DO NOT OPEN THIS EXAM UNTIL TOLD TO DO SO.

Pepperdine Math Day

October 2, 2010

Exam Instructions and Rules

1. **Write the following information on your Scantron form:**
   Name in NAME box
   Grade in SUBJECT box
   School name in DATE box (and into PERIOD box, if necessary)
   On the back of your Scantron form, write this same information on the first line of the green shaded area.

2. **This exam will last 90 minutes**. It is a **45 question** multiple choice exam. Each question is followed by answers marked A, B, C, D and E. Exactly one answer is correct for each problem. You will use the first 45 spots on the front page of the scantron form to record your answers. **Your answer to the tie-breaker question should be written on the backside of the Scantron form below your name, etc. in the green shaded area.** Your answer to this problem does not count toward your score—it will be used only for tie-breaking.

3. On this exam, **there is no penalty for incorrect answers**, so it is to your advantage to put an answer for each question, especially if you are able to eliminate one or more of the answers as incorrect. Credit will be given only for answers on your scantron form, not for any work written on the exam itself.

4. **Use a number 2 pencil to mark your answer.** Be sure to completely darken each of your penciled-in answers. Extra pencils are available from proctors.

5. There should be enough space between problems to work your solutions (or use the backsides of the exam sheets). Credit is given only for answers on your Scantron answer form, not for any work written on the exam.

6. Figures are not necessarily drawn to scale.

7. While we certainly don't expect it, any sort of cheating will be dealt with at the discretion of the proctors, and will likely include at least disqualification.

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1. If $s = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}$ and $t = 1 + \frac{1}{2}s$, then $t - s =$

(a) $\frac{1}{16}$ (b) $\frac{1}{32}$ (c) $\frac{1}{64}$ (d) $\frac{1}{128}$ (e) $\frac{1}{256}$

2. If it is now 10 a.m., then in 2010 hours it will be (disregard Daylight Savings)?

(a) 4 a.m. (b) 6 a.m. (c) 2 p.m. (d) 4 p.m. (e) 6 p.m.

3. Peter stopped at his favorite store to buy a hamburger for \$2. Then he spent half of what he had left plus \$3 for a drink. Then he stopped at his favorite baker and bought cupcakes, and spent half of what he had left plus \$4. At this point he had \$2 remaining. How much money did Peter have to begin with?

(a) 18 (b) 24 (c) 26 (d) 28 (e) 32

4. If a, b and c are numbers such that $\frac{a}{b} = 3$ and $\frac{b}{c} = 7$, then $\frac{a+b}{b+c} =$

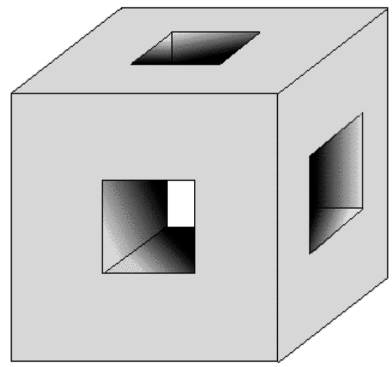
(a) $\frac{3}{7}$ (b) $\frac{7}{8}$ (c) $\frac{7}{2}$ (d) $\frac{1}{7}$

(e) It cannot be determined from the given information.

5. How many real values which satisfy the expression $|x| - |x+1| = 0$?

(a) 0 (b) 1 (c) 2 (d) 3 (e) More than 3.

6. In the given figure, a wood cube has edges of length 3. Square holes, each of width 1, centered in each face are cut through to the opposite face. The edges of the holes are parallel to the edges of the cube. The entire surface area, including the inside, is:



(a) 72 (b) 73 (c) 76

(d) 77 (e) 78

7. A *triangular number* is a number of the form $T_n = 1 + 2 + \cdots + n$. Which expression is equivalent to $T_n + T_{n+1}$?

(a) $n^2 + 2n + 1$ (b) $n^2 + n$ (c) $\frac{n(n+1)(n+2)}{2}$

(d) $\frac{n^2+n}{2}$ (e) None of (a) – (d)

8. The Golden Ratio is a positive number φ that satisfies $\varphi = 1 + 1/\varphi$. Which of the following is an equivalent expression for φ?

(a) $\frac{1+\sqrt{5}}{2}$ (b) $1 + \frac{1}{1+\frac{1}{1+\frac{1}{1+\cdots}}}$ (c) e

(d) Both (a) and (b) (e) All of (a), (b) and (c)

9. What is the probability that a randomly chosen real number does NOT have the digit 4 appearing in its decimal expansion?

(a) 0 (b) $\frac{1}{10}$ (c) $\frac{1}{9}$ (d) $\frac{4}{10}$ (e) None of (a) – (d)

10. How many integers between 1 and 2010 have either a 2 or 9 (or both) as one of its digits?

(a) 688 (b) 690 (c) 976 (d) 978 (e) 987

11. Assume that when a certain ball is dropped from any given height it rebounds to 60% of that height. If the ball is dropped initially from 10 feet, then what is the total distance traveled by the ball.

(a) 25 (b) 30 (c) 40 (d) 50 (e) ∞

12. If $\{a_k\}_{k=-\infty}^{\infty} = \ \ldots, a_{-2}, a_{-1}, a_0, a_1, a_2, \ldots$ is a sequence of positive numbers where

$$\sum_{k=-\infty}^{\infty} 2^k a_k = 5$$

then

$$\sum_{k=-\infty}^{\infty} \left(2^k \sum_{n=k}^{\infty} a_n \right) =$$

(a) 10 (b) 20 (c) 25 (d) 40 (e) ∞

13. If $n! = n \cdot (n-1) \cdot \ \cdots \ \cdot 2 \cdot 1$, how many of the following are perfect squares?

$98! \cdot 99!$ $98! \cdot 100!$ $99! \cdot 100!$ $99! \cdot 101!$ $100! \cdot 101!$

(a) None (b) 1 (c) 2 (d) 3 (e) 4 or 5

14. Suppose a circle of radius 1 is cut into 2010 equally sizes wedges, like slices of pie. What is the sum of the perimeters of these 2010 wedges?

(a) 2010 (b) 4020 (c) $2010 + 2\pi$ (d) $4020 + 2\pi$ (e) $4020 + 2010\pi$

15. A friend rolls two dice: An 8-sided die with sides numbered 1 – 8, and a 6 sided die with sides numbered 1 – 6. What is the most likely sum of these two dice?

(a) 7 (b) 8 (c) 9 (d) 7, 8 and 9 are equally likely

(e) None of (a) – (d)

16. Let x be a positive integer. Consider the following six expressions:

$$x^2 \qquad x^3 \qquad \frac{x(x+1)}{2} \qquad x^2 - x \qquad x^x \qquad x^{2010} - 3$$

Let y be the number of the above expressions that *must* be <u>even</u> (depending on what the value of x is) and let z be the number of the above expressions that *must* be <u>odd</u>. What is $y - z$?

(a) –2 (b) –1 (c) 0 (d) 1 (e) None of (a) – (d)

17. If the sum of the three 3-digit numbers below is 789,

$$\begin{array}{r} 1\,2\,3 \\ a\,b\,c \\ +\,d\,e\,f \\ \hline 7\,8\,9 \end{array}$$

and if $1 \leq a, d \leq 9$ and $0 \leq b, c, e, f \leq 9$, what is the maximum possible value of $a + b + c + d + e + f$?

(a) 31 (b) 34 (c) 36 (d) 38 (e) 54

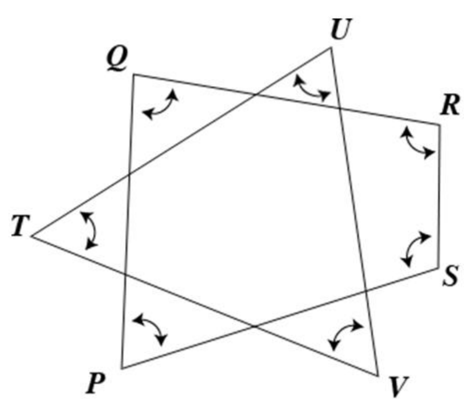
18. The sum of the positive odd integers less than 50 is subtracted from the sum of the positive even integers less than or equal to 50. What is the resulting difference.

(a) 0 (b) 25 (c) 50 (d) 100 (e) 200

19. When the decimal point of a certain positive number is moved four places to the right, the new number is four times the reciprocal of the original number. What is the original number?

(a) 0.0002 (b) 0.002 (c) 0.02 (d) 0.2 (e) 2.0

20. In the figure at right, if PQRS is a quadrilateral and TUV is a triangle, what is the sum of the degree measures of the marked angles?



(a) 420 (b) 490 (c) 540

(d) 560 (e) 900

21. A function f is defined as

$$f(n) = \begin{cases} n+3 & \mathit{if\ n\ is\ odd} \\ \dfrac{n}{2} & \mathit{if\ n\ is\ even} \end{cases}$$

Suppose k is odd and $f\left(f\left(f(k)\right)\right) = 27$. What is the sum of the digits of k?

(a) 3 (b) 6 (c) 9 (d) 12

(e) It cannot be determined from the given information.

22. Given that $0 < a < b < c < d$, which of the following is the largest?

(a) $\frac{a+b}{c+d}$ (b) $\frac{a+d}{b+c}$ (c) $\frac{b+c}{a+d}$ (d) $\frac{b+d}{a+d}$

(e) It cannot be determined from the given information.

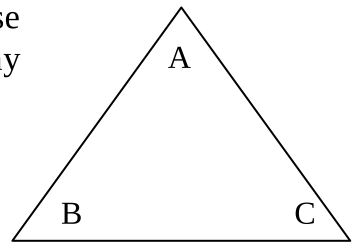
23. What is the maximum number for the possible points of intersection of a circle and a triangle?

(a) 3 (b) 4 (c) 5 (d) 6 (e) More than 6

24. The solution of the equation $7^{x+7} = 8^x$ can be expressed in the form $x = \log_b 7^7$. What is b?

(a) $\frac{7}{15}$ (b) $\frac{7}{8}$ (c) $\frac{8}{7}$ (d) $\frac{15}{8}$ (e) $\frac{15}{7}$

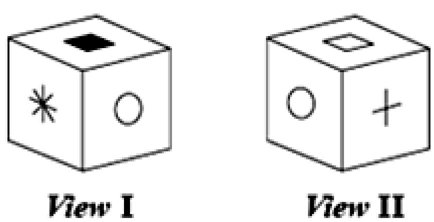
25. In the triangle at right (**not drawn to scale**), suppose $\angle BAC = 30^{\circ}$, $\overline{AC} = 6$ inches, $\overline{BC} = 4$ inches. How many possible lengths could $\overline{AB}$ have?



(a) 0 (b) 1 (c) 2

(d) 3 (e) Infinity many

26. In the figure at right, two views of the same cube are shown. If each face of the cube has a different symbol on it, how many faces of the cube have not been shown in either view?



(a) 0 (b) 1 (c) 2 (d) 3

(e) It cannot be determined from the given information.

27. How many prime numbers are there between 10 and 100?

(a) 21 (b) 22 (c) 23 (d) 24 (e) 25

28. Miguel is 180 centimeters tall. At 2:00 p.m. one day, his shadow is 60 centimeters long, and the shadow of a nearby fence post is t centimeters long. In terms of t, what is the height, in centimeters, of the fence post.

(a) $t + 120$ (b) $\frac{t}{3}$ (c) $3t$ (d) $\frac{2}{3}t$ (e) $\left(\frac{t}{3}\right)^2$

29. If $x + 2y$ is 5 more than $y + 2x$, then $x - y =$

(a) -5 (b) $-\frac{5}{3}$ (c) $\frac{5}{3}$ (d) 5

(e) It cannot be determined from the given information.

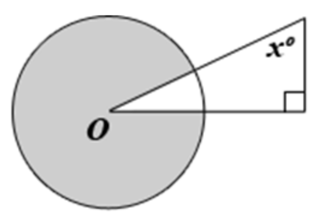
30. Cannon balls are stacked in the form of a pyramid with a square base. Suppose that each edge of the square base consists of eight cannon balls (so a total of 64 balls on the bottom layer of the pyramid). If the radius of each ball is 1, what is the total height of the pyramid?

(a) $8\sqrt{2}$ (b) $8\sqrt{3}$ (c) $2+8\sqrt{3}$ (d) $2+7\sqrt{2}$ (e) $2+7\sqrt{3}$

31. In a class of 80 seniors, there are 3 boys for every 5 girls. In the junior class, there are 3 boys for every 2 girls. If the two classes combined have an equal number of boys and girls, how many students are in the junior class?

(a) 72 (b) 80 (c) 84 (d) 100 (e) 120

32. The circle shown at right has center O and a radius of 5. If the area of the shaded region is 20π, what is the angle x? (Figure not drawn to scale.)



(a) 18 (b) 36 (c) 45 (d) 54 (e) 72

33. Add $8x$ to $2x$ and then subtract 5 from the sum. If x is a positive integer, the result must be an integer multiple of

(a) 2 (b) 5 (c) 8 (d) 10 (e) 15

34. Alice bought m pens for n dollars each, and Ben bought n pens for m dollars each. Which of the following is the average price per pen, in dollars, for all of the pens that Alice and Ben bought?

(a) $\frac{mn}{m+n}$ (b) $\frac{m+n}{2mn}$ (c) $\frac{m+n}{mn}$ (d) $\frac{2(m+n)}{mn}$ (e) $\frac{2mn}{m+n}$

35. The sum of the digits (all of which of course are positive integers between 0 and 9) of a four-digit number is 17. If the hundreds digit is 3 times the tens digit and the tens digit is $\frac{1}{2}$ the units digit, and the thousands digit is at least twice the tens digit, what is the sum of the four-digit number formed by *reversing* the order of the four digits of the original number?

(a) 14 (b) 16 (c) 18 (d) None of (a) – (c)

(e) It cannot be determined from the given information.

36. All numbers divisible by both 12 and 30 are also divisible by which of the following?

(a) 36 (b) 60 (c) 72 (d) 240

(e) More than one of (a) – (d)

37. A list of 2010 integers has the property that the average (arithmetic mean) a of the integers is greater than the median m of the integers. Which of the following *must* be true?

I. More of these integers are greater than a then are less than a.
II. More of these integers are greater than m than are less than m.
III. More of these integers are less than m than are greater than m.

(a) None (b) I only (c) II only (d) I and II (e) I and III

38. In a supermarket, Shakira bought 5 items from aisles 1 through 7, and 7 items from aisles 4 though 10. Which of the following could be the total number of items Shakira bought?

I. 9
II. 10
III. 11

(a) II only (b) I and II only (c) I and III only

(d) II and III only (e) I, II and III

39. A photocopy machine can reduce or increase the size of a document. If a page is reduced to 80% of its original size, what percent enlargement is needed to return the document to its original size?

(a) 20% (b) 25% (c) 120% (d) 125% (e) 180%

40. How many ways can you arrange three boys and three girls in six side-by-side seats if no two boys may sit next to each other and no two girls may sit next to each other?

(a) 6 (b) 9 (c) 21 (d) 36 (e) 72

41. Let n be the smallest three-digit (i.e. > 100) number such that the sum of the three digits of n divides n and the product of the three digits divides n. What is the sum of the digits of n?

(a) 2 (b) 3 (c) 4 (d) 6 (e) 9

42. Given ten lines in the plane, what is the maximum number of regions the plane is divided into by the lines?

(a) 20 (b) 36 (c) 46 (d) 56 (e) 96

43. A plane is flying at a speed of 240 mph due north and at noon it passes over a car traveling due east at 70 mph. What time is closest to the time at which they are 100 miles apart?

(a) 12:19 (b) 12:20 (c) 12:24 (d) 12:25 (e) 1:00

44. Let A be the set {1,2,3} and B be the set {1,3,4,5,6}. How many one-to-one functions are there from A to B? Note: a function is one-to-one if no two objects from one set are sent to the same object in the other set.

(a) 3 (b) 6 (c) 10 (d) 60 (e) 125

45. How many points on the unit circle $x^2 + y^2 = 1$ are both x and y rational?

(a) 0 (b) 4 (c) 8 (d) 16 (e) Infinitely many

Tie-breaker problem

TB. For how many real values of x is $\frac{x}{2010} = \sin x$? Assume x is in radians.