical distances from the center of the goal posts (0,0).

Where $\phi = \arctan\frac{x+g}{y}$ and $\alpha = \arctan\frac{x-g}{y}$

then $\theta = \phi - \alpha$:

$$\theta = \arctan\frac{x+g}{y} - \arctan\frac{x-g}{y} .$$

We find $\frac{\partial\theta}{\partial y} = -\frac{x+g}{y^2+(x+g)^2} + \frac{x-g}{y^2+(x-g)^2}$

and $\frac{\partial\theta}{\partial x} = \frac{y}{y^2+(x+g)^2} - \frac{y}{y^2+(x-g)^2}$.

It is not hard to show that $\frac{\partial\theta}{\partial x} < 0$ for all *x*, *y*,

while $\frac{\partial\theta}{\partial y} > 0$ for $y < \sqrt{x^2-g^2}$,

$\frac{\partial\theta}{\partial y} = 0$ for $y = \sqrt{x^2-g^2}$,

and $\frac{\partial\theta}{\partial y} < 0$ for $y > \sqrt{x^2-g^2}$.

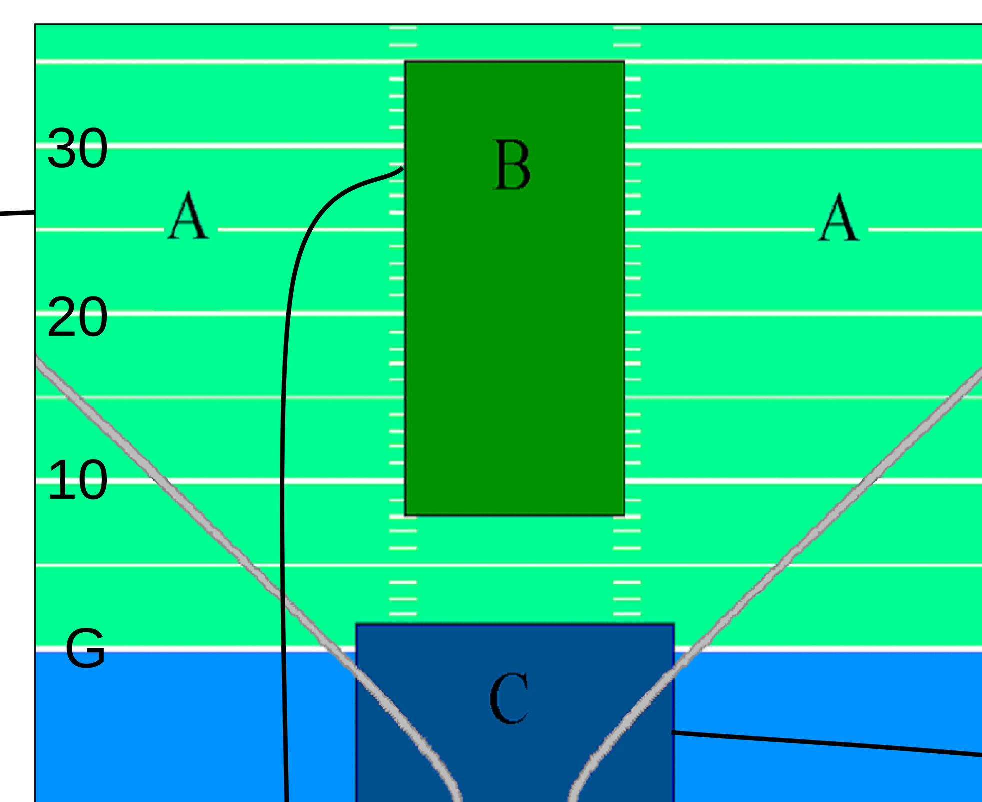
$\frac{\partial\theta}{\partial y} < 0$ Moving *toward* endzone increases kicking angle.

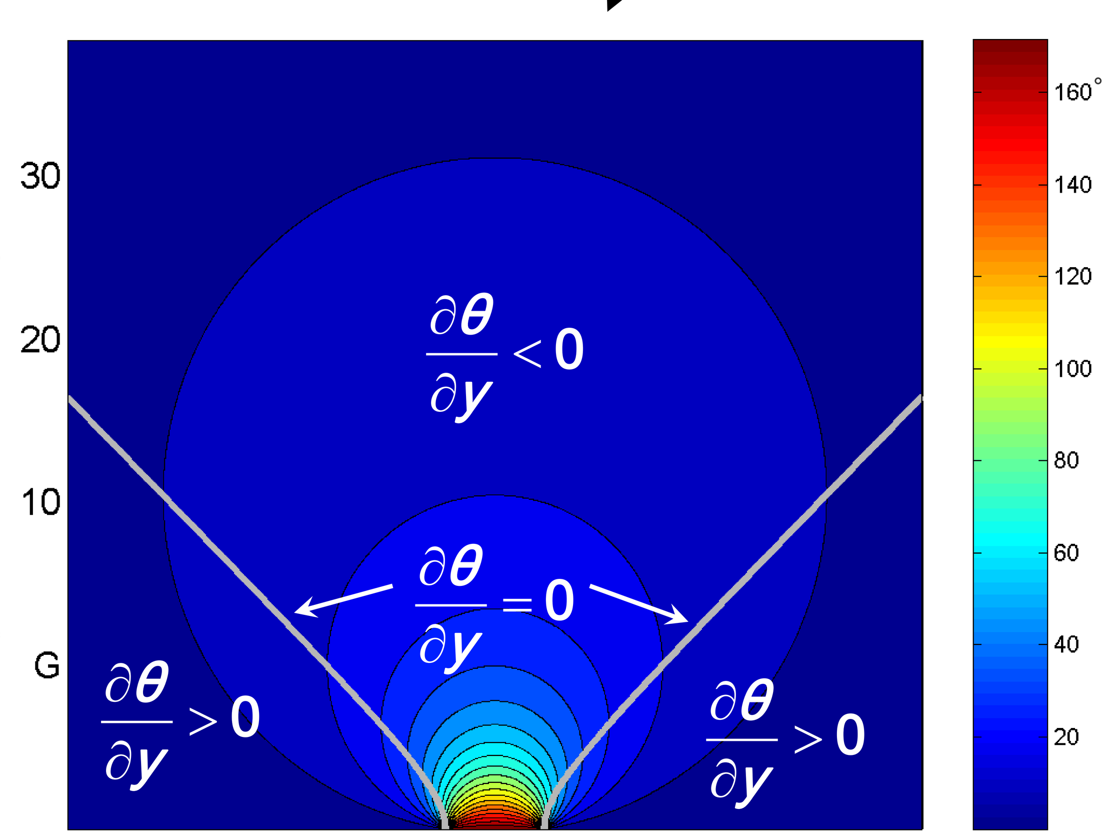
$\frac{\partial\theta}{\partial y} > 0$ Moving *away* from endzone increases kicking angle.

The difficulty level of kicking a field goal is determined primarily by the kicking angle. Intuitively one might expect that the smaller the distance to the goal posts, the greater the angle, but do certain regions of the playing field provide exceptions to this generality? In certain regions of the field could it actually be beneficial to move away from the endzone in order to increase the kicking angle, and do these regions overlap with kicking regions in any organized football leagues?

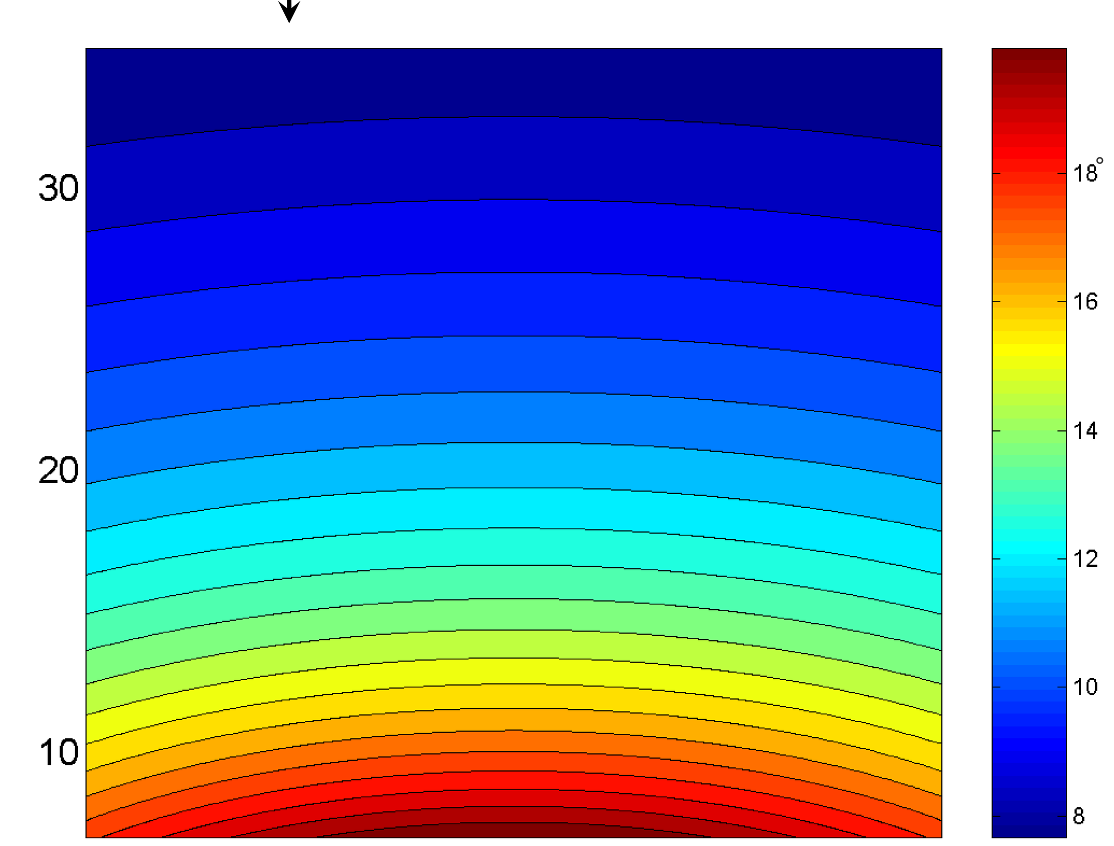
Three regions of interest:

- The entire field (A)
- The approximate range of an NCAA/NFL kicker (B)
- Inside the endzone (C)

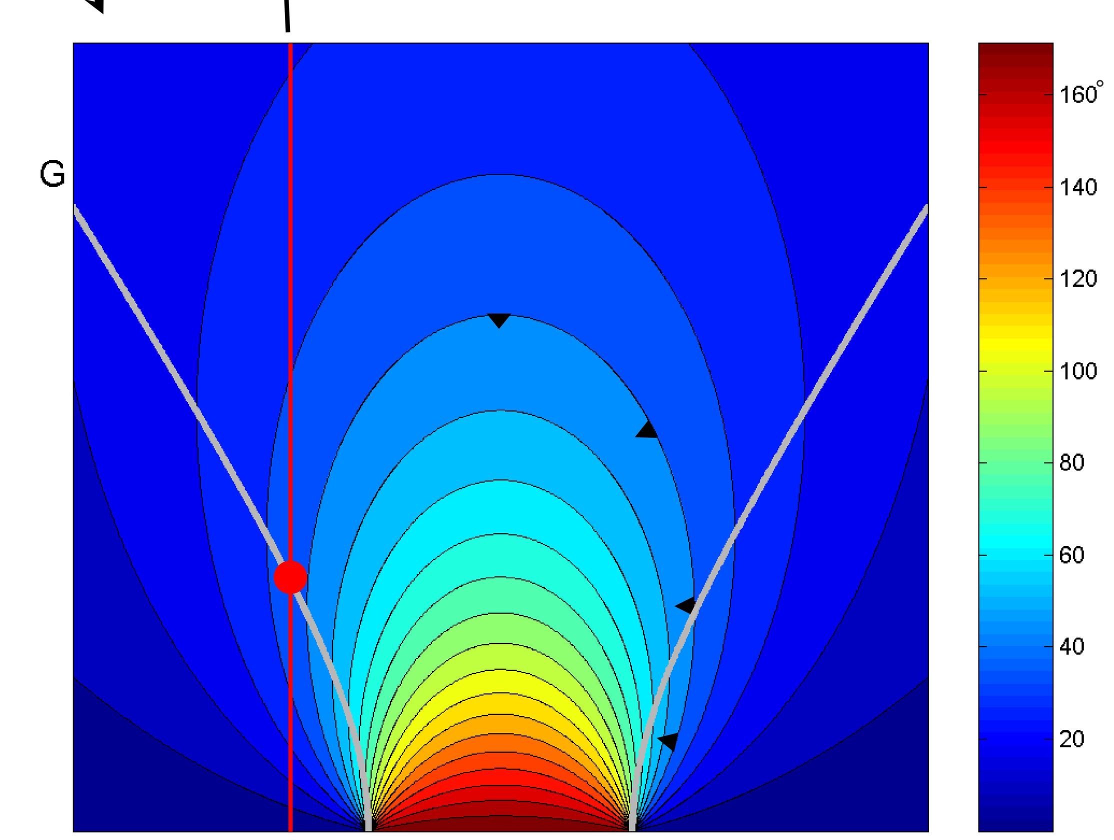




Region A: The entire field.

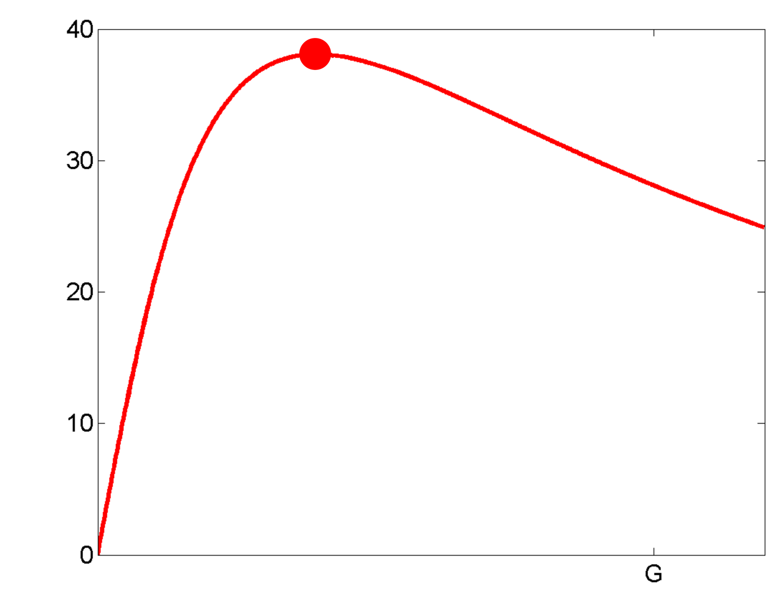


Region B: The feasible range of NCAA field goals (region within the hash marks) — the kicking angle always *increases* with *decreased* distance to goal posts.



Region C: The endzone.

Here there are clear examples of locations at which it is beneficial to move back to improve kicking angle. Movement perpendicular to the contour lines results in the greatest increase in the kicking angle (see arrows in contour plot).



This graph plots the kicking angle as a function of vertical distance *y* from the goal post where horizontal distance from the goal post *x* is fixed. It corresponds to the red line below for which *x* = 15 feet. Notice it reaches its maximum exactly when it intersects the gray curve below:

$y = \sqrt{x^2-g^2}$ which is where $\frac{\partial\theta}{\partial y} = 0$.

Conclusions:

It has been proven that there are regions on the field in which moving away from the endzone increases kicking angle. However, these regions do not overlap with the range of field goal attempts on NCAA or NFL football fields, as seen in the figures below.

In relation to to other sports such as soccer or hockey, these points represent regions for which a backward pass or horizontal cross is more advisable than a shot on goal in order to improve shooting angle. This idea could be used to program computer decisions for a soccer or hockey computer game.

Also, this solution, with its employment of trigonometric identities and partial derivatives, could serve as an appropriate exercise for a calculus student or for someone learning to use mathematical software such as Matlab.