

## My Favorite MAA Articles for Linear Algebra



January 11, 2015

Jeremy Case  
Taylor University  
jrcase@taylor.edu

## Assumptions

Not all great applications can be included in a textbook.

Linear Algebra services a variety of disciplines.

Journal articles are a good thing.

Incorporating journal articles is not automatically easy.

### Applications of Linear Algebra in Calculus

Jack W. Rogers, Jr.

**1. INTRODUCTION.** The concepts of basis, matrix for a linear transformation relative to bases, and change-of-basis matrix are fundamental in linear algebra, but students in an introductory class often have trouble understanding the point of applying these concepts for bases other than the standard basis for  $\mathbb{R}^n$ . Our object is to illustrate some applications of these concepts in solving problems with which students who have recently completed the calculus sequence should be familiar. The spaces are abstract vector spaces—finite subspaces of function spaces—not

**Applications of Linear Algebra in Calculus**  
Jack W. Rogers, Jr.,  
*The American Mathematical Monthly*,  
Vol. 104, No. 1  
(Jan., 1997), pp. 20-26.

For this problem, the matrix  $D = M_{\mathcal{B}} \mathcal{A} \mathcal{B}^{-1}$  for  $\mathcal{D}$  relative to  $\mathcal{B}$  is

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \text{ and } D^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix}$$

The single matrix inversion provides all the following antiderivatives, with constants given by the constants of  $D^{-1}$ .

$\int t^2 e^{-t} dt = -t^2 e^{-t} - 2t e^{-t} + 2e^{-t}$ ,  $\int t^2 e^{-t} dt = t^2 - e^{-t}$ , and  $\int t^2 e^{-t} dt = e^{-t}$ .

Matrix inversion has replaced the use of integration by parts for this problem. The same technique can be used to provide an alternative approach for standard integration by parts problems. For  $t^2 e^{-t} dt$ , for example, let

### PITFALLS IN COMPUTATION, OR WHY A MATH BOOK ISN'T ENOUGH

GEORGE E. FORSYTHE, Computer Science Dept., Stanford University

**1. Introduction.** Why does a student take mathematics in college? I see two reasons: (i) To learn the structure of mathematics itself so he (or she) finds it interesting, (ii) To prepare to apply mathematical solution of problems he expects to encounter in his own field, whether engineering, physics, economics, or whatever. Surely (ii) motivates far more students than (i). Moreover, most so

**Pitfalls in Computation, or Why a Math Book Isn't Enough,**  
George E. Forsythe  
*The American Mathematical Monthly*  
(Nov. 1970)  
20-26

of linear algebraic equations by Gauss' method of eliminating unknowns. A little systematization, it becomes another algorithm for general use. I like to examine it in the simple case of two equations in two unknowns, but on a computer with  $\beta = 10$ ,  $z = 3$ .

The equation system is one treated by Forsythe and Moler [3]:

$$\begin{bmatrix} 0.000100x + 1.00y = 1.00 \\ 1.00x + 1.00y = 2.00 \end{bmatrix}$$

The solution, rounded correctly to the number of decimals shown, is

$$x = 1.00010, y = 0.99990 \text{ (truly rounded).}$$

The Gauss elimination algorithm uses the first equation (if possible) to set the first variable,  $x$ , from the second equation. Here this is done by dividing the first equation by 0.000100 and then subtracting it from the second

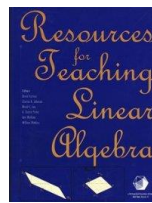
## More Numerical Issues

**Numerical Linear Algebra on the HP-28 or How to Lie With Supercalculators**  
Yves Nievergelt  
*The American Mathematical Monthly*  
Vol. 98, No. 6 (Jun. - Jul., 1991), pp. 539-544

**Six, Lies, and Calculators**  
R. M. Corless  
*The American Mathematical Monthly*  
Vol. 100, No. 4 (Apr., 1993), pp. 344-350

## My Favorite MAA Journal for Linear Algebra

College Mathematics Journal, Vol 24, Jan 1993



### CONTENTS

<b>INTRODUCTION</b>	
<b>PART I THE ROLE OF LINEAR ALGEBRA</b>	Classical Education in Single Mathematics 527
The Growing Importance of Linear Algebra in Undergraduate Mathematics 530	An Application of the Spectral Theorem to the Theory of Linear Algebra 533
David C. Mack, University of New York, State University of New York, Stony Brook	David B. Loffler, Colgate University
<b>PART II LINEAR ALGEBRA AS SEEN FROM OTHER DISCIPLINES</b>	Richard D. Cline, University of Tennessee, Knoxville
Linear Algebra in Economics 545	William Koon, University of Tennessee, Knoxville
Linear Algebra in Chemistry 548	William Koon, University of Tennessee, Knoxville
Linear Algebra in Biology 551	William Koon, University of Tennessee, Knoxville
Linear Algebra in Physics 554	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Language 557	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Music 560	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Art 563	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Law 566	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Medicine 569	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Education 572	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of History 575	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Philosophy 578	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Psychology 581	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Sociology 584	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Anthropology 587	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Linguistics 590	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Geography 593	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Meteorology 596	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Oceanography 599	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Atmospheric Science 602	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Environmental Science 605	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Earth Science 608	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Space Science 611	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Planetary Science 614	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Astrophysics 617	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Cosmology 620	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Particle Physics 623	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Nuclear Physics 626	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Physics 629	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Relativity 632	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of String Theory 635	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Superstring Theory 638	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of M-Theory 641	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Brane Theory 644	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Matrix Theory 647	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Operator Theory 650	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Representation Theory 653	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Algebraic Geometry 656	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Differential Geometry 659	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Topology 662	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Knot Theory 665	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Knot Theory 668	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Field Theory 671	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Gravity 674	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Cosmology 677	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Black Holes 680	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Wormholes 683	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Spacetime 686	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Time 689	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Space 692	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Matter 695	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Energy 698	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Entropy 701	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Information 704	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Communication 707	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Cryptography 710	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Computing 713	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Simulation 716	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Emulation 719	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Verification 722	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Certification 725	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Authentication 728	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Authorization 731	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Accounting 734	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Auditing 737	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Taxation 740	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Finance 743	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Insurance 746	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Banking 749	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Commerce 752	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Industry 755	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Services 758	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Retail 761	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Wholesale 764	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Distribution 767	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Logistics 770	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Supply Chain 773	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 776	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 779	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 782	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 785	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 788	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 791	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 794	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 797	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 800	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 803	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 806	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 809	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 812	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 815	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 818	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 821	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 824	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 827	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 830	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 833	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 836	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 839	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 842	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 845	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 848	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 851	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 854	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 857	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 860	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 863	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 866	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 869	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 872	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 875	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 878	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 881	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 884	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 887	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 890	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 893	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 896	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 899	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 902	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 905	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 908	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 911	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 914	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 917	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 920	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 923	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 926	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 929	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 932	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 935	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 938	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 941	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 944	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 947	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 950	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 953	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 956	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 959	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 962	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 965	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 968	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 971	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 974	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 977	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 980	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 983	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 986	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 989	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 992	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 995	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Procurement 998	William Koon, University of Tennessee, Knoxville
Linear Algebra in the Study of Quantum Sourcing 1001	William Koon, University of Tennessee, Knoxville

# Provocative...

**Down with Determinants!**  
**Sheldon Axler**  
*The American Mathematical Monthly*  
 Vol. 102, No. 2 (Feb., 1995), pp. 139-154

# Everyone ought to have a card trick up their sleeve...

**Arithmetic Matrices and the Amazing Nine-Card Monte**  
 Dean Clark and Dilip K. Datta, University of Rhode Island, Kingston, RI 02881

With few exceptions, mathematical analysis of the trick raises no interest. Generally, the trick is much more significant on some combinatorial principle. Here we deal with one of the escape properties of certain matrix subspaces themselves. An "ignored set" in the 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000.

**Arithmetic Matrices and the Amazing Nine-Card Monte**  
 Dean Clark and Dilip K. Datta  
*The College Mathematics Journal*  
 Vol. 24, No. 1 (Jan., 1993), pp. 52-56

If you were not there, you missed out....

# Projects

## Visualization of Matrix Singular Value Decomposition

Cliff Long  
 Bowling Green State University  
 Bowling Green, OH 43403

A real matrix is frequently used as a finite representation of a real function of two variables, especially as a tool for analyzing continuous functions on topological analysis and computer graphics. It is also advantageous to use continuous functions to provide visualization for matrix techniques such as singular value decomposition (SVD). We will illustrate how the factorization technique can be thought of as providing linear equations for the approximations to functions of two variables. The basic theory of SVD (sometimes called basic structure of a matrix) will be presented, one simple example given for illustration similar to those found in [1], and then a matrix representation of a sculptured head of Aho Lincoln will be used to illustrate the geometry involved. For ease in understanding, we'll restrict our attention to real matrices and refer the reader to [2], [3], [4], [5], and [6] for the proofs.



Figure 1

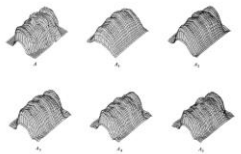


Figure 2: Another view decomposition of "Aho"

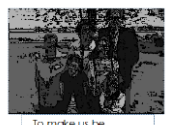
**Visualization of Matrix Singular Value Decomposition**  
 Cliff Long  
*Mathematics Magazine*  
 Vol. 56, No. 3 (May, 1983), pp. 161-167



Out of seeming chaos...



...you impart your love...



...to make us be...



...One happy family.  
 We would not be who we are without you.  
*Happy Mother's Day*

# Lacking Success: Reading articles

- Failure
- Expected too much
- What do you do?

# Why Have Students Read Articles?

- Enhance Analytic Abilities
- Gain confidence in reading the literature
- Provide insight into the research process
- Facilitate the transition to graduate school
- "It is well recognized that the use of primary literature improves undergraduate teaching..."

Hoskins, Sally G., David Iapichino, and Leslie M. Stevens. 2011. "The CREATE Approach to Primary Literature Shifts Undergraduates' Self-Assessed Ability to Read and Analyze Journal Articles, Attitudes about Science, and Epistemological Beliefs." *CBE—Life Sciences Education* 10(3):368-378.

Further: JONES, A., LEVINE, C., WILSON, C., PUTZICKI, LESLIE A., BOWEN, B., and MOHRMAN, A. 2010. "Integration of Information and Scientific Literacy: Promoting Literacy in Undergraduates." *CBE—Life Sciences Education* 9(4):545-554.

Kobonicki, Carol A., Michael F. Conroy, John Callahan, and Marc Lyle. February 2006. "An Innovative Primary Literature-based Teaching Program Develops Research Undergraduate Science Majors and Facilitates Their Transition to Doctoral Programs." *CBE—Life Sciences Education* 5(2):347-354.

# Lights Out

**Turning Lights Out with Linear Algebra**  
 MARLOW ANDERSON  
 Colorado College  
 Colorado Springs, CO 80903  
 TODD FEIL  
 Denver University  
 Golden, CO 80402

The game Lights Out, commercially available from Tiger Electronics, consists of a  $5 \times 3$  array of 15 lighted buttons, each light may be on or off. A move consists of pushing one or more buttons, which causes each of the lights on the buttons to toggle.

**Turning Lights Out with Linear Algebra**  
 Marlow Anderson and Todd Feil  
 Mathematics Magazine  
 Vol. 71, No. 4 (Oct., 1998), pp. 300-303

**Theorem 1.** A configuration  $b$  is winnable if and only if  $b$  is perpendicular to the two vectors  $h_1$  and  $h_2$ .

Therefore, to see if a configuration is winnable, we simply compute the dot product of that configuration with  $h_1$  and  $h_2$ . For example, consider the configurations below (which we have shaped as  $5 \times 3$  arrays):

$$f = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad g = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Then  $f$  is winnable, while  $g$  is not (it is not perpendicular to  $h_1$ ).

Since the dimension of the real space is 2 and the scalar field is  $\mathbb{Z}_2$ , it follows from this theorem that of the  $2^{15}$  possible configurations, only one-fourth of them are winnable. Furthermore, if  $b$  is a winnable configuration with winning strategy  $x$ , then  $\bar{x} + x$ ,  $x + x_1$ , and  $x + x_2$  are also winning strategies.

Suppose now that  $f$  is a winnable configuration. We would like to find one of the four strategies  $x$  for which  $Ax = f$ . But since we need only find one solution, we may as well set the two free variables  $x_1$  and  $x_2$  equal to zero. In this case  $x = EX$ ,  $f = EX = BAx = Bf$ . Explicitly, we have a winning strategy given by  $x = Bf$ . We thus have the following theorem:

**Theorem 2.** Suppose that  $b$  is a winnable configuration. Then the four winning strategies for  $b$  are

$$Bf, \quad Bf + h_1, \quad Bf + h_2, \quad Bf + h_1 + h_2.$$

We observed above that the configuration  $f$  is winnable. To find a winning strategy, we compute  $Bf$  (where we reshape  $f$  as a column vector):

$$Bf = (0, 0, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0)^T.$$

# Provide Context

- lights out 2\lights out 2.mp4
- www.whitman.edu/mathematics/lights\_out/

# Prepare and Set Expectations

- Questions to answer on the article:**
1. What does the vector  $b$  represent?
  2. What does the vector  $x$  represent?
  3. By definition, when is a configuration  $b$  winnable. By the theorem, when is a configuration winnable?
  4. What does the matrix  $R$  represent? (Remember that these matrices are in mod 2.)
  5. What does  $2^{23}/2^{25}$  equal? How does this show that 1/4 of the configurations are winnable?
  6. What is the rank( $E$ )?
- There are two main parts to the article.
- In Linear Algebra terms, when is a configuration winnable? In Linear Algebra terms, given a winnable configuration, what calculation can be made to determine a winning strategy?
  - Given a winning strategy for a particular winnable configuration, how many other winning strategies are there? How can they be found?

# Additional Articles

**An Easy Solution to Mini Lights Out**  
 Jennie Missigman and Richard Weida  
 Mathematics Magazine  
 Vol. 74, No. 1 (Feb., 2001), pp. 57-59

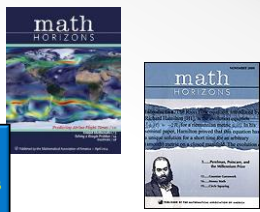
**Two Reflected Analyses of Lights Out**  
 Óscar Martín-Sánchez and Cristóbal Pareja-Flores  
 Mathematics Magazine  
 Vol. 74, No. 4 (Oct., 2001), pp. 295-304

# Provide Choices

- Projects (Select one of the four options.) Each of these are in two parts.**
1. Analyzing student errors from the NAEP. (Designed for education majors)
  2. Error Correcting codes and the hat problem
  3. Democracy and the SVD. How many judges or senators do we really need?
  4. The Linear Algebra behind search engines

# Error Correcting codes and the Hat Problem

**A Dozen Hat Problems**  
 Ezra Brown and James Tanton  
 Math Horizons  
 Vol. 16, No. 4 (April 2009), pp. 22-25



## Error Correcting codes and the Hat problem

FOCUS

November 2001

### The Hat Problem and Hamming Codes

By Mira Bernstein

Consider the following two problems—one, an entertaining new puzzle, the other, an important practical question:

**Problem 1:** At a mathematical game show with  $n$  players, the host blindfolds the contestants and puts colored hats on their

our discussion with Problem 1, a recent hit in the mathematical community. An article in last April's *Science Times* [2] dates the puzzle's first appearance to 1996 and tells of its "spreading through networks of mathematicians like a juicy bit of gossip". Since the publication of the article, the gossip has spread ever faster and further. While the "hat problem" itself is just a restatement of an old cases-

plete set of instructions for each player: if you observe  $k$  blue  $T$ 's we consider only deterministic strategies). Given  $S$ , each possible hat configuration will be either "winning" or "losing". Denote the set of winning configurations by  $W$ , and the set of losing configurations by  $L$ . Since all configurations are equally likely, the team's probability of winning is  $|W|/2^n$ . The goal, therefore, is to find  $S$  such that  $|W|$  is maximal.



M. Bernstein, *The hat problem and Hamming codes*, MAA Focus, November 2001, 4-8.

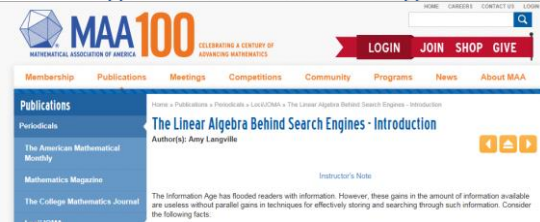
Write something a brief summary including a) Whether you read all three articles, b) how you could get your \_\_\_\_ friends to win at the hat game. (Fill in the blank with the optimal number, and then go through the strategy) c) Some remark on whether you enjoyed this result and why.

## Option 3: Singular Vectors' Subtle Secrets

1. Write a one page summary of the article including the methodology, the other applications of the SVD and how many judges we actually need according to the article examining the 2008 court.) What judges vote like each other? As in part 1, list the current judges and divide them into conservative, moderate and liberal.
2. Find your own set of data and do an SVD analysis as done in the article. Ask and answer an interesting question regarding the data.

**Singular Vectors' Subtle Secrets**  
David James, Michael Lachance and Joan Remski *The College Mathematics Journal* Vol. 42, No. 2 (March 2011), pp. 86-95

## Option 4: Loci Article "The Linear Algebra Behind Search Engines"



<http://www.maa.org/node/115883>

## Why MAA Articles?

- Accessibility
- Accessibility
- Narrowing the topic

## Annotated Reading

- Print out the article. You will turn in a copy of the article with annotations.
  - Highlight all of the terms we used in the class.
    - Suggested color: yellow.
  - Highlight all of the terms you do not know.
    - Suggested color: red.
- Reflect on your experiences in studying Linear Algebra and the connections between Linear Algebra and your earlier mathematical or anticipated quantitative experiences.
- After reflecting, write a short response describing your impressions of the article and describing your experiences in Linear Algebra. How has this class been helpful in developing your thinking? What connections do you see? What is so important about Linear Algebra?

### The Growing Importance of Linear Algebra in Undergraduate Mathematics

Alan Tucker



Alan Tucker is SUNY Distinguished Teaching Professor of Applied Mathematics at the State University of New York-Stony Brook. He obtained his Ph.D. in mathematics from Stanford University in 1969. Dr. Tucker has been at Stony Brook since 1970 except for sabbaticals at Stanford and UC-San Diego. He is current chair of the MAA Education Council and was MAA First Vice-President during 1988-1989. He comes from a very mathematical family, from grandfathers up to his two daughters. His father A. W. Tucker and maternal grandfather D. R. Curtis were MAA Presidents, his brother Tom is past First Vice-President.

**The Growing Importance of Linear Algebra in Undergraduate Mathematics**  
Alan Tucker  
*The College Mathematics Journal*  
Vol. 24, No. 1 (Jan., 1993), pp. 3-9

Linear algebra stands today as the epitome of accessible, yet powerful mathematical theory. Linear algebra has many appealing facets which radiate in different directions. In the 1960s, linear algebra was positioned to be the first real mathematics course in the undergraduate mathematics curriculum in part because its theory is so well structured and comprehensive, yet requires limited mathematical prerequisites. A mastery of finite vector spaces, linear transformations, and their extensions to function spaces is essential for a practitioner or researcher in most areas of pure and applied mathematics. Linear algebra is the mathematics of our modern technological world of complex multivariable systems and computers.

## Suggestions for Reading

- Provide Context and Guidance
- Provide Choice
- Assign Annotated Reading

the ground rules for much of higher analysis, advanced geometry, statistics, operations research, and computational applied mathematics. For example, one of these ground rules is that it suffices to understand the action of a linear transformation on a set of basis functions and then let linearity do the rest.

Linear algebra really is a model for what a mathematical theory should be!

4

THE COLLEGE MATHEMATICS JOURNAL

## Kaitlin Russ quote

*Linear Algebra is the most powerful thing I have held in my hand.*

*Kaitlin Russ*

## Contact Information

Jeremy Case, [jrcase@taylor.edu](mailto:jrcase@taylor.edu)

[faculty.taylor.edu/jrcase/Favorites15.docx](http://faculty.taylor.edu/jrcase/Favorites15.docx)

[To appear Thursday, January 15, 2015]