

Flipping Linear Algebra

Teaching a Majors-Level Linear Algebra Course in a Flipped Learning Environment

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Flipped Classes

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A video lecture may be the **BEST WAY** to present mathematics, because it shows mathematics as a process, not a finished product.

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- Promote student exploration of mathematics.

A flipped environment is ideally suited for these goals!

A Day In The Life: Before Class

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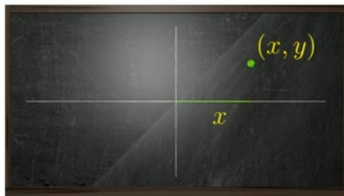
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Reflections

Problem

For $P \in \mathbb{R}^2$, write the transformation matrix for $M_x : P \rightarrow P'$, where P' is the reflection of P across the x -axis.



We'll go x units horizontally and then y units vertically.

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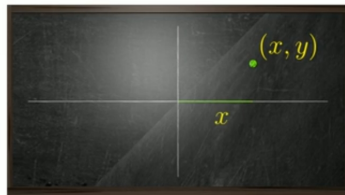
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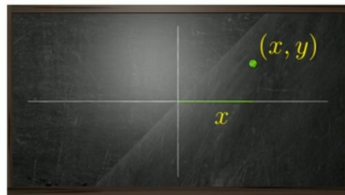
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- Comprehension questions.

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Let M_y be the transformation matrix for a reflection across the y -axis. Find M_y .

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Let R_{90° be the transformation matrix for a rotation by 90° counterclockwise. Find R_{90° .

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$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae & bf \\ cg & dh \end{pmatrix}$$

Explain.

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Defend your conclusion.

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- Find $(M_y)^{1000}$ and $(R_{90^\circ})^{15}$.
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- Find $(M_y)^{1000}$ and $(R_{90^\circ})^{15}$.
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- Find $(M_y R_{90^\circ})^{-1}$. Express your answer in terms of M_y^{-1} and $R_{90^\circ}^{-1}$.

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- Find $(M_y R_{90^\circ})^{-1}$. Express your answer in terms of M_y^{-1} and $R_{90^\circ}^{-1}$.
- Let A, B be linear transformations from $\mathbb{R}^2 \rightarrow \mathbb{R}^2$. Find AB .

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- Students often make mistakes in proofs that they don't learn about until they get their papers back,
- Starting “the wrong way” can make it impossible to complete a proof,
- Students don't see the point of proof.

Developing a Proof

We teach:

Theorem (Product of Determinants)

The determinant of a product is the product of the determinants.

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- Find $\det M^{-1}$ **without** finding M^{-1} . Defend your conclusion.
- Find $\det M^{-1}M$ **without** computing it. Defend your conclusion.

Three Shameless Plugs

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But mine are on YouTube: “Jeff Suzuki linear algebra”.

Example
Let m be a scalar, and \vec{b} a non-zero vector. Prove or disprove: $T(\vec{x}) = m\vec{x} + \vec{b}$ is a linear transformation.

$$T(\vec{u} + \vec{v}) =$$

Definition
Let $T: A \rightarrow B$. We say T is a **linear transformation** if for all vectors \vec{u}, \vec{v} and all scalars c ,

$$T(\vec{u} + \vec{v}) = T(\vec{u}) + T(\vec{v})$$
$$T(c\vec{u}) = cT(\vec{u})$$

We write $T\vec{x} = \vec{y}$.

Linear Algebra
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- Linear Transformations, Part 1 (Jeff Suzuki) 5:08
- Linear Transformations, Part 2 (Jeff Suzuki) 3:09
- Geometric Transformations (Jeff Suzuki) 5:43
- Range of a Linear Transformation (Jeff Suzuki) 5:52
- Null Space of a Linear Transformation (Jeff Suzuki) 6:18
- Matrix Addition (Jeff Suzuki) 7:04
- Khan's Multidimension
- Circular Purple & Pink Particles Moving! 4K Relaxing... (vimeo/Relax) 10:28

Linear Transformations, Part 1
431 views

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MyOpenMath

This is a free, open source LMS with a well-integrated mathematics OHM:

- www.myopenmath.com
- Library of courses available to copy and modify (including mine)
- Library of problems available to copy and modify (including mine) ever seen)
- No “in-house” server needed (long story ...)

The screenshot shows the MyOpenMath mobile app interface. At the top, there's a status bar with icons for a calendar, a 'T' icon, a download icon, signal strength, 94% battery, and the time 8:35 AM. Below this, the course structure is displayed. The first unit is 'Unit 2: Elementary Matrix Operations' with a gear icon and a dropdown arrow, and the text 'Showing Collapsed Always'. The second unit is 'Unit 3: Linear Transformations', also with a gear icon, a dropdown arrow, and the text 'Showing Collapsed Always'. Under 'Unit 3', there's a section for 'Lectures' with a gear icon and a dropdown arrow, and the text 'Showing Collapsed Always'. Below 'Lectures', there's an 'Add An Item...' dropdown. The first item is 'Linear Transformations, Part One' with a globe icon, a plus icon, and the text 'Showing Always'. The second item is 'Comprehension: Linear Transformations, Part One' with a pencil icon, a gear icon, and a dropdown arrow. The text for this item reads: 'Past Due Date of Tue 6/12/18, 12:25 pm. Showing as Review. LP This assessment is in review mode - no scores will be saved'. Below this, there's a paragraph: 'This quiz is based on the material in the preceding video. You should be able to complete it in under 5 minutes.'

Shameless Plug

Patently Mathematical (Johns Hopkins University Press, 2019)

- Mathematics and recent patents,
- Lots of basic applications of linear algebra,
- Google is based on pre-midterm material.

