Validation of an Assessment for Introductory Linear Algebra Concepts

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Introduction

- Work is part of broader TIMES project aimed at supporting instructors who want to teach in inquiry-oriented ways.
  - http://times.math.vt.edu
  - AA, DE, and Linear Algebra
  - TIMES instructor used specially designed inquiry-oriented curricular materials
  - http://iola.math.vt.edu

Introduction

- My dissertation work focuses to measure if students taught with inquiry-oriented curricular materials gain better conceptual understanding of linear algebra concept
- This presentation is the first step of my work
  - Development and Validation of a linear algebra assessment

Assessment

- Two characteristics of a quality assessment
  - Measure what it purposes to measure (validity)
  - Produce consistent results repeatedly (reliability)
- Classroom assessments are loosely structured and depend upon several factors
  - Instructors' investment of time and effort
  - Instructors' expertise in the subject (Thissen-Roe, Hunt, and Minstrell, 2004).

Why Validation?

- Measurement is all about generalized conclusions based on the limited observations
  - How credible are the generalized conclusions?
- Validated assessment can do more
  - Evaluate the efforts to improve learning
  - Measure the quality and achievements of instructional innovations
  - Diagnostic tool and placement criteria

Kane's Validation Framework

- Goal of this work is to ensure the validity and reliability of the LA assessment that I will use to measure the conceptual understanding
- I used Kane(2006) validation framework
  - Validation is the measure of credibility and appropriateness of proposed interpretations and uses (Kane, 2006)
  - Content Model and Construct model

Data Source

- Assessment data of almost 450 students and counting (Haider et al., 2016)
  - 18 linear algebra classes at 15 institutions across the country in two academic years (2015-2017)
  - Assessment covers four focal areas aligned with IOLA material
    - span and linear (in)dependence
    - solutions to systems of linear equations
    - matrices as linear transformations
    - eigenvectors & eigenvalues

1. Answer the following questions regarding the set of vectors $V = \left\{\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}0\\1\\2\end{bmatrix}\right\}$.

a. Which of the following best describes the span of the set V?

- ☐ i: A point
- ☐ ii: Two points
- ☐ iii: A line
- ☐ iv: Two lines
- ☐ v: A plane
- ☐ vi: Two planes
- ☐ vii: A 3-dimensional space

b. Justify your response to part a.

c. Which of the following are in the span of $V = \left\{\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}0\\1\\2\end{bmatrix}\right\}$? (Check all that apply.)

- ☐ i: $\begin{bmatrix}1\\2\\0\end{bmatrix}$
- ☐ ii: $\begin{bmatrix}1\\2\end{bmatrix}$
- ☐ iii: $\begin{bmatrix}0\\-2\\-4\end{bmatrix}$
- ☐ iv: $\begin{bmatrix}1\\0\\0\end{bmatrix}$
- ☐ v: $3.1\begin{bmatrix}1\\2\\0\end{bmatrix} - \frac{4}{5}\begin{bmatrix}0\\1\\2\end{bmatrix}$
- ☐ vi: any vector in $\mathbb{R}^3$

d. Explain in general how you can determine if a given vector is in the span of some other set of vectors.

Validity in the Development of the Assessment

- Established the relevance of the test items to the four focal topics covered by the test
  - Expert validated
    - 3 rounds of expert's feedback and refinement
- Ensured that the LA test measures the concept or construct that it is intended to measure
  - Analysis of clinical interviews

Validity in the Scoring of the Assessment Data

- Developed the scoring rubric to score OEQs.
  - Developed initial scoring key by using students' approaches from clinical interviews
  - Four researchers scored same 10 copies
    - Correct, partially correct, and incorrect
    - Updated initial solution key and documented criteria for all categories of responses
    - Finalized scoring rubric

Scoring of Assessment Data..

- Pilot tested of the scoring rubric
  - Four researchers again scored six assessment copies independently to ensure the reliability of the scoring rubric
    - 85% agreement and refined the rubric again
      - Three researchers scored 10 copies and agreement was 91%
    - Final Version of the rubric is ready to score entire data
- Planned to double-code 25% of the data to ensure reliability of scoring and rubric.

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Question/ Testlet	Topics	Item/ Part	Category/ Type	Possible Score	Type
1	Span and Linear Combination of Vectors	a	Conceptual	1	MCQ (Pick one)
		b	Conceptual	3	Open-Ended
		c	Procedural	6	MCQ (Pick All that Apply)
		d	Procedural & Conceptual	3	Open-Ended
2	Linear Independence	a	Conceptual	1	Circle One
		b	Conceptual	1	Circle One
		c	Conceptual	1	Fill in the Blank
3	Interpretation of RREF	a	Procedural	1	MCQ (Pick one)
		b	Conceptual	3	Open-Ended
4	Product of Matrices	-	Procedural	5	MCQ (Pick All that Apply)
5	System of Linear Equations	a	Procedural	3	Open-Ended
		b	Conceptual	3	Open-Ended
		c	Conceptual	1	Circle One
6	Linear Transformation	a	Conceptual	1	MCQ (Pick one)
		b	Conceptual	3	Open-Ended
7	Invertible Matrices	-	Conceptual	3	Open-Ended
8	Eigenvalues	-	Procedural	3	Open-Ended
9	Eigenvectors	a	Conceptual	6	MCQ (Pick All that Apply)
		b	Conceptual	3	Open-Ended

	Procedural	Conceptual
1a	0	1
1b	0	1
1c	1	0
1d	1/2	1/2

$$[1 \;\; 2 \;\; 3 \;\; 1] \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1/2 & 1/2 \end{bmatrix} = [3.5 \;\; 3.5]$$

Preliminary Results

- Test and items analysis
  - Scored 50 assessment copies
- Test reliability (Cronbach α) of MCQ & OE
  - Together α = .74
  - MCQ and OE α = .49 and .66
- Inter-item correlation
  - None is negative
  - Corrected item-total correlation range .34 - .68

Preliminary Results…

- Item level analysis
  - Average score of all question Range .54 - .83
  - Average of MCQ .69 and OE .65

- Focal areas performance
  - Span, LI, & System of Eqs Average: ~.75
  - Transformation and Eigen concept: ~.65

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THANK YOU