

Helping Students Master Linear Algebra through Writing

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Overview

- Linear Algebra at Shippensburg University
- Where students struggle in Linear Algebra
- Resources
- Writing Assignments
- Assessment Results

Linear Algebra at SU

- The prerequisite for Linear Algebra is *either* Calculus II or Discrete Mathematics (an intro-to-proof “bridge course”)
- Students in this course have a wide variety of experience with logical reasoning (and especially writing, depending on how their Calculus II was taught)

Linear Algebra Vocabulary

- One of the main struggles in Linear Algebra is in the vocabulary
- Students very quickly learn how to row-reduce a matrix
- They have a hard time interpreting the result

Early Points of Confusion

- Solution
- Span (as a verb and as a noun)
- Linear independence

Resources

- During Summer 2018, I created a series of 43 video lectures to supplement the course
- Each class period, I handed out a packet of notes and worked through them
- The students were given practice problems for each section (with solutions)

YouTube Video Series



Example

Let $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$, and define $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ by $T(\mathbf{x}) = A\mathbf{x}$.

(a) Find the image of $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ under T .

$$T\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} (1)(2) + (-3)(-1) \\ (3)(2) + (5)(-1) \\ (-1)(2) + (7)(-1) \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ -9 \end{bmatrix}$$

Linear Algebra Lectures

James Hamblin - 18 / 44



Linear Algebra - Lecture 12 - Applications to Economics
James Hamblin



Linear Algebra - Lecture 13 - Applications to Chemistry
James Hamblin



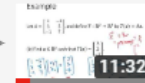
Linear Algebra - Lecture 14 - Applications to Networks
James Hamblin



Linear Algebra - Lecture 15 - Linear Independence
James Hamblin



Linear Algebra - Lecture 16 - Characterization of Linearly Independent Sets
James Hamblin



Linear Algebra - Lecture 17 - Matrix Transformations
James Hamblin

Linear Algebra - Lecture 18 -

5:21 / 11:31



Linear Algebra - Lecture 17 - Matrix Transformations

426 views

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Writing Assignments

- From the beginning of the semester, I insisted on students writing explanations, even for computational problems
- For many of them, it took time to convince them that I really did want them to write some words for each problem

Writing Assignments

- For a typical homework problem, my expectation was that the student would break this into three steps:
 - Setup: Explain what equation must be solved to answer the question
 - Computation (usually row-reducing a matrix)
 - Conclusion: Based on the result of the computation, answer the original question

Language Errors

- Students would often write things that indicated a lack of understanding of the underlying vocabulary
 - “This matrix has a free variable”
 - “These vectors have a nontrivial solution”
- In each case, I gave verbose written feedback on their assignments

Assessment

- At the end of the semester, I gave a 32-question multiple choice assignment as extra credit on the final exam
- 46 of the 53 students completed the assignment

Assessment Results

- The mean score on the concept assessment was 24.4/32
- Out of the 46 students:
 - There were 2 questions on which 10 or fewer students got the correct answer
 - For each of the other questions, at least 25 students answered correctly, and on 9 of the questions, 40 or more students got the correct answer

Most Successful Questions

- Suppose that A is an invertible $n \times n$ matrix. Which of the following statements about A is false?
 - A has n pivots
 - $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions
 - The columns of A are linearly independent
 - The columns of A span $\mathbb{R}^n$

Most Successful Questions

- Suppose that A is a $3 \times n$ matrix such that $A\mathbf{x} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ has a solution. Which of the following statements is true?
 - $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ is one of the columns of A
 - $[1 \quad -1 \quad 0]$ is one of the rows of A
 - The third row of A is all zeroes
 - $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ is a linear combination of the columns of A

Least Successful Questions

- Suppose that $\{v_1, v_2, \dots, v_p\}$ is a linearly dependent set of vectors in $\mathbb{R}^n$. Which of the following statements must be true?
 - $p > n$
 - This set contains the zero vector
 - One of the vectors in the set is a multiple of one of the other vectors in the set
 - None of the above

Least Successful Questions

- Suppose that T is a transformation, though not necessarily a linear transformation. Which of the following statements about T would imply that T is a linear transformation?
 - T is one-to-one
 - T is onto
 - $T(\mathbf{0}) = \mathbf{0}$
 - None of the above

Conclusions

- Generally, students were successful at answering the questions on the assessment
- Anecdotally, on their final exams, many students still struggled with writing their own explanations to interpret the meaning of row-reduced matrices

Lessons for Next Time

- Next time I teach this course, I will spend more time at the beginning of the semester working through examples involving span and linear independence
- I will also allow students to submit re-writes when their explanations are insufficient

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