The Flip Side of Linear Algebra

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Linear Algebra I at WOU

- In one ten-week term, we cover the typical topics, up to and including a brief brush with eigenthings
- Class consists of 25-35 Mathematics majors and minors, and Mathematics / Computer Science majors
- Typically sophomores, fresh out of the Calculus sequence
- No proofs (at least not explicitly)

Strang's Intro. to Linear Algebra 4^{th} Ed.

- Discussions have a largely conversational / informal tone
- Homework problems in Strang are well-designed to crystallize concept knowledge rather than develop computational skills

I *Like* Lecturing

- Had lectured, typically, from Linear Algebra I from Strang for several years
- Comfortable for me; comfortable for students
- Tortured by Nagging Doubts regarding the relative efficacy of lecture
  - Good students had done typically well; poor students typically poorly
  - Average students: $\frac{1}{\sqrt{2\pi}\sigma} e^{-\left(\frac{x-\mu}{\sigma}\right)^2}$
- My effect on student learning appeared largely marginal even if my lectures were good

The *Feedback* Problem

The lecture-homework feedback loop is too slow, and because of this, lossy

- Students take notes for use as reference while they work on homework
- Even if I return papers quickly, there's a lag between student knowledge demonstration and assessment, in which
- Students lose the opportunity to *sufficiently complete* learning on the topic *efficiently*
- Ideally: Students submit homework problems which they already know are complete and correct (Moore!)
- A greater measure of real-time feedback would be welcome

The *Thinking* Problem

Too little active listening during lectures

- Opportunity for active listening, but students typically feel that they must take verbatim notes
- One cannot expect to think while writing any more than one might expect to learn while talking.
- But this, or to inefficiently review their notes after class, is what we expect of them in a typical lecture format

The *Communication* Problem

Communicating mathematics is crucial to developing an understanding of it

- Lecture: Little verbal communication from students in class
- Class presentations help, but to help solidify communication skills, would need to be much more frequent
- With class sizes around 30 and a tight schedule, (substantial) individual or even group presentations are not practical for the purpose of enhancing communication skills

The *Language* Problem

There is a "language barrier" in mathematics! (Not breaking news)

- Much of students' time is spent decrypting notation and vocabulary - growing in number and abstraction
- To maintain currency in this course students ought to *verbally* communicate more of their growing mathematical vocabulary
- Students may find it more comfortable to talk about mathematics among themselves rather than to their professor

Two-Pronged Solution in Fall Term 2012

Flip the Class Employ Group Work

- Used Khan Academy-like "pencasts," to be viewed before class
- Had students work in groups on homework problems while in class (most days)
- Submit group homeworks; individual quizzes and exams

Pencasts

- Notes are written in permanent ink on magical paper with a magical pen
- Magical pen records voice and writing in sync to form a magical pencast
- Magical pen magically uploads magical pencasts for regular (Muggle) student viewing
- Students can afford to concentrate on the content, without the distraction of transcribing it

Pencast Example

Section 4.4 - Orthonormal Bases and Gram-Schmidt

The Group Work

- Had 29 students grouped into groups of three (one group of two)
- Groups reconstituted at the end of Weeks 4 and 7
- No overlap in groups

- Each group was responsible for 6 or 9 written homework problems for each section covered in a pencast
- Immediately as class met, students got into groups and worked on the homework problems
- Group got the grade for the entire homework

The Group Protocol

1. Work on one problem at a time, together.
2. Discuss how the problem could be solved and choose a method. Each student must be an active participant in this process.
3. Solve the problem verbally or by scratchwork; only after which
4. The student in the group who is tasked with writing the problem up does so.
5. Once complete, before submission the group reads the written solution and makes necessary corrections

Enforce/emphasize the group protocol as rebuttal to the concern of being graded on others' work

Class Meetings

- Class was more like an office hour
- Tried to answer their questions with leading questions of my own
- "I don't know where to start" was always met with "You haven't thought about it enough."
- Occasionally (once or twice per class) I would stop the group work to cover a point which needed attention

Does it Help Solve the Four Problems?

- Feedback: Students necessarily get feedback from classmates in group work, and more readily from me in the "in-class office hour"
- Thinking: Students are free to listen to lecture
- Communication: Students communicate the material continually to each other in class (monitor this)
- Language: Through increased verbal communication, the language barrier is lowered (but not sufficiently; see below)

What Will I Do Differently?

Better address the *Language* Problem

- High-stakes, "no-excuse" notation, vocabulary, theorem statement quizzes
- Must get 100%; unlimited re-takes permitted in office hours within 167 hours
- Lose 5% of final course grade for each quiz not finished with a 100%
- If question n is missed on a quiz, question n plus an additional (specified) question becomes the re-take for question n

Summary Reflections

- The pencasts really did free up students' minds to think
- The ability to rewind and fast-forward was a welcome feature
- Students prefer to be engaged in the material; group work enables this quite well
- In Linear Algebra the vocabulary is crucial, very unfamiliar, and requires immediate and ongoing attention
- High-stakes "no-excuse" quizzes further reduce the Language Problem

Will it Work for You and Your Students?

It's more work up-front than delivering familiar lecture notes but less than a new prep

- Must deliver the lectures into the pencasts - But just like traditional notes, once they're done sufficiently, they're done
- Must carefully develop in-class protocols to ensure class time is used efficiently

It's *absolutely* worth trying

Contact Information

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