Flipping Linear Algebra

Teaching a Majors-Level Linear Algebra Course in a Flipped Learning Environment

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Flipped Classes

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A video lecture may be the BEST WAY to present mathematics, because it shows mathematics as a process, not a finished product.

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A flipped environment is ideally suited for these goals!

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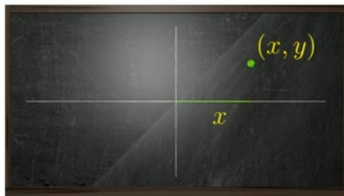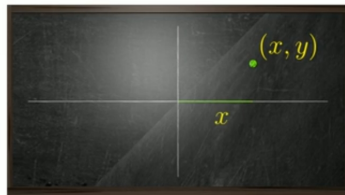
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Reflections

Problem

For $P \in \mathbb{R}^2$, write the transformation matrix for $M_x : P \to P'$, where P' is the reflection of P across the x-axis.

(x, y)

x

We'll go x units horizontally and then y units vertically.

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- Comprehension questions.
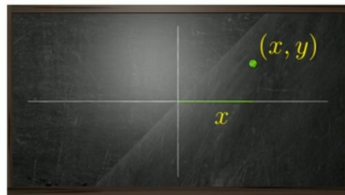
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Problem

Let M_y be the transformation matrix for a reflection across the y-axis. Find M_y.

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Let M_y be the transformation matrix for a reflection across the y-axis. Find M_y.

Problem

Let $R_{90°}$ be the transformation matrix for a rotation by $90°$ counterclockwise. Find $R_{90°}$.

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- Find $M_y R_{90^\circ}$ and $R_{90^\circ} M_y$. Is matrix multiplication commutative?

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- Is

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae & bf \\ cg & dh \end{pmatrix}$$

Explain.

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Defend your conclusion.

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- Find $(M_y)^{1000}$ and $(R_{90°})^{15}$.

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- Let A, B be linear transformations from $\mathbb{R}^2 \to \mathbb{R}^2$. Find AB.

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- Starting "the wrong way" can make it impossible to complete a proof,
- Students don't see the point of proof.

Developing a Proof

We teach:

Theorem (Product of Determinants)

The determinant of a product is the product of the determinants.

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- Find $\det M^{-1}M$ without computing it. Defend your conclusion.

Three Shameless Plugs

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But mine are on YouTube: "Jeff Suzuki linear algebra".

MyOpenMath

This is a free, open source LMS with a well-integrated mathematics OHM:

- www.myopenmath.com
- Library of courses available to copy and modify (including mine)
- Library of problems available to copy and modify (incuding mine) ever seen)
- No "in-house" server needed (long story . . .)

Shameless Plug

Patently Mathematical (Johns Hopkins University Press, 2019)

- Mathematics and recent patents,
- Lots of basic applications of linear algebra,
- Google is based on pre-midterm material.