

# **DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO.**

Pepperdine Math Day

November 15, 2008

## **Exam Instructions and Rules**

**1. Write the following information on your Scantron form:**

**Name** in NAME box

**Grade** in SUBJECT box

**School name** in DATE box (into PERIOD box, if necessary)

On the back of your Scantron form, put this same information.

- 2. This exam will last approximately 90 minutes.** It is a **45 question** multiple choice exam. Each question is followed by answers marked A, B, C, D and E. Exactly one answer is correct for each problem. You will use the first 45 spots on the front page of the scantron form to record your answers. **Your answer to the additional tie-breaker problem should be written into the Test No. box on the backside of the Scantron form.** Your answer to this problem does not count toward your score—it will be used only for tie-breaking.
- 3. On this exam, there is no penalty for incorrect answers,** so it is to your advantage to put an answer for each question, especially if you are able to eliminate one or more of the answers as incorrect. However, **in the case of a tie, the person with the answer closest to the correct answer for the tie-breaking problem wins.** If there is still a tie, your answer to problem 45 will determine who wins, then problem 44, and so on, if necessary. Credit will be given only for answers on your scantron form, not for any work written on the exam itself.
- 4. Use a number 2 pencil to record your answer.** Be sure to completely darken each of your penciled-in answers. Extra pencils are available from proctors.
5. There should be enough space between problems to work your solutions. If needed, extra scratch paper is available from the proctors. Credit is given only for answers on your Scantron answer form, not for any work written on the exam or scratch paper.
6. Figures are not necessarily drawn to scale.
7. While we certainly don't expect it, any sort of cheating will be dealt with at the discretion of the proctors, and will likely include at least disqualification.

# **DO NOT OPEN THIS EXAM UNTIL TOLD TO DO SO.**

1. If 84 inches of wire are used to build the skeleton (i.e the frame) of a cube, what is the surface area of the cube, in square inches?

(a) 7

(b) 42

(c) 84

(d) 294

(e) 483

2. In rectangle  $ABCD$ , point  $E$  is chosen on side  $BC$  so that  $BE = 4$ , side  $EC = 4$ , and  $\angle AED$  is a right angle. What is the length of  $ED$ , rounded to the nearest integer?

(a) 4

(b) 5

(c) 6

(d) 7

(e) 8

3. Which of the following points is contained within the circle  $x^2 + 2x + y^2 - 4y = 95$ ?

(a) (6,9)

(b) (-11,3)

(c) (8,7)

(d) (9,6)

(e) More than one of these points is inside the circle.

4. At the Trader Joes in Malibu, 3 bananas cost as much as 2 apples, and 6 apples cost as much as 4 oranges. How many oranges cost as much as 18 bananas?

(a)  $\frac{4}{3}$

(b) 7

(c) 8

(d) 10

(e) 12

5. A triangle has sides of length 10, 10 and 13. What is the area of this triangle?

- (a) 30                      (b) 60                      (c) 65                      (d) 120                      (e)  $\frac{13\sqrt{231}}{4}$

6. What is the area of the trapezoid bounded by the  $x$ -axis, the vertical lines  $x = 1$  and  $x = 4$ , and the line  $y = -3x + 15$ ?

- (a) 22.5                      (b) 36                      (c) 37.5                      (d) 45                      (e) 75

7. Where  $n! = n \cdot (n-1) \cdot \dots \cdot 2 \cdot 1$ , how many of the following are perfect squares?

98!·99!      98!·100!      99!·100!      99!·101!      100!·101!

- (a) None                      (b) 1                      (c) 2                      (d) 3                      (e) 4 or 5

8. How many two-digit numbers have digits whose sum is a perfect square? (This would include, for example, the number 18, since  $1 + 8 = 9$ , and 9 is a perfect square.)

- (a) 14                      (b) 15                      (c) 16                      (d) 17                      (e) 18

9. Two standard dice are rolled. Which of the following is most likely? Rolling the sum of ...

- (a) 7      (b) 5 or 8      (c) 2, 3 or 4      (d) 8 or 12      (e) 9 or 11

10. A certain bouncy ball, when dropped from any height, bounces  $\frac{1}{3}$  of the original height. If the ball is dropped from 54 feet, bounces back up and continues to bounce up and down, what is the total distance, in feet, that the ball has traveled when it hits the ground fourth time?

- (a) 52      (b) 102      (c) 104      (d) 106      (e) 160

11. In the previous problem, if the ball continued to bounce up and down forever (bouncing up  $\frac{1}{3}$  of the height it falls from with each additional bounce), what is the total distance, in feet, the ball travels before it comes to rest?

- (a) 79      (b) 81      (c) 104      (d) 108      (e)  $\infty$

12. A train station is 8 miles exactly southwest of one city and 10 miles exactly southeast of a second city. Which of the following is closest to the number of miles between the two cities?

- (a) 13      (b) 14      (c) 15      (d) 16      (e) 17

13. What is the smallest positive integer with exactly 6 positive integer divisors (including 1 and the number itself)?

(a) 12

(b) 30

(c) 60

(d) 120

(e) 720

14. An aquarium has a rectangular base that measures 100 cm by 50 cm and has a height of 40 cm. The aquarium is filled with water to a depth of 37 cm. A rock with volume  $1000 \text{ cm}^3$  is then placed in the aquarium and completely submerged. By how many centimeters does the water level rise?

(a) 0.2

(b) 0.5

(c) 1

(d) 2

(e) 5

15. Five straight lines are drawn on a plane. What is the largest possible number of points of intersection?

(a) 4

(b) 5

(c) 8

(d) 10

(e) 20

16. Circle A has diameter of  $2\pi$ . Circle B has area  $16\pi$ . Circle C has a circumference of  $7\pi$ . List the circles in order from smallest to largest.

(a) A, B, C

(b) B, A, C

(c) C, B, A

(d) C, A, B

(e) None of (a) – (d)

17. How many prime numbers are there between 90 and 110?

(a) 3

(b) 4

(c) 5

(d) 6

(e) 7

18. If  $4^x = 9$ , then  $8^x = ?$

(a)  $9\sqrt{2}$

(b)  $9\sqrt{3}$

(c) 18

(d)  $12\sqrt{3}$

(e) 27

19. Which number is closest to  $\frac{3}{17} + \frac{17}{3}$ ?

(a) 5.5

(b) 5.75

(c) 6.0

(d) 6.25

(e) 6.5

20. What is the volume of the largest cube that will fit inside of a sphere of radius 1?

(a)  $\sqrt{2}$

(b)  $\sqrt{3}$

(c)  $2\sqrt{2}$

(d)  $4\sqrt{2}$

(e)  $\frac{8}{3\sqrt{3}}$

21. Given the following number of students taking the SAT at Timpview High School, between what two consecutive years was there the largest percentage increase?

Year	Number of students
2003	600
2004	661
2005	700
2006	763
2007	788
2008	849

- (a) 2003 and 2004                      (b) 2004 and 2005                      (c) 2005 and 2006  
(d) 2006 and 2007                      (e) 2007 and 2008

22. How many numbers between 50 and 500 have the property that when divided by 23 the remainder is 1 and when divided by 13 the remainder is also 1?

- (a) 0                      (b) 1                      (c) 2                      (d) 3                      (e) 4

23. A wooden cube of edge 4 inches is painted red. The cube is then cut into 64 one-inch cubes by making 9 saw cuts. If  $x$  is the number of one-inch cubes with no red faces,  $y$  is the number of cubes with exactly 1 red face, and  $z$  is the number of cubes with exactly 2 red faces, what is  $x - y + z$ ?

- (a) 4                      (b) 8                      (c) 12                      (d) 16                      (e) 20

24. If  $f(x) = x + \frac{1}{x}$ , what is  $f(f(f(1)))$ ?

- (a) 1                      (b) 2                      (c)  $\frac{5}{2}$                       (d)  $\frac{29}{10}$                       (e) 4

25. Consider the rule: If  $n$  is odd then replace  $n$  by  $3n+1$ ; otherwise replace  $n$  by  $\frac{n}{2}$ .  
If initially  $n=3$ , what is  $n$  after 100 iterations of this rule?

- (a) 1                      (b) 2                      (c) 4                      (d) 8                      (e) 16

26. How many points are there that are equidistant from the following three points?

$$(-2, -1), (10, 1), (1, -1)$$

- (a) 0                      (b) 1                      (c) 2                      (d) 3                      (e)  $\infty$

27. What positive number is twice as large as its reciprocal?

- (a)  $\frac{1}{2\sqrt{2}}$                       (b)  $\frac{1}{\sqrt{2}}$                       (c)  $\sqrt{2}$                       (d) 2                      (e)  $2\sqrt{2}$



28. What value of  $C$  result(s) in the *difference* between the two roots of the equation  $x^2 - 3x + C$  being 1?

- (a)  $-3$       (b)  $-2$       (c)  $1$       (d)  $2$       (e)  $3$

29. The product of all real integer solutions of the equation  $x^4 - 13x^2 - 48 = 0$  is

- (a)  $-48$       (b)  $-16$       (c)  $0$       (d)  $13$       (e)  $48$

30. How many of the integers in the set  $\{100, 101, \dots, 999\}$  do not contain any of the digits 2, 5, 7 or 8?

- (a)  $180$       (b)  $216$       (c)  $410$       (d)  $500$       (e) Other

31. On a five-question true/false exam, correct answers are worth 3 points, incorrect answers are worth 0, and each unanswered question is worth 1 point. How many different sets of responses will result in a total score of 4?

- (a)  $9$       (b)  $10$       (c)  $13$       (d)  $15$       (e)  $25$

32. Integers  $a$ ,  $b$ ,  $c$  and  $d$ , not necessarily distinct, are chosen independently and at random from 1 to 2008, inclusive. What is the probability that  $ad - bc$  is odd?

- (a)  $\frac{1}{4}$       (b)  $\frac{3}{8}$       (c)  $\frac{1}{2}$       (d)  $\frac{5}{8}$       (e)  $\frac{3}{4}$

33. What non-zero real value  $x$  satisfies  $(7x)^{14} = (14x)^7$ ?

- (a)  $\frac{1}{7}$       (b)  $\frac{2}{7}$       (c) 1      (d) 7      (e) 14

34. If  $\sin x = \frac{1}{3}$ , what is  $\sin 2x$ ?

- (a)  $\frac{16}{27}$       (b)  $\frac{2}{3}$       (c)  $\frac{2\sqrt{2}}{9}$       (d)  $\frac{4\sqrt{2}}{9}$       (e)  $\frac{4}{3\sqrt{3}}$

35. Two positive integers have a sum of 9. What is the smallest possible value for the sum of their cubes?

- (a) 40.5      (b) 81      (c) 189      (d) 513      (e) 729

36. If

$$\begin{aligned} -x + y + z &= 1 \\ x - y + z &= 1 \\ x + y - z &= 1 \end{aligned}$$

then what is  $x + y + z$ ?

- (a) -3      (b) 0      (c) 3      (d) 6  
(e) Can't tell from the information given.

37. If  $x^2 + y^2 = 10$  and  $xy = 10$ , then  $|x + y| =$

- (a) 10      (b)  $\sqrt{30}$       (c) 20      (d) 30      (e) 42

38. How many of the 128 subsets of  $\{1,2,3,4,5,6,7\}$ , including the empty set and the complete set itself, include the number 2.

- (a) 32      (b) 48      (c) 56      (d) 60      (e) 64

39.  $(a^{-1} - b^{-1})(a - b)^{-1} =$

- (a) 1      (b)  $-ab$       (c)  $-\frac{1}{ab}$       (d)  $-\frac{a}{b}$       (e)  $\frac{1}{a^2 + b^2}$

40. Marty is going to the store to buy candy that will cost up to 26 cents. What is the fewest number of coins he can bring in order to be certain to have exact change, regardless of the cost (up to and including 26 cents)?

(a) 5

(b) 6

(c) 7

(d) 8

(e) 9

41. Suppose the following two statements are true:

If it is raining, then I am sad.

If it is sunny, then it is not raining.

Which of the following must be true?

(a) If it is sunny, then I am not sad.

(b) If I am sad, then it is raining.

(c) If I am not sad, then it is sunny.

(d) If it is raining, then it is not sunny.

(e) More than one of the statements in (a) – (d) are true.

42. Suppose the 24 ways to write rearrangements of the letters MATH are listed in alphabetical order. At what position in this list does the word MATH appear?

(a) 1

(b) 7

(c) 8

(d) 9

(e) 14

43. What is the product  $\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \dots \times \frac{2008}{2007}$ ?

- (a) 1                      (b) 1003.5                      **(c) 1004**                      (d) 2007                      (e) 2008

44. If  $y = \frac{3x^2+2x-8}{x+2}$ , then  $y$  may assume any real value except for:

- (a) -10**                      (b) -2                      (c) 0                      (d) Any positive value  
(e) None of (a) – (d):  $y$  may assume all real values.

45.  $\sqrt{7+3\sqrt{5}} - \sqrt{7-3\sqrt{5}} =$

- (a)  $\sqrt{10}$**                       (b)  $\sqrt{3+3\sqrt{5}}$                       (c)  $\sqrt{3-3\sqrt{5}}$                       (d)  $\sqrt{3+7\sqrt{5}}$                       (e)  $\sqrt{5+3\sqrt{5}}$

### Tie-breaker Question

Write your answer in the green shaded area on the backside of your Scantron form.

A car odometer proceeds from digit 3 to 5, always skipping the digit 4, regardless of position. For example, on one occasion the odometer changed from 000039 to 000050 and on another occasion it changed from 000123 to 000125. If the odometer now reads 2008, how many miles has the car actually traveled? (Hint: the answer is between 1 and 2008.)

**1465**