Bilbo and the Last Moon of Autumn

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WHAT IS THIS?
THERE ARE MOON-LETTERS HERE, BESIDE THE PLAIN RUNES WHICH SAY "FIVE FEET HIGH THE DOOR AND THREE MAY WALK ABREAST."
WHAT ARE MOON-LETTERS?

They must arrive in the waning light on the first day of the last moon of autumn

Thorin: This fact will not help us much---it passes our skill in these days to guess when such a time will come again.

Two Rules of Toe

- A new moon will occur on the same day as it did 19 years ago
- A new moon will occur two days later than it did 160 years ago

$T_m \approx 2,360,591.5$ seconds

$T_s \approx 31,558,149.5$ seconds

$\sigma_0 = T_s/T_m \approx 13.368747$ rotations per year

$\sigma = \sigma_0 - 1 \approx 12.368747$ times in one year

$t = 0 \longleftrightarrow$ noon on Nov 7

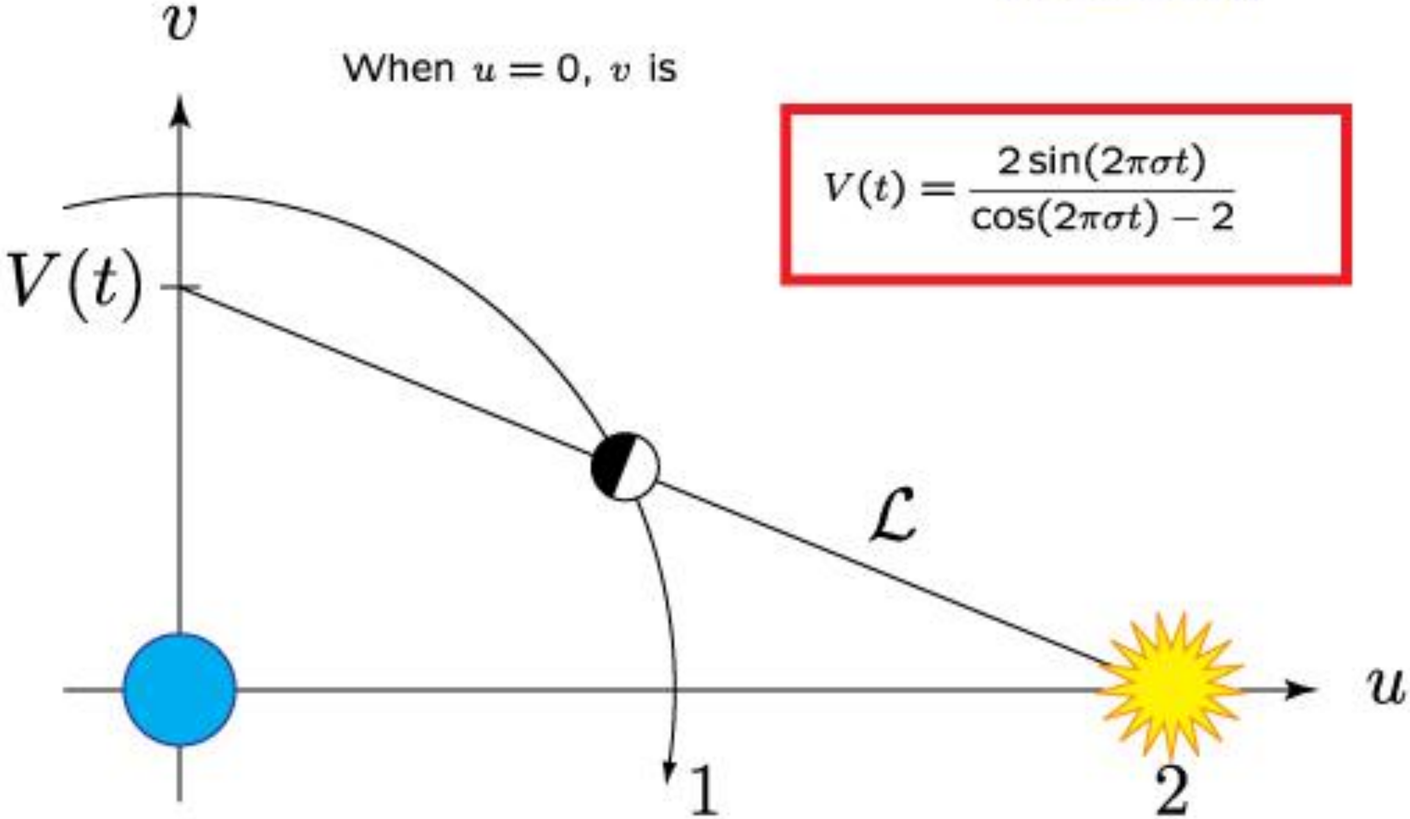
t in years

$(u, v) = (\cos(2\pi\sigma t), -\sin(2\pi\sigma t))$

$$v = \frac{\sin(2\pi\sigma t)}{2 - \cos(2\pi\sigma t)}(u - 2)$$

When $u = 0$, v is

$$V(t) = \frac{2\sin(2\pi\sigma t)}{\cos(2\pi\sigma t) - 2}$$



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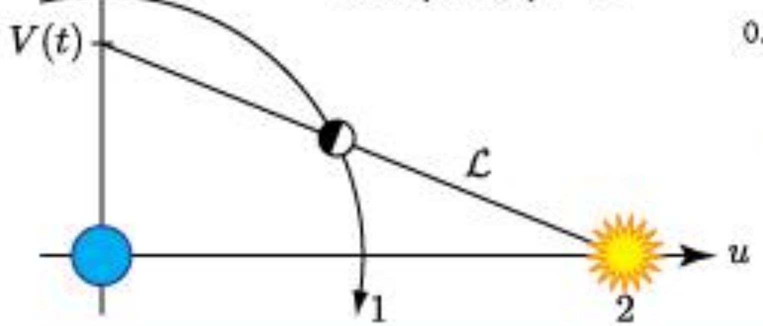
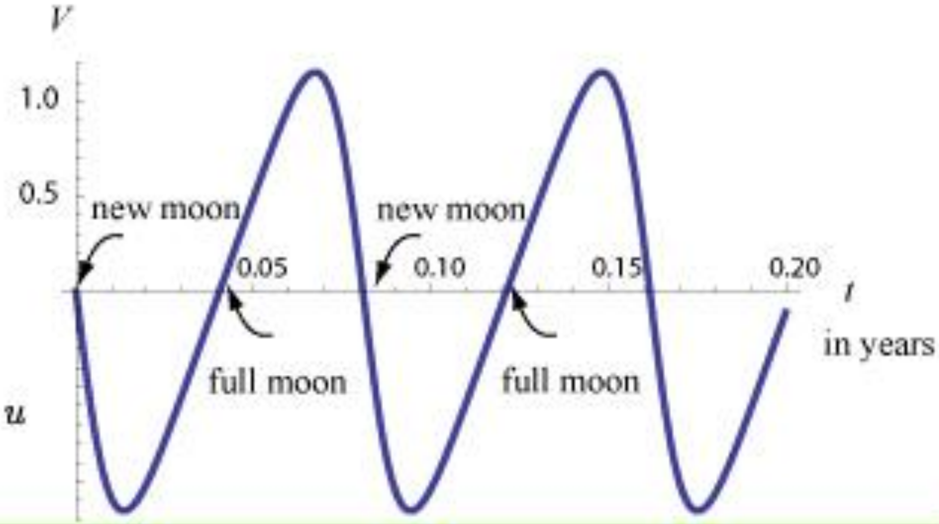
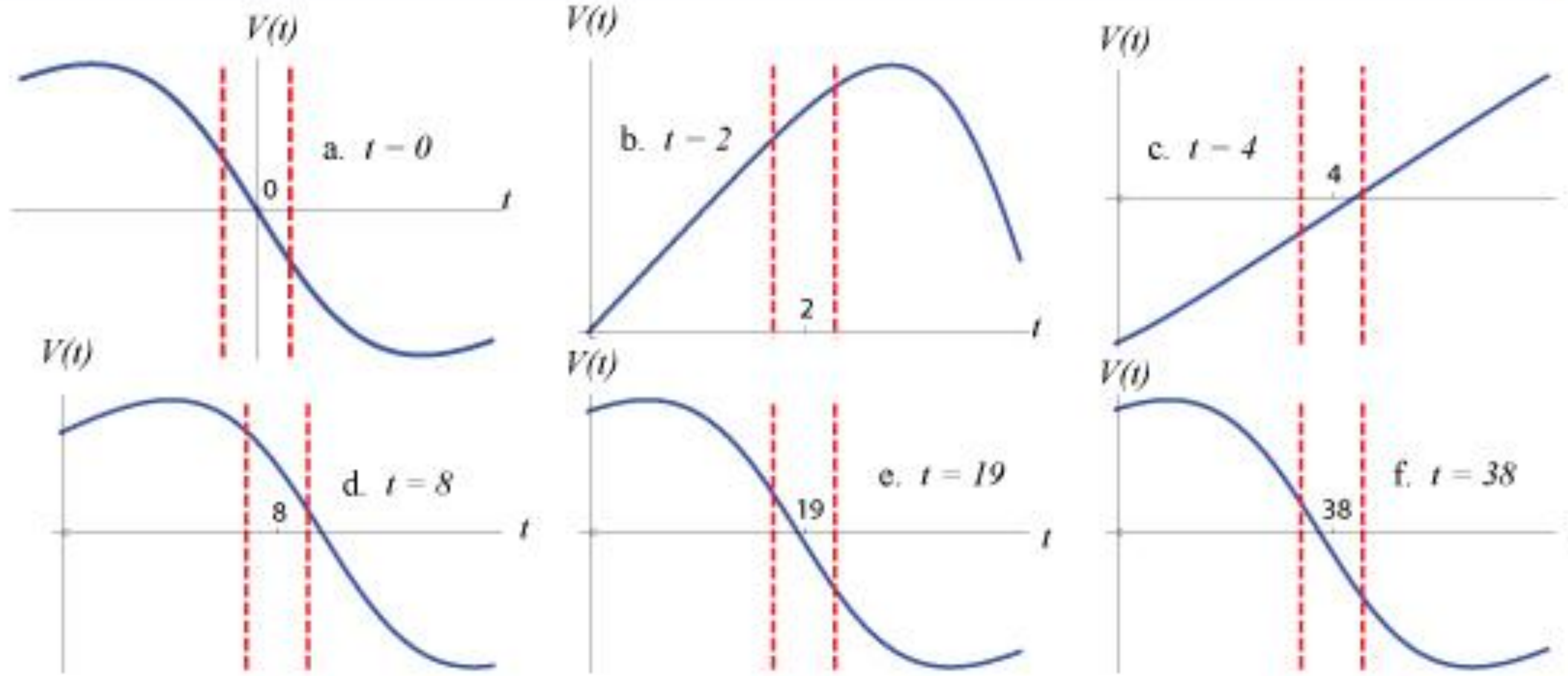


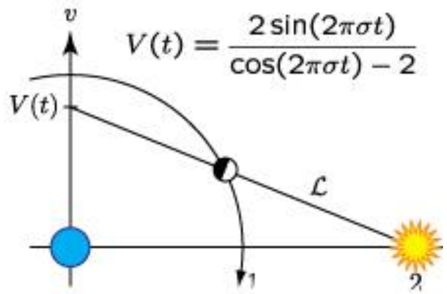




Year	New Moon	First Quarter	Full Moon	Last Quarter
2014	Jan 1 11:14	Jan 8 03:39	Jan 16 04:52	Jan 24 05:19
	Jan 30 21:39	Feb 6 19:22	Feb 14 23:53	Feb 22 17:15
	Mar 1 08:00	Mar 8 13:27	Mar 16 17:09	Mar 24 01:46
	Mar 30 18:45	Apr 7 08:31	Apr 15 07:42 t	Apr 22 07:52
	Apr 29 06:14 A	May 7 03:15	May 14 19:16	May 21 12:59
	May 28 18:40	Jun 5 20:39	Jun 13 04:11	Jun 19 18:39
	Jun 27 08:09	Jul 5 11:59	Jul 12 11:25	Jul 19 02:08
	Jul 26 22:42	Aug 4 00:50	Aug 10 18:09	Aug 17 12:26
	Aug 25 14:13	Sep 2 11:11	Sep 9 01:38	Sep 16 02:05
	Sep 24 06:14	Oct 1 19:33	Oct 8 10:51 t	Oct 15 19:12
	Oct 23 21:57 P	Oct 31 02:48	Nov 6 22:23	Nov 14 15:16
	Nov 22 12:32	Nov 29 10:06	Dec 6 12:27	Dec 14 12:51
	Dec 22 01:36	Dec 28 18:31		

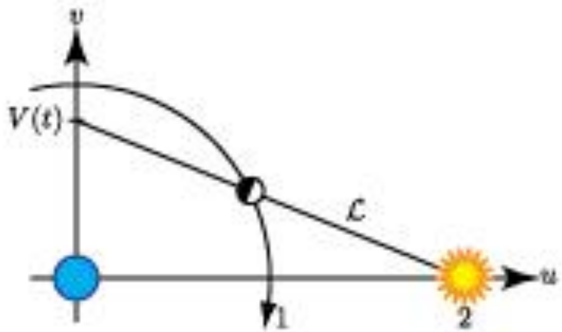
Over a 6000 year period

8 October ← Autumn → 7 November

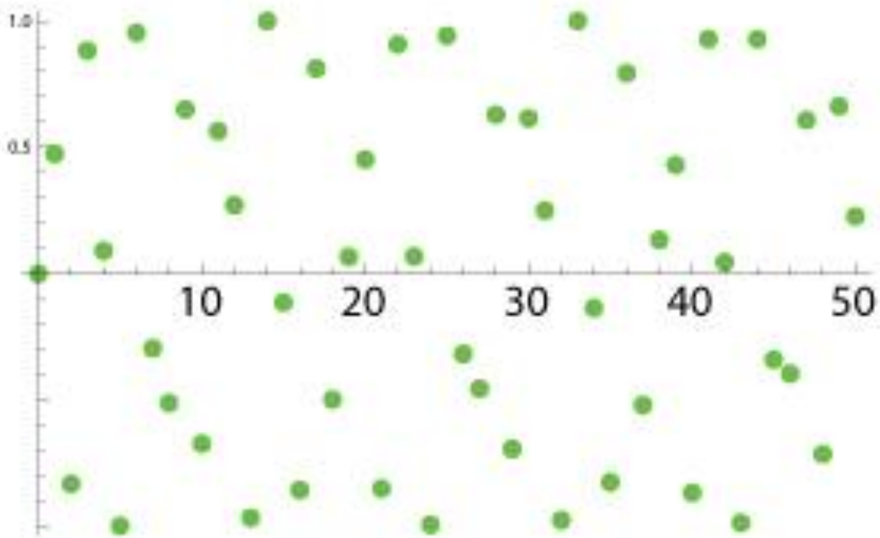


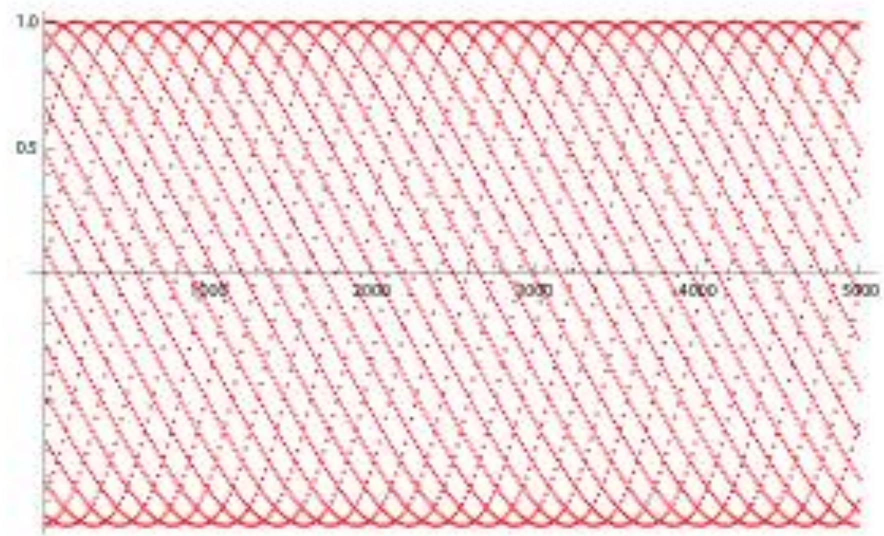
19 year cycle?

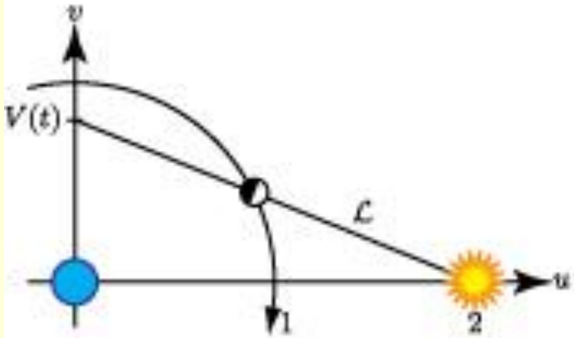
year	day & time	Δ days	day & time	year
1994	11/03/13:36	−0.03	11/03/12:50	2013
1995	10/24/04:36	−0.28	10/23/21:57	2014
1996	10/12/23:07	0.04	10/13/00:06	2015
1997	10/31/10:01	−0.68	10/30/17:38	2016
1998	10/20/10:09	−0.62	10/19/19:12	2017
1999	10/09/11:34	0.28	10/09/18:11	2018
2000	10/27/07:58	0.82	10/28/03:38	2019
2001	10/16/19:23	0.01	10/16/19:31	2020
2002	11/04/20:34	0.03	11/04/21:15	2021
2003	10/25/12:50	−0.08	10/25/10:49	2022
2004	10/14/02:48	0.63	10/14/17:55	2023
2005	11/02/01:25	−0.53	11/01/12:47	2024
2006	10/22/05:14	−0.16	10/21/19:54	2025
2007	10/11/05:01	−0.22	10/10/15:50	2026
2008	10/28/23:14	0.24	10/29/13:36	2027
2009	10/18/05:33	−0.40	10/18/02:57	2028
2010	11/06/04:52	−0.10	11/06/04:24	2029
2011	10/26/19:56	0.10	10/26/20:17	2030
2012	10/15/12:02	0.34	10/16/08:21	2031
2013	11/03/12:50	−0.12	11/03/05:45	2032
	mean Δ days	−0.04		

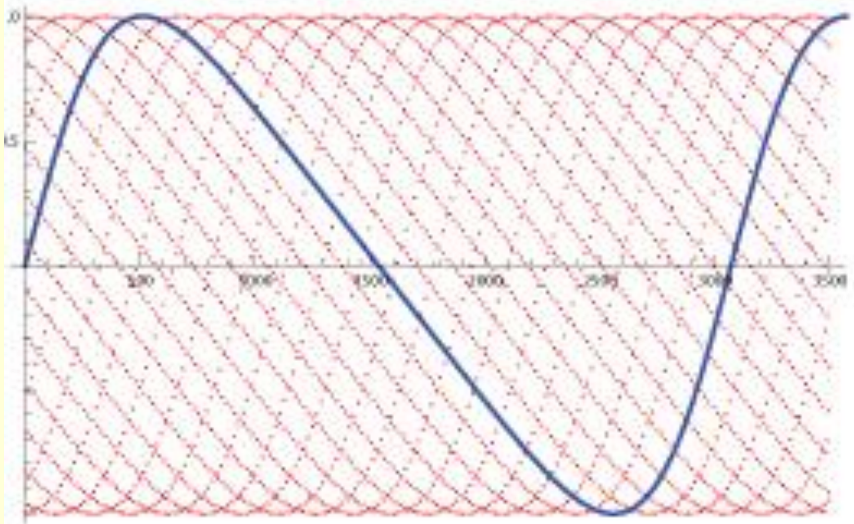


$$\mathcal{D} = \{(n, W(n)) | n \in \mathcal{Z}\}, \text{ where } W(n) = -\frac{\sqrt{3}}{2} V(n)$$

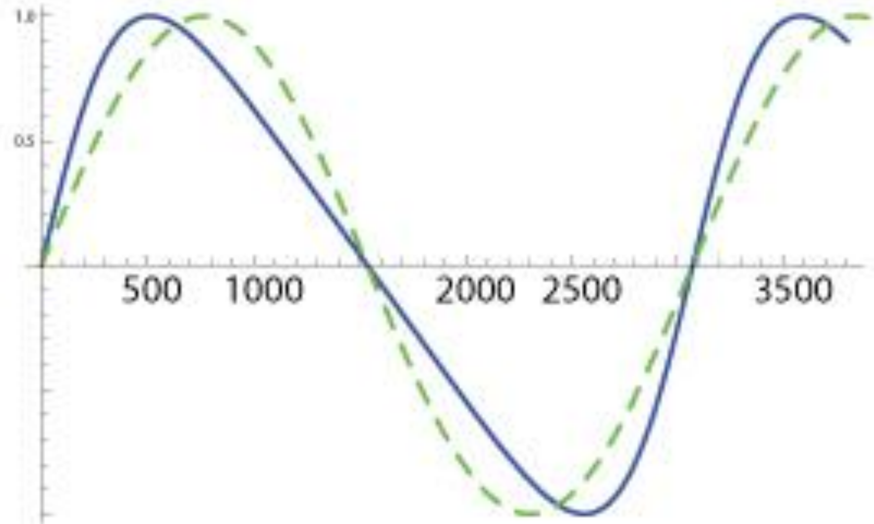






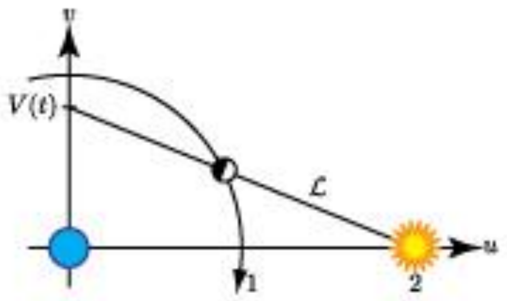


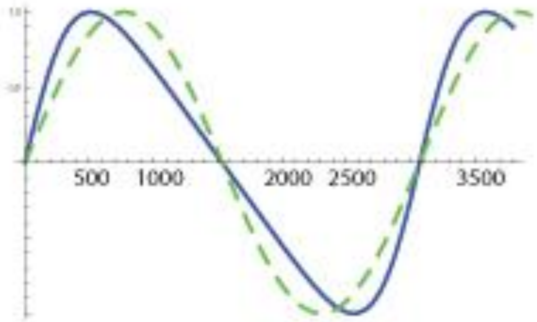
Branch-0 of 19



Branch-0 versus
$(19n, \sin(2\pi\sigma 19n))$

$T \approx 3100$ years



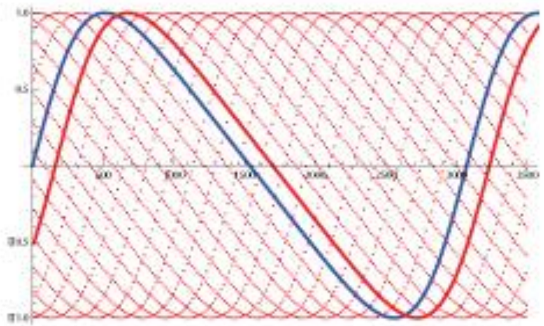


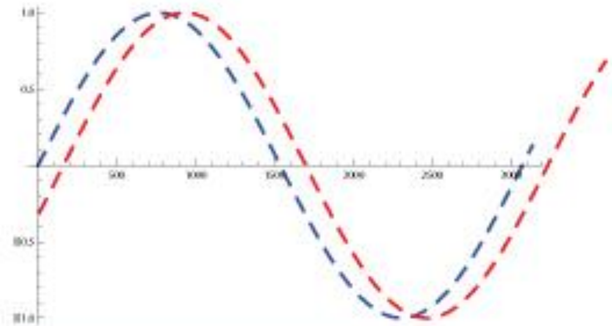
Periods of $W(19n) = -\frac{\sqrt{3}}{2}\frac{2\sin(2\pi\sigma 19n)}{\cos(2\pi\sigma 10n)-2}$ and $\sin(2\pi\sigma 19n)$ are the same T.

$$\sin\left(\frac{2\pi 19n}{T}\right) = \sin(2\pi\sigma 19n)$$

$$\frac{1}{T} = \sigma - \frac{235}{19} \approx 12.3687467 - 12.3684211 \approx 0.0003256$$

$$T \approx 3072.7 \text{ years!}$$





$$\frac{1}{T} = \sigma - \frac{235}{19}$$

Find r so that

$$\sin\left(\frac{2\pi}{T}\left(19n + r - \frac{T}{19}\right)\right) \quad = \quad \sin(2\pi\sigma(19n + r))$$

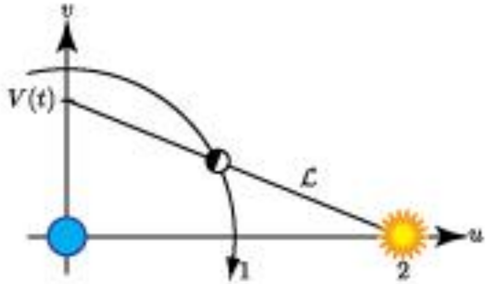
$$\sin\left(2\pi\sigma(19n + r) - 2\pi\left(\frac{235r + 1}{19}\right)\right)$$

$$r = 8$$

Find m, $m = 19n + 8$ near $T/19 \approx 161.7$ years

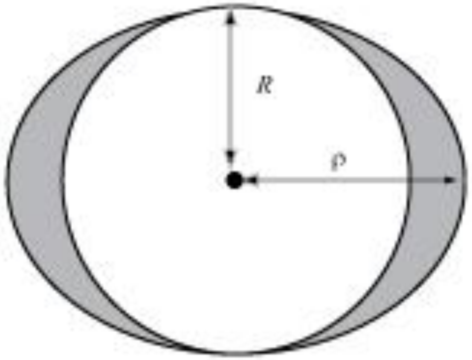
$$161.7 = (19 \cdot 8 + 8) + 1.7 = 160 + 1.7$$

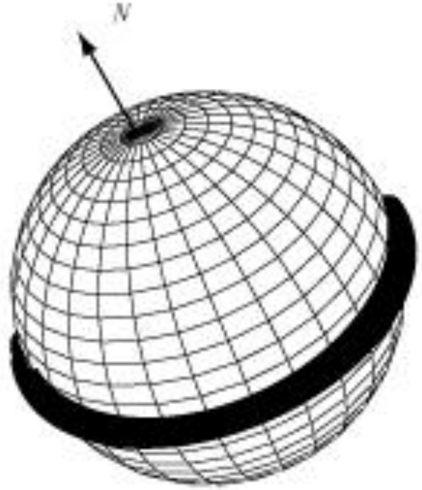
160 year cycle?



160 year cycle?

year	day & time	Δ days	day & time	year
2011	10/26/19:56	2.22	10/29/01:19	2171
2012	10/15/12:02	2.16	10/17/15:57	2172
2013	11/03/12:50	2.09	11/05/15:00	2173
2014	10/23/21:57	1.91	10/25/19:52	2174
2015	10/13/00:06	1.82	10/14/19:48	2175
2016	10/30/17:38	1.83	11/01/13:36	2176
2017	10/19/19:12	1.98	10/21/18:46	2177
2018	10/06/18:11	1.54	10/11/07:06	2178
2019	10/28/03:38	2.19	10/30/08:17	2179
2020	10/16/19:31	2.21	10/19/00:36	2180
	mean Δ days	2.00		

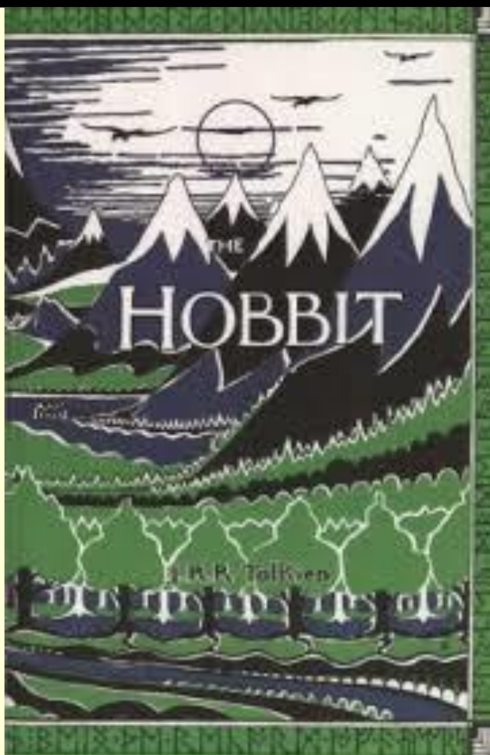




precession period 26,000 years

in 160 years

$$\frac{160 \cdot 365}{26\,000} \approx 2.25 \text{ days}$$



long-span $\equiv$ a span of 160 years

a *standard* long-span $\equiv$ long-span, such as 1990–2150, containing one deficient century $\equiv$ 60%

30% of long-spans contain two deficient centuries, such as 2050–2210;

10% have no deficient centuries, such as 1910–2070.

Let Δ be the average day-lapse between new moons 160 years apart for standard long-spans.

for 30% of long-spans, this lapse will be $\Delta + 1$; and for 10% of long-spans, this lapse will be $\Delta - 1$. Altogether, the average day-lapse for new moons 160 years apart should be

$$0.60\Delta + 0.30(\Delta + 1) + 0.10(\Delta - 1) \approx 2.25,$$

which means $\boxed{\Delta \approx 2.05}$