

DO NOT OPEN THIS EXAM UNTIL TOLD TO DO SO.

Pepperdine Math Day

October 29, 2011

Exam Instructions and Rules

1. **Write the following information on your Scantron form:**

Name in NAME box

Grade in SUBJECT box

School name in DATE box (and into PERIOD box, if necessary)

On the back of your Scantron form, write this same information on the first line of the green shaded area.

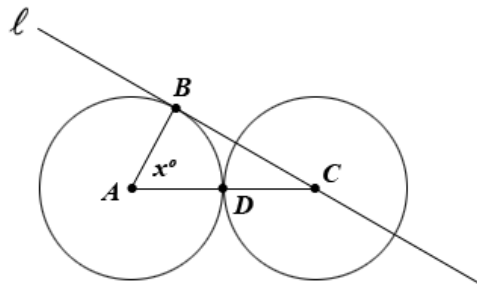
2. **This exam will last 90 minutes.** It is a **45 question** multiple choice exam. Each question is followed by answers marked A, B, C, D and E. Exactly one answer is correct for each problem. You will use the first 45 spots on the front page of the scantron form to record your answers. **Your answer to the tie-breaker question should be written on the backside of the Scantron form below your name, etc. in the green shaded area.** Your answer to this problem does not count toward your score—it will be used only for tie-breaking.
3. On this exam, **there is no penalty for incorrect answers**, so it is to your advantage to put an answer for each question, especially if you are able to eliminate one or more of the answers as incorrect. Credit will be given only for answers on your scantron form, not for any work written on the exam itself.
4. **Use a number 2 pencil to mark your answer.** Be sure to completely darken each of your penciled-in answers. Extra pencils are available from proctors.
5. There should be enough space between problems (or the backside of pages) to work your solutions. Credit is given only for answers on your Scantron answer form, not for any work written on the exam or scratch paper.
6. Figures are not necessarily drawn to scale.
7. While we certainly don't expect it, any sort of cheating will be dealt with at the discretion of the proctors, and will likely include at least disqualification.

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1. If 3^{2011} is divided by 5, what is the remainder?

- (a) 0 (b) 1 (c) 2 (d) 3 (e) 4

2. In the figure below, the circle with center A and the circle with center C are tangent at point D . If the circles each have radius 10, and if line ℓ is tangent to the circle with center A at point B , what is the value of angle x (in degrees)?



- (a) 55 (b) 60 (c) 63 (d) 65
(e) It cannot be determined from the information given.

3. What is the largest value of $|x + 3| - |x - 3| - |2x - 3|$ for $-5 \leq x \leq 5$?

- (a) 3 (b) 6 (c) 12 (d) 17 (e) None of (a) – (d)

4. Each of 5 men played one game of chess with each of 5 women, and then each woman played one game of chess with each of the other women. How many games of chess were played?

- (a) 20 (b) 25 (c) 35 (d) 45 (e) 50

5. The two cities Albany and Buffalo are 288 miles apart. Suppose that you travel the first 144 miles from Albany to Buffalo at 100 mph and the second 144 miles at x mph, and then return at a speed of 300 mph for the entire 288 miles back. Approximately what would the speed x need to be in order for the average speed for the entire trip to be 200 mph?

- (a) 150 mph (b) 167 mph (c) 200 mph (d) 217 mph (e) None of (a) – (d)

6. If 84 inches of wire are used to build the skeleton (i.e the frame) of a cube, what is the surface area of the cube, in square inches?

- (a) 7 (b) 42 (c) 84 (d) 294 (e) 483

7. How many different words (including nonsense words) can be made of the letters in BEACHES if all seven letters are used for each word?

- (a) 49 (b) 128 (c) 2520 (d) 5,040 (e) 823,543

8. Seven cards in a pile are numbered 1 through 7. One card is drawn. The units digit of the *sum* of the numbers on the remaining cards is 7. What is the number on the drawn card?

- (a) 1 (b) 3 (c) 5 (d) 6 (e) 7

9. Suppose at a certain bank your investment earns 1% at the end of each month. Rounded to the nearest percent, how much would your investment increase after 12 months of this interest (assuming the interest is compounded each month)?

- (a) 12% (b) 13% (c) 14% (d) 15% (e) 16%

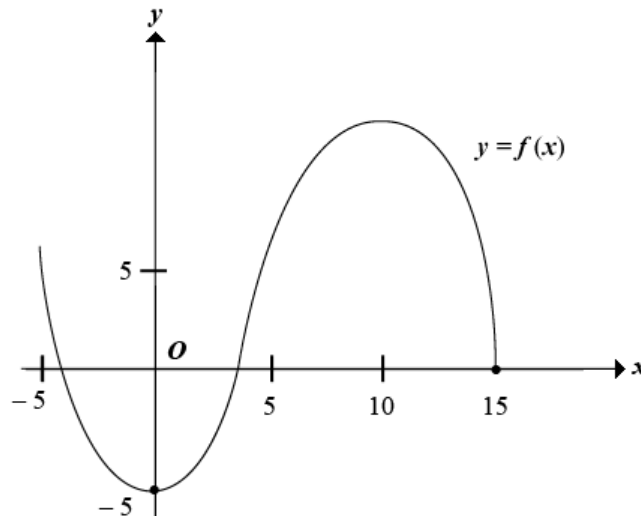
10. At how many values of x are the functions $y = \sin x$ and $y = \frac{x}{2011}$ equal?
 (Assume x is in radians.)

- (a) 638 (b) 639 (c) 640 (d) 1,279 (e) 2,011

11. The addition below is incorrect. We want to change just one digit to make the expression correct. There is more than one way to accomplish this. What is the value of the largest digit that can be changed to make the addition correct?

$$\begin{array}{r} 641 \\ 852 \\ + 973 \\ \hline 2456 \end{array}$$

- (a) 4 (b) 5 (c) 6 (d) 7 (e) 8



12. The function f is shown in the figure above. If $g(x) = |f(x)| - 4$, for how many values of x between -5 and 15 is $g(x) = 0$?

- (a) 0 (b) 1 (c) 2 (d) 3 (e) 4 or more

13. Roger was supposed to subtract 3 from a certain number and then divide the result by 9. He instead subtracted 9 and then divided the result by 3 and got an answer of 43. Would have his answer been had she worked the problem correctly?

- (a) 15 (b) 34 (c) 43 (d) 51 (e) 138

14. A person bought 12 cards for 30 cents. If the next day the price of the cards was 5 cents each, how much did the person save per card by buying at the earlier price?

- (a) 2 cents (b) $2\frac{1}{2}$ cents (c) 3 cents (d) $3\frac{1}{2}$ cents (e) 5 cents

15. The *sum*, *product*, and *average* (arithmetic mean) of four integers are all equal. This information uniquely determines exactly how many of the four integers?

- (a) 0 (b) 1 (c) 2 (d) 3 (e) 4

16. Ten cars containing a total of 32 persons passed through a checkpoint. If none of these cars contained more than 4 persons, what is the greatest number of these cars that could have contained exactly 2 persons?

- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

17. If $3 < x < 7$ and $4 < y < 6$, which of the following best describes the range of values of $x - y$?

- (a) $-3 < x - y < 1$ (b) $-3 < x - y < 3$ (c) $-1 < x - y < 1$
(d) $-1 < x - y < 3$ (e) $7 < x - y < 13$

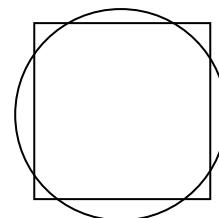
18. A standard die (with six faces numbered 1 through 6) will be rolled twice. What is the probability that the number that comes up on the first roll will be less than the number that comes up on the second roll?

- (a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{5}{12}$ (d) $\frac{7}{12}$ (e) $\frac{1}{2}$

19. Suppose that an electric fan is rotating at 8 revolutions per second when it is switched off and that it continually loses 20% of its speed with each ensuing second. The total number of revolutions the fan blade makes after being turned off until coming to rest is in which of the following intervals?

- (a) 0 to 10 (b) 10 to 20 (c) 20 to 30 (d) 30 to 40 (e) 40 or more

20. A square with side length 2 and a circle share the same center. The total area of the regions that are inside the circle and outside the square is equal to the total area of the regions that are outside the circle and inside the square. What is the radius of the circle?



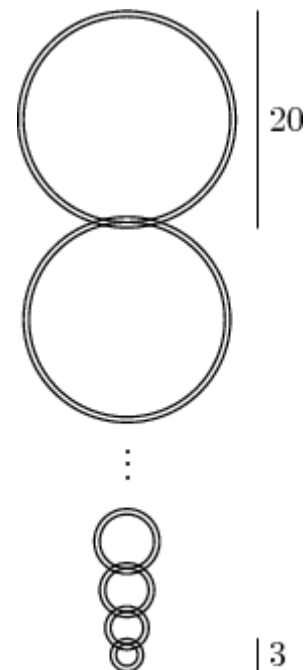
- (a) $\frac{2}{\sqrt{\pi}}$ (b) $\frac{1+\sqrt{2}}{2}$ (c) $\frac{3}{2}$ (d) $\sqrt{3}$ (e) $\sqrt{\pi}$

21. Four distinct lines lie in a plane, and exactly two of them are parallel. Which of the following could be the number of points where at least two of the lines intersect?

- I. Three
II. Four
III. Five

- (a) I only (b) III only (c) I and II only (d) I, II and III (e) I and III only

22. A number of linked rings, each 1 cm thick, are hanging on a peg. The top ring has an outside diameter of 20 cm. The outside diameter of each of the outer rings is 1 cm less than that of the ring above it. The bottom ring has an outside diameter of 3 cm. What is the distance, in cm, from the top of the top ring to the bottom of the bottom ring?



- (a) 171 (b) 173 (c) 182
 (d) 188 (e) 210

23. Let x be the smallest number that is divisible by 6, 10, 15 and 70. What is the sum of the digits of x ?

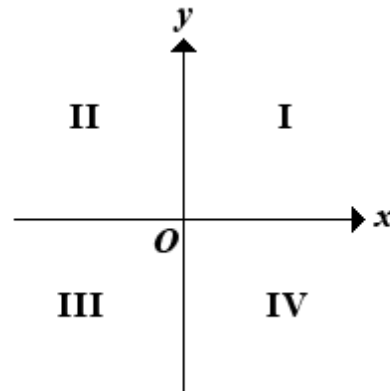
- (a) 3 (b) 4 (c) 6 (d) 9 (e) 10

24. If $x^{-2/3} = 9$, what is the value of $x^{-3/2}$?

- (a) $\frac{1}{81}$ (b) $\frac{1}{27\sqrt{3}}$ (c) $27\sqrt{3}$ (d) 81 (e) $81\sqrt{3}$

25. In the figure at right, which quadrants could contain pairs (x, y) that satisfy the condition $\frac{x}{y} = \frac{3}{7}$?

- (a) I only
(b) I and II only
(c) I and III only
(d) II and IV only
(e) I, II, III and IV



26. For how many values of n from 1 to 1000 is

$$\sum_{i=1}^n i = 1 + 2 + \cdots + n$$

divisible by 5?

- (a) 0 (b) 100 (c) 200 (d) 400 (e) 1000
27. In the *Harry Potter* series, 1 galleon is worth 17 sickles, and 1 sickle is worth 29 knuts. At one time the author of the books mentioned that a book which sells for \$4.00 is worth 14 sickles and 3 knuts. How much (to the nearest cent) is 1 galleon worth in dollars?
- (a) \$4.63 (b) \$4.82 (c) \$4.95 (d) \$5.13 (e) \$5.26

28. $f(x) = x^3 + x - 9$ has one zero on the interval $[0, 5]$. In which of the following intervals is the zero located?

- (a) $[0, 1)$ (b) $[1, 2)$ (c) $[2, 3)$ (d) $[3, 4)$ (e) $[4, 5]$

29. Which of the following numbers is *smallest*?

- (a) $(2010!)^{2012}$ (b) $(2011!)^{2011}$ (c) $(2012!)^{2010}$
(d) $(2011)!$ (e) $(2011)^{2011!}$

30. If $\frac{14x-3y}{x+2y} = 4$, what is the value of $\frac{x+y}{x-y}$?

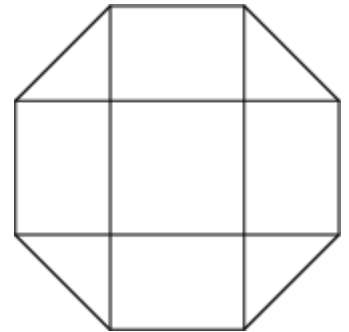
- (a) 12 (b) 18 (c) 20 (d) 21
(e) It cannot be determined from the information given.

31. When a clock (with minute and hour hands) shows the time 4:18, what is the approximate measure (in degrees) of the acute angle between the hour and minute hands?
- (a) 15 (b) 18 (c) 21 (d) 24 (e) None of (a) – (d)
32. If $a + b = 2$ and $a^3 + b^3 = 2$, what is the value of $a(a + b)b$?
- (a) -4 (b) -1 (c) 1 (d) 2
- (e) It cannot be determined from the given information.
33. Keiko walks once around a track at exactly the same constant speed every day. The sides of the track are straight, and the ends are semicircles. The track has width (from outside edge to inside edge) of 6 meters, and it takes her 36 seconds longer to walk around the outside edge of the track than around the inside edge. What is Keiko's speed in meters per second?
- (a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) π (d) $\frac{4\pi}{3}$ (e) $\frac{5\pi}{3}$

34. Rectangle $ABCD$ has $AB = 6$ and $BC = 3$. Point M is chosen on side AB so that $\angle AMD = \angle CMD$. What is the degree measure of $\angle AMD$?

- (a) 15 (b) 30 (c) 45 (d) 60 (e) 75

35. A dart board is a regular octagon divided into regions as shown at right. Suppose that a dart thrown at the board is equally likely to land anywhere on the board. What is the probability that the dart lands within the center square?



- (a) $\frac{\sqrt{2}-1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{2-\sqrt{2}}{2}$ (d) $\frac{\sqrt{2}}{4}$ (e) $2 - \sqrt{2}$

36. A pyramid has a square base with side of length 1 and has lateral faces that are equilateral triangles. A cube is placed within the pyramid so that one face is on the base of the pyramid and its opposite face has all its edges on the lateral faces of the pyramid. What is the volume of this cube?

- (a) $5\sqrt{2} - 7$ (b) $7 - 4\sqrt{3}$ (c) $\frac{2\sqrt{2}}{27}$ (d) $\frac{\sqrt{2}}{9}$ (e) $\frac{\sqrt{3}}{9}$

37. Let

$$f(x) = 10^{10x}$$

$$g(x) = \log_{10} \left(\frac{x}{10} \right)$$

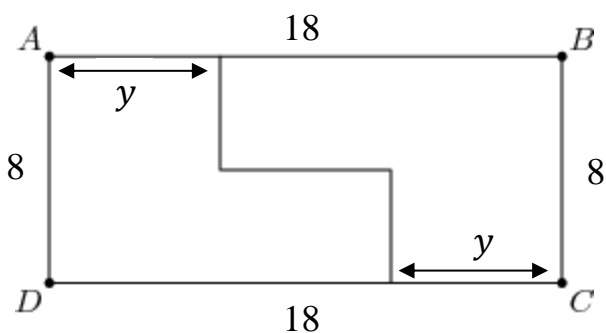
$$h_1(x) = g(f(x))$$

$$h_n(x) = h_1(h_{n-1}(x))$$

for integers $n \geq 2$. What is the sum of the digits of $h_{2011}(1)$?

- (a) 16,081 (b) 16,089 (c) 18,089 (d) 18,098 (e) 18,099

38. An 8×18 rectangle is cut into two congruent hexagons, as shown, in such a way that the two hexagons can be repositioned without overlap to form a square. What is y ?



- (a) 6 (b) 7 (c) 8 (d) 9 (e) 10

39. The square of the nine-digit number 987,654,321 is an n -digit number. What is the value of n ?

- (a) 16 (b) 17 (c) 18 (d) 19 (e) 20

40. Mary is 20% older than Sally, and Sally is 40% younger than Danielle. The sum of their ages is 23.2 years. How old will Mary be on her next birthday?

- (a) 7 (b) 8 (c) 9 (d) 10 (e) 11

41. How many real values of x are there for which $\sqrt{120 - \sqrt{x}}$ is an integer?

- (a) 11 (b) 119 (c) 120 (d) 121 (e) Infinitely

42. If $a \uparrow b$ means a^b and $a \downarrow b$ means $\sqrt[b]{a}$, what is the value of $[(2 \uparrow 6) \downarrow 3] \uparrow 2$?

- (a) 10 (b) 12 (c) 14 (d) 16 (e) 20

43. If A , B and C are the angles of a right triangle, what is the numerical value of $\sin^2 A + \sin^2 B + \sin^2 C$?

- (a) 0 (b) 1 (c) $\sqrt{2}$ (d) 2
(e) The value is not uniquely determined by the given information.

44. If

$$x = 3 + 6 + 9 + 12 + \dots + 291 + 294$$

and

$$y = 4 + 8 + 12 + 16 + \dots + 388 + 392$$

what is the value of $\frac{x}{y}$ expressed in simplest form?

- (a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$ (e) $\frac{4}{5}$

45. If $3^{33} + 2 \cdot 9^{17} + 2 \cdot 27^{11} = 3^x$, what is x ?

- (a) 12 (b) 34 (c) 35 (d) 36 (e) 99

Tie-breaker Question (free response)

TB. The expression $(x + y + z)^{2011} + (x - y - z)^{2011}$ is simplified by expanding it and combining like terms. How many terms are in the simplified expression?

1,012,036