Preparing Students for Linear Algebra Concepts with Pre-class Work

Feryal Alayont
alayontf@gvsu.edu

Grand Valley State University

January 17, 2014

Overview

Overview

- Background of the linear algebra course

Overview

- Background of the linear algebra course
- How I design the course

Overview

- Background of the linear algebra course
- How I design the course
- Student reaction

Linear Algebra

Linear Algebra

- Only prerequisite Calculus 2

Linear Algebra

- Only prerequisite Calculus 2
- Required for math, physics and statistics majors,

Linear Algebra

- Only prerequisite Calculus 2
- Required for math, physics and statistics majors, draws a few students from other majors

Linear Algebra

- Only prerequisite Calculus 2
- Required for math, physics and statistics majors, draws a few students from other majors
- Topics covered: Systems of linear equations, vectors, matrix algebra, transformations, vector spaces, eigenvalues/eigenvectors

Challenges

Challenges

- A different level of abstraction than calculus tackled with only calculus background

Challenges

- A different level of abstraction than calculus tackled with only calculus background
- Unfamiliarity with evaluating truth of mathematical statements

Challenges

- A different level of abstraction than calculus tackled with only calculus background
- Unfamiliarity with evaluating truth of mathematical statements
- A lot of seemingly unrelated topics

Challenges

- A different level of abstraction than calculus tackled with only calculus background
- Unfamiliarity with evaluating truth of mathematical statements
- A lot of seemingly unrelated topics
- Questioning the value of abstraction and conceptual understanding

How do I teach?

How do I teach?

- Students do learning outside of class using pre-class activities

How do I teach?

- Students do learning outside of class using pre-class activities
- Significant amount of group work in the classroom

How do I teach?

- Students do learning outside of class using pre-class activities
- Significant amount of group work in the classroom
- Rest: pair homework, exams and project

How do pre-class activities work?

How do pre-class activities work?

- Handed out at the end of each class period, due next class period

How do pre-class activities work?

- Handed out at the end of each class period, due next class period
- Material ties to the topic of the day and the in-class activities

How do pre-class activities work?

- Handed out at the end of each class period, due next class period
- Material ties to the topic of the day and the in-class activities
- Prepares students for class:

How do pre-class activities work?

- Handed out at the end of each class period, due next class period
- Material ties to the topic of the day and the in-class activities
- Prepares students for class: Examples/counterexamples,

How do pre-class activities work?

- Handed out at the end of each class period, due next class period
- Material ties to the topic of the day and the in-class activities
- Prepares students for class: Examples/counterexamples, conjecturing,

How do pre-class activities work?

- Handed out at the end of each class period, due next class period
- Material ties to the topic of the day and the in-class activities
- Prepares students for class: Examples/counterexamples, conjecturing, connections,

How do pre-class activities work?

- Handed out at the end of each class period, due next class period
- Material ties to the topic of the day and the in-class activities
- Prepares students for class: Examples/counterexamples, conjecturing, connections, applying simple definitions,

How do pre-class activities work?

- Handed out at the end of each class period, due next class period
- Material ties to the topic of the day and the in-class activities
- Prepares students for class: Examples/counterexamples, conjecturing, connections, applying simple definitions, visualization

How do pre-class activities work?

- Handed out at the end of each class period, due next class period
- Material ties to the topic of the day and the in-class activities
- Prepares students for class: Examples/counterexamples, conjecturing, connections, applying simple definitions, visualization
- Discussion of pre-class in the classroom

How do pre-class activities work?

- Handed out at the end of each class period, due next class period
- Material ties to the topic of the day and the in-class activities
- Prepares students for class: Examples/counterexamples, conjecturing, connections, applying simple definitions, visualization
- Discussion of pre-class in the classroom
- Assign participation points

Why do I use the pre-class activities?

Why do I use the pre-class activities?

- Save class time

Why do I use the pre-class activities?

- Save class time
- Have students ready for the new material

Why do I use the pre-class activities?

- Save class time
- Have students ready for the new material
- Fits CCSS of Standards for Mathematical Practice:

Why do I use the pre-class activities?

- Save class time
- Have students ready for the new material
- Fits CCSS of Standards for Mathematical Practice: Provides practice for mathematician skills including conjecturing, recognizing patterns and using appropriate tools

Why do I use the pre-class activities?

- Save class time
- Have students ready for the new material
- Fits CCSS of Standards for Mathematical Practice: Provides practice for mathematician skills including conjecturing, recognizing patterns and using appropriate tools
- Fits constructivist learning theory:

Why do I use the pre-class activities?

- Save class time
- Have students ready for the new material
- Fits CCSS of Standards for Mathematical Practice: Provides practice for mathematician skills including conjecturing, recognizing patterns and using appropriate tools
- Fits constructivist learning theory: Build new knowledge on existing knowledge

Why do I use the pre-class activities?

- Save class time
- Have students ready for the new material
- Fits CCSS of Standards for Mathematical Practice: Provides practice for mathematician skills including conjecturing, recognizing patterns and using appropriate tools
- Fits constructivist learning theory: Build new knowledge on existing knowledge
- Fits transfer theory:

Why do I use the pre-class activities?

- Save class time
- Have students ready for the new material
- Fits CCSS of Standards for Mathematical Practice: Provides practice for mathematician skills including conjecturing, recognizing patterns and using appropriate tools
- Fits constructivist learning theory: Build new knowledge on existing knowledge
- Fits transfer theory: Adaptive experts have well-structured rich knowledge bases and possess developed metacognitive skills

How are students convinced to do the activities?

How are students convinced to do the activities?

- Emphasize skills development in addition to content

How are students convinced to do the activities?

- Emphasize skills development in addition to content
- Previous student advice

How are students convinced to do the activities?

- Emphasize skills development in addition to content
- Previous student advice
- Start early

How are students convinced to do the activities?

- Emphasize skills development in addition to content
- Previous student advice
- Start early
- Participation points

How are students convinced to do the activities?

- Emphasize skills development in addition to content
- Previous student advice
- Start early
- Participation points
- Sell the activities: interesting and fun,

How are students convinced to do the activities?

- Emphasize skills development in addition to content
- Previous student advice
- Start early
- Participation points
- Sell the activities: interesting and fun, clearly linked to class material,

How are students convinced to do the activities?

- Emphasize skills development in addition to content
- Previous student advice
- Start early
- Participation points
- Sell the activities: interesting and fun, clearly linked to class material, enthusiasm about the activities

Theorem 4 provided us with an easy criterion to determine whether a given set of vectors span the whole space or not. In this activity, you will use this criterion to determine which vector sets are optimal spanning sets.

1. For each of the following sets of vectors, determine whether they span $\mathbf{R}^2$ or not. Make sure to include the REF or RREF of the matrix A that you used while applying Theorem 4.

a. $\mathbf{v_1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

b. $\mathbf{v_1} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

c. $\mathbf{v_1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

d. $\mathbf{v_1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}, \mathbf{v_3} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

2. For each of the following sets of vectors, determine whether they span $\mathbf{R}^3$ or not. Make sure to include the REF or RREF of the matrix A that you used while applying Theorem 4.

a. $\mathbf{v_1} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$

b. $\mathbf{v_1} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \mathbf{v_3} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$

c. $\mathbf{v_1} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \mathbf{v_3} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$

d. $\mathbf{v_1} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \mathbf{v_3} = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}, \mathbf{v_4} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$

When Does the Inverse of a Matrix Exist?
Columns/Rows Perspective
Pre-class due Monday, October 28

We saw earlier that an invertible matrix has a non-zero determinant. But which matrices have non-zero determinants? In this activity, we will try to figure out an invertibility criterion(criteria) in terms of the columns/rows of the matrix. This will help us in determining an algorithm for finding the inverse of a matrix.

Problem 1: Find the inverses (or determinants) of the following matrices using a calculator. If the inverse does not exist, indicate so.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad , \quad \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix} \quad , \quad \begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix} \quad , \quad \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} \quad , \quad \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix}$$

Using the results, conjecture a property of rows and/or columns for a 2×2 matrix whose inverse does not exist.

Problem 2: Find the inverses (or determinants) of the following matrices using a calculator. If the inverse does not exist, indicate so.

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ -1 & 1 & 1 \end{bmatrix} , \begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & -1 \\ 2 & -2 & 3 \end{bmatrix} , \begin{bmatrix} 2 & -1 & 1 \\ 4 & -2 & 2 \\ 1 & 1 & 1 \end{bmatrix} , \begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & 2 \\ 2 & 1 & 3 \end{bmatrix} , \begin{bmatrix} -2 & 1 & 3 \\ 1 & 2 & -1 \\ 2 & 2 & -3 \end{bmatrix} , \begin{bmatrix} 1 & 1 & -1 & 2 \\ 2 & -1 & 2 & 1 \\ 4 & 1 & 1 & 1 \\ 3 & 0 & 1 & 3 \end{bmatrix}$$

Using the results, conjecture a property of rows and/or columns for a 3×3 matrix whose inverse does not exist.

Student attempt and completion rates

Student attempt and completion rates

- A vast majority complete the activities

Student attempt and completion rates

- A vast majority complete the activities (and attend class)

Student attempt and completion rates

- A vast majority complete the activities (and attend class)
- By providing intermediate answers, completion rate increases

Student attempt and completion rates

- A vast majority complete the activities (and attend class)
- By providing intermediate answers, completion rate increases
- Mid-end of semester, 3-4 out of 30 students might skip

Evaluations

Evaluations

- She provided many examples to help with concepts.

Evaluations

- She provided many examples to help with concepts.
- She has been very effective in that in order to succeed in the class, you must be involved and know whats going on for yourself.

Evaluations

- She provided many examples to help with concepts.
- She has been very effective in that in order to succeed in the class, you must be involved and know whats going on for yourself.
- Pre-class, in-class, and handouts were very helpful. Working with other people in the class instead of just listening to lecture was beneficial.

Evaluations

- She provided many examples to help with concepts.
- She has been very effective in that in order to succeed in the class, you must be involved and know whats going on for yourself.
- Pre-class, in-class, and handouts were very helpful. Working with other people in the class instead of just listening to lecture was beneficial.
- I personally liked/benefited from the self-paced discovery learning assignments, they helped me reach a greater understanding of topics by uncovering results on my own. Pre-class activities helped me stay engaged and are a great resource when studying for exams.

Evaluations

- She provided many examples to help with concepts.
- She has been very effective in that in order to succeed in the class, you must be involved and know whats going on for yourself.
- Pre-class, in-class, and handouts were very helpful. Working with other people in the class instead of just listening to lecture was beneficial.
- I personally liked/benefited from the self-paced discovery learning assignments, they helped me reach a greater understanding of topics by uncovering results on my own. Pre-class activities helped me stay engaged and are a great resource when studying for exams.
- My instructor could have gone over more examples in class instead of just having the students try to figure it out in pre-class activities, or give more clarification.

Important points to consider

Important points to consider

- Pre-class activity only when it is really useful

Important points to consider

- Pre-class activity only when it is really useful
- Clear connection between the pre-class and in-class work

Important points to consider

- Pre-class activity only when it is really useful
- Clear connection between the pre-class and in-class work
- Reasonably long and reasonably interesting assignments

Important points to consider

- Pre-class activity only when it is really useful
- Clear connection between the pre-class and in-class work
- Reasonably long and reasonably interesting assignments
- Students are allowed to be not completely correct

Important points to consider

- Pre-class activity only when it is really useful
- Clear connection between the pre-class and in-class work
- Reasonably long and reasonably interesting assignments
- Students are allowed to be not completely correct
- Can be combined with reading

Important points to consider

- Pre-class activity only when it is really useful
- Clear connection between the pre-class and in-class work
- Reasonably long and reasonably interesting assignments
- Students are allowed to be not completely correct
- Can be combined with reading
- Building a pre-class activity collection over time

Thank you for listening!