

Transforming Linear Algebra Education with GeoGebra Applets
NSF TUES Grant Award ID: 1141045

ALVERNO COLLEGE
ESTABLISHED 1887

Visualization of each Step of the Solution of Gauss-Jordan Elimination using GeoGebra

JMM 2018
Jim Factor

Mathematics and Computing
Alverno College, Milwaukee, WI

James.Factor@Alverno.edu

This presentation used an interactive GeoGebra applet to show the visual geometric changes for each step of the Gaussian–Jordan elimination process as the solution unfolds in achieving reduced row echelon form. The cases of one solution, no solution, and infinite solutions (linear and planar) will be illustrated dynamically.

This applet, along with others, is freely available for instructor demonstration and student discovery of the meaning of linear algebra concepts. Associated activities have been designed to guide students in using the interactive applets to enhance learning.

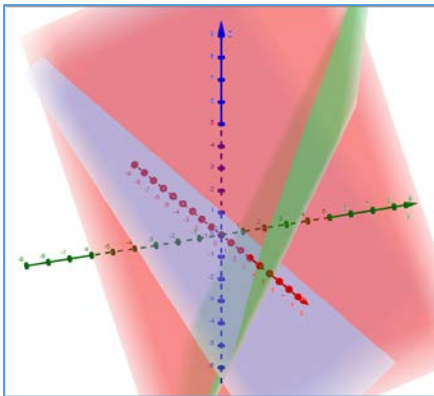
These resources are in the GeoGebra Book entitled *Transforming Linear Algebra Education with GeoGebra* located at <https://www.geogebra.org/m/XnfUWvvp>

Note this GeoGebra Book is presently being populated with other applets and activities covering a first course in Linear Algebra. It will be completed by September 1, 2018.

Below are a collection of systems of equations that were presented at the 2018 JMM.

Gauss-Jordan Elimination Examples

Unique solution



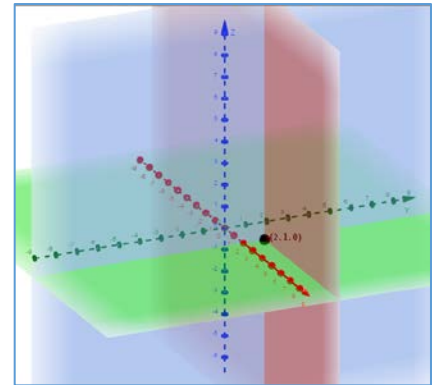
Eq1: $1x + 0y + 0z = 2$ $\begin{bmatrix} 1 & 0 & 0 & | & 2 \\ 0 & 1 & 0 & | & 1 \\ 0 & 0 & 1 & | & 0 \end{bmatrix}$
 Eq2: $0x + 1y + 0z = 1$
 Eq3: $0x + 0y + 1z = 0$

Elementary Row Operations: $R_i + cR_j \rightarrow R_i$ $R_i \leftrightarrow R_j$ $c \cdot R_i \rightarrow R_i$

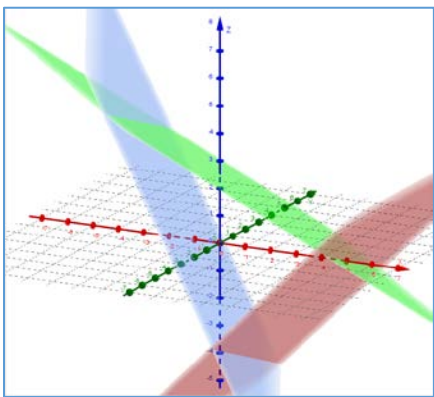
Row Changed: R1 R2 R3

Row Used: R1

Show Solution (2, 1, 0)



No Solution



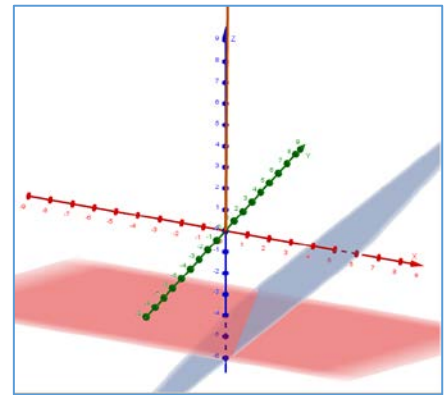
Eq1: $1x + 0y + -1z = 5$ $\begin{bmatrix} 1 & 0 & -1 & | & 5 \\ 0 & 1 & 3 & | & -12 \\ 0 & 0 & 0 & | & 24 \end{bmatrix}$
 Eq2: $0x + 1y + 3z = -12$
 Eq3: $0x + 0y + 0z = 24$

Elementary Row Operations: $R_i + cR_j \rightarrow R_i$ $R_i \leftrightarrow R_j$ $c \cdot R_i \rightarrow R_i$

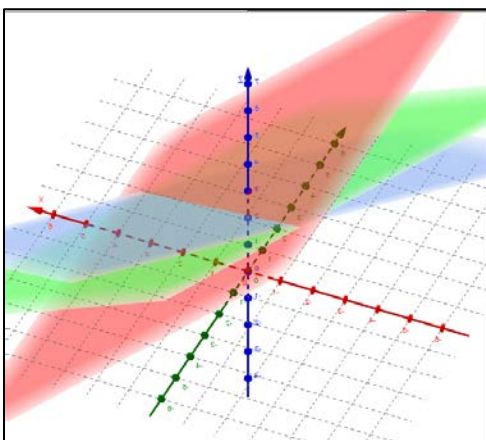
Row Changed: R1 R2 R3

Row Used: R2

Show Solution a No Solution



Infinitely many solutions (Linear)



Eq 1: $x + z = 3$ $\begin{bmatrix} 1 & 0 & 1 & | & 3 \\ 0 & 1 & 1 & | & 2 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$
 Eq 2: $y + z = 2$
 Eq 3: ?

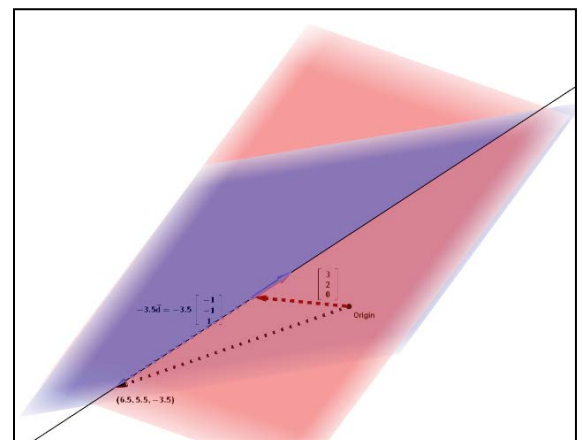
Elementary Row Operations: $R_i + cR_j \rightarrow R_i$ $R_i \leftrightarrow R_j$ $c \cdot R_i \rightarrow R_i$

Row Changed: R1 R2 R3

Row Used: R1

Show Solution (7, 7, 7)

Example 1 Example 2 Example 3 Example 4 Example 5
 $x = 3 + -1z \Rightarrow x = 3 + -1t$
 $y = 2 + -1z \Rightarrow y = 2 + -1t$ $-\infty < t < \infty$
 $z = t$
 $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$
 What is this geometrically?
 $\begin{bmatrix} 6.5 \\ 5.5 \\ -3.5 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} + -3.5 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$
 $t = 0$ $t = 1$ $t = -1$



Infinitely many solutions (Planar)

Create and Solve your own system of equations. Click

Note on the page arrived at to **Define a System of Equations** any system of equations can be created. Once it is defined then you are taken back to the home page where it is solved using the Gauss-Jordan Elimination Process as was done previously.

Consider this example

- + - + - + - +

Eq1: $1x + 2y + 0z = 2$

- + - + - + - +

Eq2: $0x + 1y - 2z = 5$

- + - + - + - +

Eq3: $-2x + 1y + 1z = 3$

Determine Solution from RREF (reduced row echelon form) obtained by Gauss-Jordan Elimination.

When

Determine Solution from RREF (reduced row echelon form) obtained by Gauss-Jordan Elimination.

is clicked, we arrive where the system of equations is solved

Eq1: $1x + 2y + 0z = 2$

 $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 2 \\ 0 & 1 & -2 & 5 \\ -2 & 1 & 1 & 3 \end{array} \right]$

Eq2: $0x + 1y - 2z = 5$

Eq3: $-2x + 1y + 1z = 3$

Elementary Row Operations:

Row Changed:

Row Used:

Show Solution in RREF.

Define a System of Equations