My Favorite MAA Articles for Linear Algebra

January 11, 2015

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Assumptions

Not all great applications can be included in a textbook.

Linear Algebra services a variety of disciplines.

Journal articles are a good thing.

Incorporating journal articles is not automatically easy.

Applications of Linear Algebra in Calculus

Jack W. Rogers, Jr.

1. INTRODUCTION. The concepts of basis, matrix for a linear transformation relative to bases, and change-of-basis matrix are fundamental in linear algebra, but students in an introductory class often have trouble understanding the point of applying these concepts for bases other than the standard basis for $\mathbb{R}^n$. Our object is to illustrate some applications of these concepts in solving problems with which students who have recently completed the calculus sequence should be familiar. The spaces are abstract vector spaces—finite subspaces of function spaces—not

Applications of Linear Algebra in Calculus
Jack W. Rogers, Jr.,
The American Mathematical Monthly,
Vol. 104, No. 1
(Jan., 1997), pp. 20-26.

For this problem, the matrix $D = M_{2;\mathscr{B}\leftarrow\mathscr{B}}$ for $\mathscr{D}$ relative to $\mathscr{B}$ is

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad \text{and} \quad D^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix}.$$

The single matrix inversion provides all the following antiderivatives, with cients given by the columns of D^{-1}.

$$\int t^2e^t\,dx = t^2e^t - 2te^t + 2e^t,\ \int te^t\,dx = te^t - e^t, \text{ and } \int e^t\,dx = e^t.$$

Matrix inversion has replaced the use of integration by parts for this proble
The same technique can be used to provide an alternative approach for standard integration by parts problems. For $\int t\sin t\,dx$, for example, let

PITFALLS IN COMPUTATION, OR WHY A MATH BOOK ISN'T ENOUGH

GEORGE E. FORSYTHE, Computer Science Dept., Stanford University

1. Introduction. Why does a student take mathematics in college sity? I see two reasons: (i) To learn the structure of mathematics its se he (or she) finds it interesting. (ii) To prepare to apply mathema solution of problems he expects to encounter in his own field, wheth ineering, physics, economics, or whatever.
Surely (ii) motivates far more students than (i). Moreover, most so

Pitfalls in Computation, or Why a Math Book Isn't Enough,"
George E. Forsythe
The American Mathematical Monthly
(Nov. 1970)
20-26.

f linear algebraic equations by Gauss' method of eliminating unknowns. a little systematization, it becomes another algorithm for general use. I like to examine it in the simple case of two equations in two unknowns, d out on a computer with $\beta=10$, $s=3$.
t the equation system be one treated by Forsythe and Moler [3]:

$$\begin{cases} 0.000100x + 1.00y = 1.00 \\ 1.00x + 1.00y = 2.00. \end{cases}$$

ue solution, rounded correctly to the number of decimals shown, is

$$x \approx 1.00010,\ y \approx 0.99990 \quad \text{(truly rounded).}$$

e Gauss elimination algorithm uses the first equation (if possible) to ate the first variable, x, from the second equation. Here this is done by lving the first equation by 10000 and then subtracting it from the second

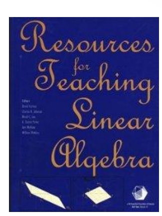
More Numerical Issues

Numerical Linear Algebra on the HP-28 or How to Lie With Supercalculators
Yves Nievergelt
The American Mathematical Monthly
Vol. 98, No. 6 (Jun. - Jul., 1991), pp. 539-544

Six, Lies, and Calculators
R. M. Corless
The American Mathematical Monthly
Vol. 100, No. 4 (Apr., 1993), pp. 344-350

My Favorite MAA Journal for Linear Algebra

College Mathematics Journal, Vol 24, Jan 1993



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Gilbert Strang, Massachusetts Institute of Technology

Provocative…

Down with Determinants!
Sheldon Axler
The American Mathematical Monthly
Vol. 102, No. 2 (Feb., 1995), pp. 139-154

Everyone ought to have a card trick up their sleeve…

Arithmetic Matrices and the Amazing Nine-Card Monte
Dean Clark and Dilip K. Datta, University of Rhode Island, Kingston, RI 02881

With few exceptions, mathematical ca
analysis of the trick raises no interest
Generally, the trick is much more enga
turns on some commonplace principle c
Here we deal with one of the except
properties of certain matrix subspaces
themselves. An "ignored set" in the L
denly takes on critical importance. K_3
houses and three utilities" fame) appea
In other words, the mathematics is at
may be. Therefore, we talk first about th
avoid the anticlimatic effect of knowing
with the second edition, The Fraud.

Arithmetic Matrices and the Amazing Nine-Card Monte
Dean Clark and Dilip K. Datta
The College Mathematics Journal
Vol. 24, No. 1 (Jan., 1993), pp. 52-56

If you were not there, you missed out….

Projects

Visualization of Matrix Singular Value Decomposition

Cliff Long
Bowling Green State University
Bowling Green, OH 43403

A real matrix is frequently used as a finite representation of a real function of two variables, especially as a tool for studying continuous functions in numerical analysis and computer graphics. It is also advantageous to use continuous functions to provide visualization for matrix techniques such as **singular value decomposition** (SVD). We will illustrate how this factorization technique can be thought of as providing least square best fit approximations to functions of two variables. The basic theory of SVD (sometimes called basic structure of a matrix) will be presented, one simple example given for clarification (similar to those found in [7]), and then a matrix representation of a sculptured head of Abe Lincoln will be used to illustrate the geometry involved. For ease in understanding, we'll restrict our attention to real matrices and refer the reader to [2], [4], [9], and [11] for the proofs.

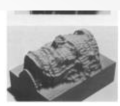


Figure 2

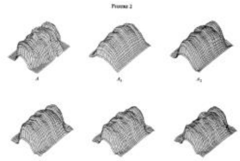
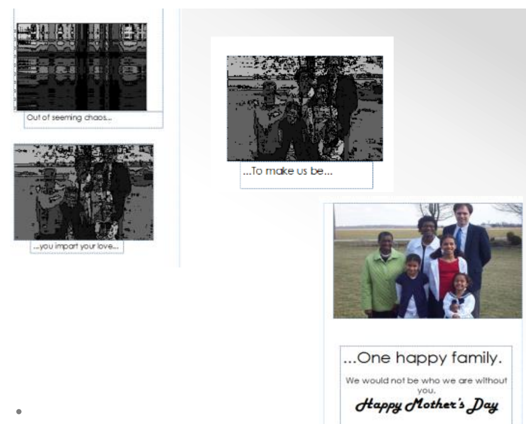


Figure 1. Singular value decomposition of "ABE."

Visualization of Matrix Singular Value Decomposition
Cliff Long
Mathematics Magazine
Vol. 56, No. 3 (May, 1983), pp. 161-167



Lacking Success: Reading articles

- Failure
- Expected too much
- What do you do?

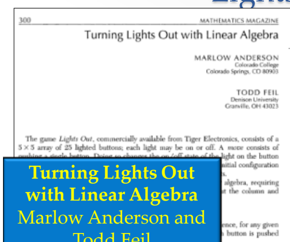
Why Have Students Read Articles?

- Enhance Analytic Abilities
- Gain confidence in reading the literature
- Provide insight into the research process
- Facilitate the transition to graduate school

- *"It is well recognized that the use of primary literature improves undergraduate teaching…"*

- Hoskins, Sally G., David Lopatto, and Leslie M. Stevens. 2011. "The CREATE Approach to Primary Literature Shifts Undergraduates' Self-Assessed Ability to Read and Analyze Journal Articles, Attitudes about Science, and Epistemological Beliefs". CBE-LSE 10:368-378
- Porter, Jason A., Kevin C. Wolbach, Catherine B. Purzycki, Leslie A. Bowman, Eva Agbada, Alison M. Mostrum. 2010. "Integration of Information and Scientific Literacy: Promoting Literacy in Undergraduates." CBE-LSE 9: 536-542
- Kozeracki, Carol A., Michael F. Carey, John Colicelli, and Marc Levis-Fitzgerald. 2006. "An Intensive Primary-Literature-based Teaching Program Directly Benefits Undergraduate Science Majors and Facilitates Their Transition to Doctoral Programs." CBE-LSE 5:340-347.

Lights Out

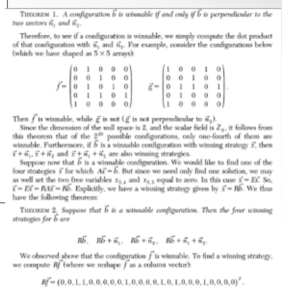

300 MATHEMATICS MAGAZINE

Turning Lights Out with Linear Algebra

MARLOW ANDERSON
Colorado College
Colorado Springs, CO 80903

TODD FEIL
Denison University
Granville, OH 43023

The game *Lights Out*, commercially available from Tiger Electronics, consists of a 5×5 array of 25 lighted buttons; each light may be on or off. A move consists of pushing a single button. Doing so changes the on/off state of the light on the button ... initial configuration ... algebra, requiring ... the column and ... ence, for any given ... button is pushed


THEOREM 1. *A configuration $\vec{b}$ is winnable if and only if $\vec{b}$ is perpendicular to the two vectors $\vec{n}_1$ and $\vec{n}_2$.*

Therefore, to see if a configuration is winnable, we simply compute the dot product of that configuration with $\vec{n}_1$ and $\vec{n}_2$. For example, consider the configurations below (which we have shaped as 5×5 arrays):

$$\vec{f} = \begin{pmatrix} 0&1&0&0&0\\ 0&0&1&0&0\\ 0&1&0&1&0\\ 0&1&1&0&1\\ 1&0&0&0&0 \end{pmatrix} \qquad \vec{g} = \begin{pmatrix} 1&0&0&1&0\\ 0&0&1&0&0\\ 0&1&1&0&1\\ 0&1&0&0&0\\ 1&0&0&0&0 \end{pmatrix}.$$

Then $\vec{f}$ is winnable, while $\vec{g}$ is not ($\vec{g}$ is not perpendicular to $\vec{n}_1$).

Since the dimension of the null space is 2, and the scalar field is Z_2, it follows from this theorem that of the 2^{25} possible configurations, only one-fourth of them are winnable. Furthermore, if $\vec{b}$ is a winnable configuration with winning strategy $\vec{x}$, then $\vec{x}+\vec{n}_1$, $\vec{x}+\vec{n}_2$ and $\vec{x}+\vec{n}_1+\vec{n}_2$ are also winning strategies.

Suppose now that $\vec{b}$ is a winnable configuration. We would like to find one of the four strategies $\vec{x}$ for which $A\vec{x}=\vec{b}$. But since we need only find one solution, we may as well set the two free variables $x_{5,4}$ and $x_{5,5}$ equal to zero. In this case $\vec{x}=E\vec{x}$. So, $\vec{x}=E\vec{x}=RA\vec{x}=R\vec{b}$. Explicitly, we have a winning strategy given by $\vec{x}=R\vec{b}$. We thus have the following theorem:

THEOREM 2. *Suppose that $\vec{b}$ is a winnable configuration. Then the four winning strategies for $\vec{b}$ are*

$$R\vec{b},\quad R\vec{b}+\vec{n}_1,\quad R\vec{b}+\vec{n}_2,\quad R\vec{b}+\vec{n}_1+\vec{n}_2.$$

We observed above that the configuration $\vec{f}$ is winnable. To find a winning strategy, we compute $R\vec{f}$ (where we reshape $\vec{f}$ as a column vector):

$$R\vec{f} = (0,0,1,1,0,0,0,0,0,1,0,0,0,0,1,0,1,0,0,0,1,0,0,0,0)^T.$$

Turning Lights Out with Linear Algebra
Marlow Anderson and Todd Feil
Mathematics Magazine
Vol. 71, No. 4 (Oct., 1998), pp. 300-303

Provide Context

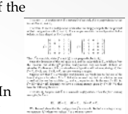
- lights out 2\lights out 2.mp4

www.whitman.edu/mathematics/lights_out/

Prepare and Set Expectations

Questions to answer on the article:

1. What does the vector b represent?
2. What does the vector x represent?
3. By definition, when is a configuration b winnable. By the theorem, when is a configuration winnable?
4. What does the matrix R represent? (Remember that these matrices are in mod 2.)
5. What does 2^23/2^25 equal? How does this show that 1/4 of the configurations are winnable?
6. What is the rank(E)?



There are two main parts to the article.

- In Linear Algebra terms, when is a configuration winnable?In Linear Algebra terms, given a winnable configuration, what calculation can be made to determine a winning strategy?
- Given a winning strategy for a particular winnable configuration, how many other winning strategies are there? How can they be found?

Additional Articles

An Easy Solution to Mini Lights Out
Jennie Missigman and Richard Weida
Mathematics Magazine
Vol. 74, No. 1 (Feb., 2001), pp. 57-59

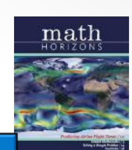
Two Reflected Analyses of Lights Out
Óscar Martín-Sánchez and Cristóbal Pareja-Flores
Mathematics Magazine
Vol. 74, No. 4 (Oct., 2001), pp. 295-304

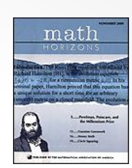
Provide Choices

Projects (Select one of the four options.) Each of these are in two parts.

1. Analyzing student errors from the NAEP. (Designed for education majors)
2. Error Correcting codes and the hat problem
3. Democracy and the SVD. How many judges or senators do we really need?
4. The Linear Algebra behind search engines

Error Correcting codes and the Hat Problem





A Dozen Hat Problems
Ezra Brown and James Tanton
Math Horizons
Vol. 16, No. 4 (April 2009), pp. 22-25

Error Correcting codes and the Hat problem

FOCUS November 2001

The Hat Problem and Hamming Codes

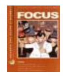
By Mira Bernstein

Consider the following two problems –one, an entertaining new puzzle, the other, an important practical question:

Problem 1: At a mathematical game show with *n* players, the host blindfolds the contestants and puts colored hats on their

our discussion with Problem 1, a recent hit in the mathematical community. An article in last April's *Science Times* [2] dates the puzzle's first appearance to 1998 and tells of it "spreading through networks of mathematicians like a juicy bit of gossip". Since the publication of the article, the gossip has spread ever faster and further. While the "hat problem" itself is just a restatement of an old ques-

plete set of instructions for each player: if you observe *X*, do *Y* (we consider only deterministic strategies). Given *S*, each possible hat configuration will be either "winning" or "losing". Denote the set of winning configurations by W_S and the set of losing configurations by L_S. Since all configurations are equally likely, the team's probability of winning is $|W_S|/2^n$. The goal, therefore, is to find *S* such that $|W_S|$ is maximal.



Write something a brief summary including a) Whether you read all three articles. b) how you could get your _____ friends to win at the hat game. (Fill in the blank with the optimal number, and then go through the strategy.) c) Some remark on whether you enjoyed this result and why.

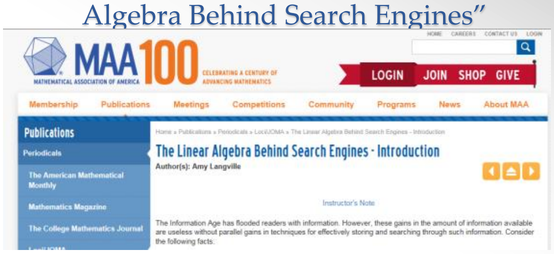
M. Bernstein, *The hat problem and Hamming codes*, MAA Focus, November, 2001, 4–6.

Option 3: Singular Vectors' Subtle Secrets

Singular Vectors' Subtle Secrets
David James, Michael Lachance and Joan Remski *The College Mathematics Journal*
Vol. 42, No. 2 (March 2011), pp. 86-95

1. Write a one page summary of the article including the methodology, the other applications of the SVD and how many judges we actually need according to the article examining the 2008 court.) What judges vote like each other? As in part 1, list the current judges and divide them into conservative, moderate and liberal.
2. Find your own set of data and do an SVD analysis as done in the article. Ask and answer an interesting question regarding the data.

Option 4: Loci Article "The Linear Algebra Behind Search Engines"



http://www.maa.org/node/115883

Why MAA Articles?

- Accessibility
- Accessibility
- Narrowing the topic

Annotated Reading

- Print out the article. You will turn in a copy of the article with annotations.
  - Highlight all of the terms we used in the class.
    - Suggested color: yellow.
  - Highlight all of the terms you do not know.
    - Suggested color: red.
- Reflect on your experiences in studying Linear Algebra and the connections between Linear Algebra and your earlier mathematical or anticipated quantitative experiences.
- After reflecting, write a short response describing your impressions of the article and describing your experiences in Linear Algebra. How has this class been helpful in developing your thinking? What connections do you see? What is so important about Linear Algebra?

The Growing Importance of Linear Algebra in Undergraduate Mathematics
Alan Tucker
The College Mathematics Journal
Vol. 24, No. 1 (Jan., 1993), pp. 3-9

The Growing Importance of Linear Algebra in Undergraduate Mathematics

Alan Tucker

Alan Tucker is SUNY Distinguished Teaching Professor of Applied Mathematics at the State University of New York-Stony Brook. He obtained his Ph.D. in mathematics from Stanford University in 1969. Dr. Tucker has been at Stony Brook since 1970 except for sabbaticals at Stanford and UC-San Diego. He is current chair of the MAA Education Council and was MAA First Vice-President during 1988–1989. He comes from a very mathematical family, from grandfathers up to his two daughters. His father A. W. Tucker and (maternal) grandfather D. R. Curtiss were MAA Presidents; his brother Tom is past First Vice-President.

Linear algebra stands today as the epitome of accessible, yet powerful mathematical theory. Linear algebra has many appealing facets which radiate in different directions. In the 1960s, linear algebra was positioned to be the first real mathematics course in the undergraduate mathematics curriculum in part because its theory is so well structured and comprehensive, yet requires limited mathematical prerequisites. A mastery of finite vector spaces, linear transformations, and their extensions to function spaces is essential for a practitioner or researcher in most areas of pure and applied mathematics. Linear algebra is the mathematics of our modern technological world of complex multivariable systems and computers

Suggestions for Reading

- Provide Context and Guidance
- Provide Choice
- Assign Annotated Reading

the ground rules for much of higher analysis, advanced geometry, statistics, operations research, and computational applied mathematics. For example, one of these ground rules is that it suffices to understand the action of a linear transformation on a set of basis functions and then let linearity do the rest.

Linear algebra really is a model for what a mathematical theory should be!

4 THE COLLEGE MATHEMATICS JOURNAL

Kaitlin Russ quote

Linear Algebra is the most powerful thing I have held in my hand.

Kaitlin Russ

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