



Sage E-books on Linear Algebra and Calculus

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2013 JOINT MATHEMATICS MEETINGS

Largest Mathematics Meeting in the World

JANUARY 9-12 (WED-SAT), 2013 | SAN DIEGO CONVENTION CENTER, SAN DIEGO, CA



Abstract



- ▶ Calculus and Linear Algebra **Books**.
- ▶ **‘E-books’** of Calculus and Linear Algebra.
- ▶ **What’s new in it?**
- ▶ My Answer : <**Movie** clip and **CAS in it**>



‘Sage’ (<http://sagemath.org>)



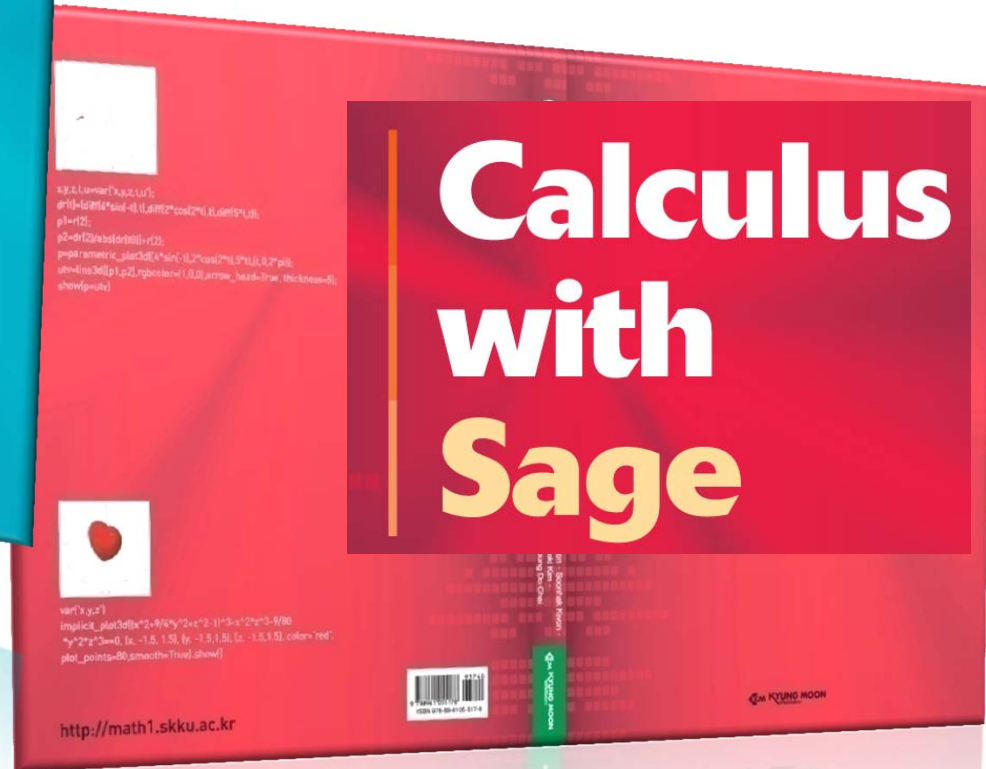
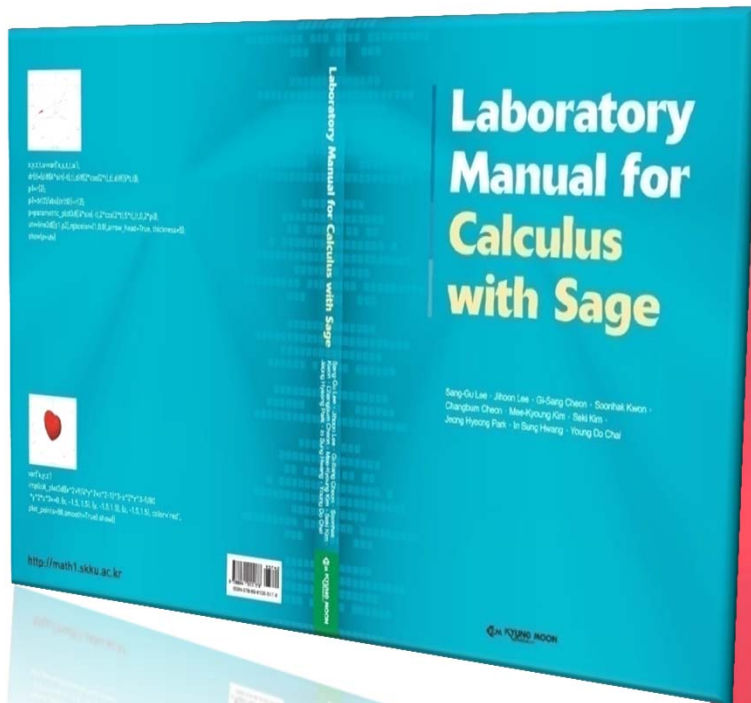
Laboratory Manual

for Calculus with Sage

ISBN: 978-89-6105-557-4, KyungMoon Pub. **Feb.2012.**



Calculus with Sage



Calculus with Sage



3.2 Derivatives of Polynomials, Exponential Functions, Trigonometric Functions, The Product Rule

In this section, we consider the derivatives of the basic functions, for example, polynomials, exponential functions, and trigonometric functions. Product rule and quotient rule are helpful to derive many derivatives of the functions which are expressed as the product and quotient. We also consider some applications of the derivative to the other sciences.

We introduce several differential rules that allow us to find the derivatives without the direct use of the limit definition.

Theorem 1 Derivative of a constant function

If f has the constant function, then

$$\frac{df}{dt} = \frac{d}{dx}(c) = 0$$

[Quick example 1] If f has the constant value $f(x) = 20$, then

$$\frac{df}{df} = \frac{d}{dx}(20) = 0$$

Similarly,

$$\frac{d}{dx}(\sqrt{3}) = 0 \quad \text{and} \quad \frac{d}{dx}(\pi) = 0$$

Note that the Constant Rule is equivalent to saying that the slope of horizontal line is zero.

The following rule tells how to differentiate x^n if n is a positive integer. To prove the following Rule we have to use the binomial expansion.

Theorem 1 The Power Rule for Positive Integer

If n is a positive integer, then $\frac{d}{dx}(x^n) = nx^{n-1}$.

molten rock cools by conduction of heat into surrounding rocks, and an engineer wants to know the rate at which water flows into or out of a reservoir.

CAS Example 10

Find an equation of the tangent line to the curve at the given point.

$$\begin{cases} x = \sin t \\ y = \cos 2t \end{cases}; t = \frac{\pi}{4}$$

Solution var(t, x, y, u)

x=sin(t)

y=cos(2*t)

dxdt=diff(x,t)

dydt=diff(y,t)

dydx=dydt/dxdt

yu=y(pi/4) + dydx(pi/4)*(u-x(pi/4))

p1=parametric_plot((x,y),(t,0,pi/2), color=blue)

p2=plot(yu, u, 0, 1.5, color=red);

show(p1+p2)

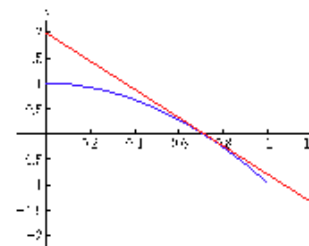


Figure 1

3.2 EXERCISES

7.1 Chapter 3: Theory of Differentiation I

CAS 7. Find the equation of the tangent line to the curve $y = x^2\sqrt{x}$ at $(4, 8)$.

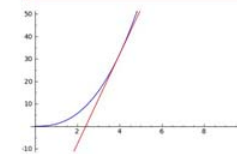
► Sol $y' = \frac{5}{2}\sqrt{x^3} \Rightarrow y'(4) = 20$. So the slope of the tangent line is 20.

$y - 8 = 20(x - 4) \Rightarrow y = 20x - 48$ ($\because y$ passes through $(4, 8)$).

```
f(x)=x^2*sqrt(x);
df(x)=diff(f(x),x);
y(x)=df(4)*(x-4)+8;
y(0)
```

20*x-48

```
p1=plot(f(x),x,0,10, color=blue);
p2=plot(y(x),x,0,10, color=red);
show(p1+p2,ymax=50,ymin=-10)
```



Laboratory Manual for Calculus with Sage



CAS

1. Introduction and Use of Sage-Math¹⁾

Mathematical tools have long held an important place in classrooms. With the innovation of information and communication technologies, many tools have appeared and been adapted for educational purposes. Sage-Math is a popular mathematical software which was released in 2005. This software has efficient features which utilize the internet and can handle most mathematical problems, including linear algebra, algebra, combinatorics, numerical mathematics and calculus. In this book, we will introduce this powerful software and discuss how it can be used in classes.

Sage-Math is a mathematical CAS tool and is based on Internet Web environment. This tool was introduced in April 2008 at University of Washington, USA. It is free and has a powerful capability that can be compared with expensive commercial softwares such as Mathematica, Maple or Matlab, but can do more than that without requiring separate installations of the program. It is more like Web-Mathematica, but has some better features. When you connect to any Web browser, you can really solve almost all calculus problems in the book by using resources that we are offering. You can easily find pre-existing command to modify for your own problems.



Figure 1 We can use others existing codes as well in Sage

Korean Sage-Math model was developed and relevant experiments were done by BK 21 Mathematical Modelling HRD division at Sungkyunkwan University. We have built Korean servers that you can use as you see below. (Instructions will be given in your first class)

1) Mobile Sage-Math for Linear Algebra and its Application, Duk-Sun Kim, Sang-Gu Lee*, Greg Markowsky, Electronic Journal of Mathematics & Technology (eJMT) <https://php.radford.edu/~ejmt/ContentIndex.php> V.4, No. 3, pp. 1-13, Oct 2010. ISSN 1933-2823.

solving this gives,

$$n > 134.3 \text{ so that } n = 135.$$

8. (a) Determine the approximations and for T_{10} and S_{10} for $\int_0^1 e^{x^2} dx$ and the corresponding errors E_T and E_S .
- (b) Compare the actual errors in part (a) with the error estimates given by \square and \square .
- (c) Determine how large do we have to choose so that the approximations T_n , M_n , and S_n to the integral in part (a) are accurate to within 0.00001?

► Sol

(a) $T_{10} = 1.280262$, $S_{10} = 1.271599$,
 $E_T = -0.008975$, $E_S = -0.008312$.

(b) Since $K = 28e \approx 76.111891$ (3) and (4) gives

$$|E_T| \leq \frac{76.111891}{12(10)^2} \approx 0.063426$$

$$|E_S| \leq \frac{76.111891}{180(10)^4} \approx 0.000042$$

(c) For T_n , find n so that $\frac{K}{12n^2} < 0.00001$, $n = 797$.

For M_n , find n so that $\frac{K}{24n^2} < 0.00001$, $n = 564$.

For S_n , find n so that $\frac{K}{180n^4} < 0.00001$, $n = 15$.

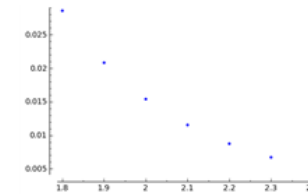
9. Given the function f at the following values,

x	1.8	1.9	2.0	2.1	2.2	2.3	2.4
$f(x)$	0.023561	0.020813	0.015384	0.011525	0.008742	0.006709	0.004079

approximate $\int_{1.8}^{2.4} f(x) dx$ using Simpson's Rule.

► Sol

Plot $f(x)$



And Use Simpson's Rule

$$\Delta x = (2.4 - 1.8) / 6 = 0.1$$

$$\Rightarrow \int_{1.8}^{2.4} f(x) dx \approx S_6$$

$$= \frac{0.1}{3} [f(1.8) + 4f(1.9) + 2f(2.0) + 4f(2.1) + 2f(2.2) + 4f(2.3) + f(2.4)]$$

$$\approx 0.007923$$

7.8 Improper Integrals

CAS 1. $\int_0^{\infty} \frac{\ln x}{x} dx$

► Sol

$$\text{integral}(\ln(x)/x,x,0,\text{infinity})$$

ValueError: Integral is divergent

CAS 2. $\int_0^{\infty} x e^{-x} dx$

► Sol

$$\text{integral}(x^0 e^{-(x)} x, 0, \text{infinity})$$

1

CAS 3. $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$

► Sol

Calculus with SKKU Sage Cell Server



<http://sage.skku.edu>

7.2 Trigonometric Integrals

CAS 1. $\int \cos^3 x dx$

CAS 2. $\int \sin^5 x \cos^4 x dx$

► Sol

`integral(sin(x)^5*cos(x)^4,x)`

$-1/9*\cos(x)^9 + 2/7*\cos(x)^7 -$

CAS 3. $\int \sin^2(4x) dx$

► Sol

`integral(sin(4*x)^2,x)`

$1/2*x - 1/16*\sin(8*x)$

CAS 4. $\int \sin^3 x \cos^{-4} x dx$

SKKU Sage-Math x SKKU Sage Single x

matrix.skku.ac.kr/2012-sage/index.html? ☆ 🔍

이 페이지는 영어 로 번역 번역 안함 옵션 x

<SKKU : Sage-Math : 수학 도구 >

► Published Data (공개된 자료) Click

```
1 | integral(sin(x)^5*cos(x)^4,x)|
```

실행 취소(U) Ctrl+Z
다시 실행(R) Ctrl+Y
잘라내기(T) Ctrl+X
복사(C) Ctrl+C
붙여넣기(P) Ctrl+V
일반 텍스트로 붙여넣기 Ctrl+Shift+V
삭제(D) Ctrl+Delete
맞춤법 검사기 옵션(S)
전체 선택(A) Ctrl+A
요소 검사(N) Ctrl+F

Run(실행)

Session d69a6b13-4b8e-472d-bdc0-c4551U21a87a

$-1/9*\cos(x)^9 + 2/7*\cos(x)^7 - 1/5*\cos(x)^5$

Session d69a6b13-4b8e-472d-bdc0-c4551021a87a done

Session Files: Powered by **SDGE**

What's more?



Library Laboratory Manual for Calculus with Sage-Chapter 14 🔍 📖

[14.1] Vector Differentiation

1-7. Sketch the vector field \vec{F} by drawing a diagram.

[CAS] 1. $\vec{F}(x, y) = 3i - 4j$
→ Sol
<http://matrix.skku.ac.kr/cal-lab/cal-14-1-1.html>

```
var(x,y)  
vf=plot_vector_field((3,4), (x,-3,3), (y,-3,3), aspect_ratio=1);  
show(vf)
```

[CAS] 2. $\vec{F}(x, y) = \frac{1}{2}xi - 2yj$
→ Sol

```
var(x,y)  
vf=plot_vector_field((1/2*x,-2*y), (x,-3,3), (y,-3,3), aspect_ratio=1);  
show(vf)
```

Back to page 12 3 of 57 11 pages left in this chapter

Library Laboratory Manual for Calculus with Sage-Chapter 11 🔍 📖

[CAS] 15. $(z^2 + \frac{9}{4}y^2 + x^2 - 1)^3 = z^2x^3 + \frac{9}{80}y^2z^3$
→ Sol
<http://matrix.skku.ac.kr/cal-lab/cal-11-6-15.html>

```
var(x,y,z)  
implicit_plot3d((x^2+9/4*y^2+z^2-1)^3-x^2*z^3-9/80*y^2*z^3=0,  
(x,-1.5,1.5), (y,-1.5,1.5), (z,-1.5,1.5), color='red',  
plot_points=80,smooth=True).show()
```

[CAS] 16. Sketch the region bounded by the surfaces $z = \sqrt{x^2 + y^2}$ and $x^2 + y^2 = 4$ for $2 \leq z \leq 4$.
→ Sol
<http://matrix.skku.ac.kr/cal-lab/cal-11-6-16.html>

```
var(x,y,z)  
z=(x^2+y^2)^(1/2)  
plot3d(z, (x,-2,2), (y,-2,2), (z,2,4))
```

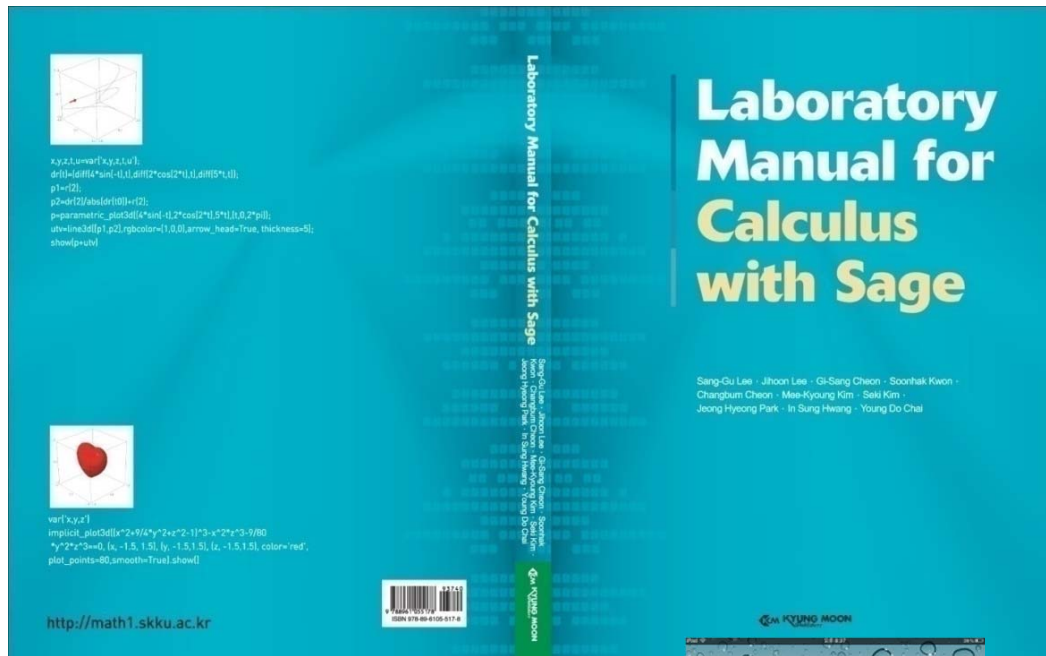
17. Find an equation for the surface obtained by rotating the parabola $z = x^2$ about the z -axis.

48 of 60 1 page left in this chapter



CAS Solutions





Session 1

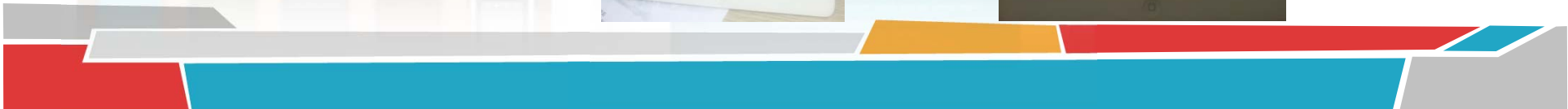
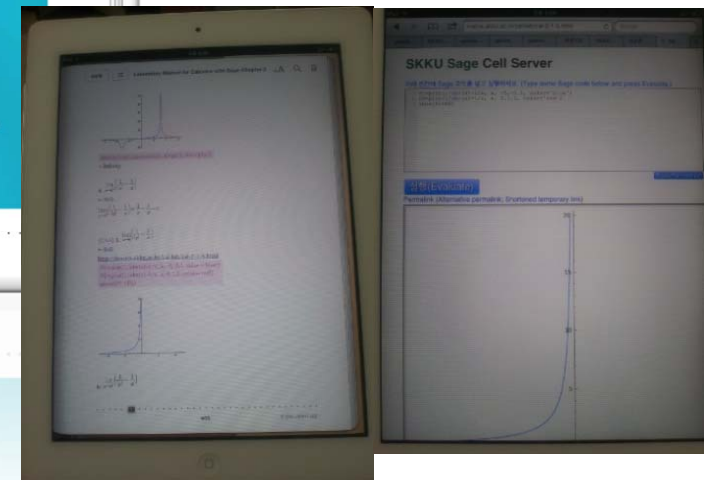
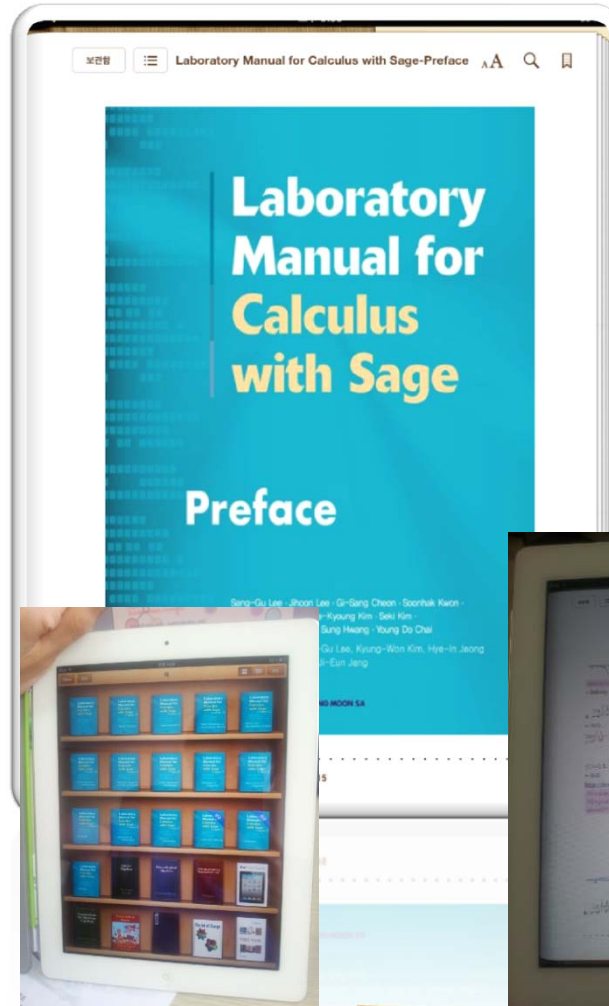
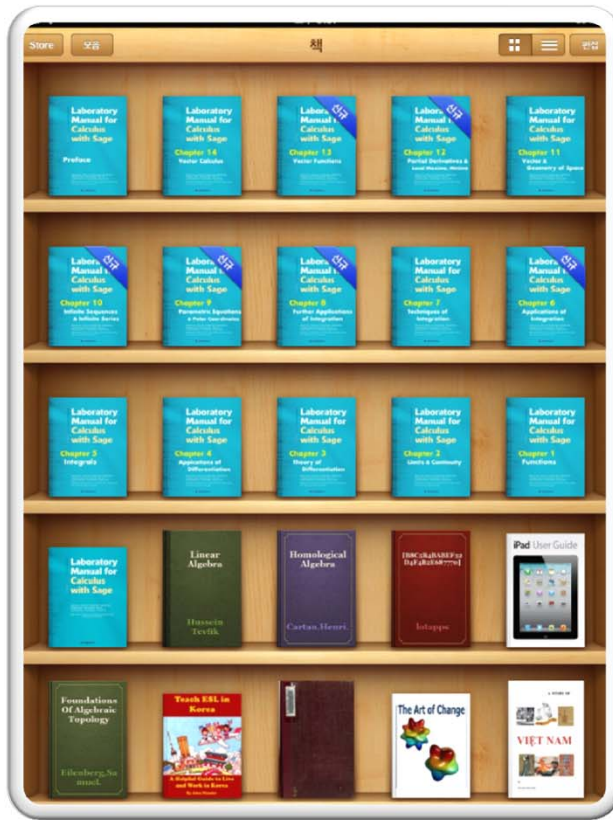
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There are each Chapters 1 ~ 14 of the e-book for

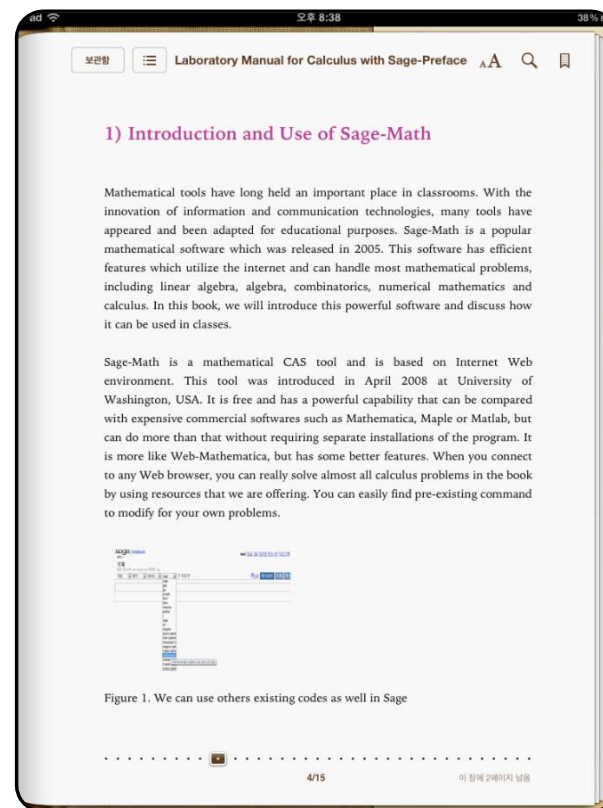
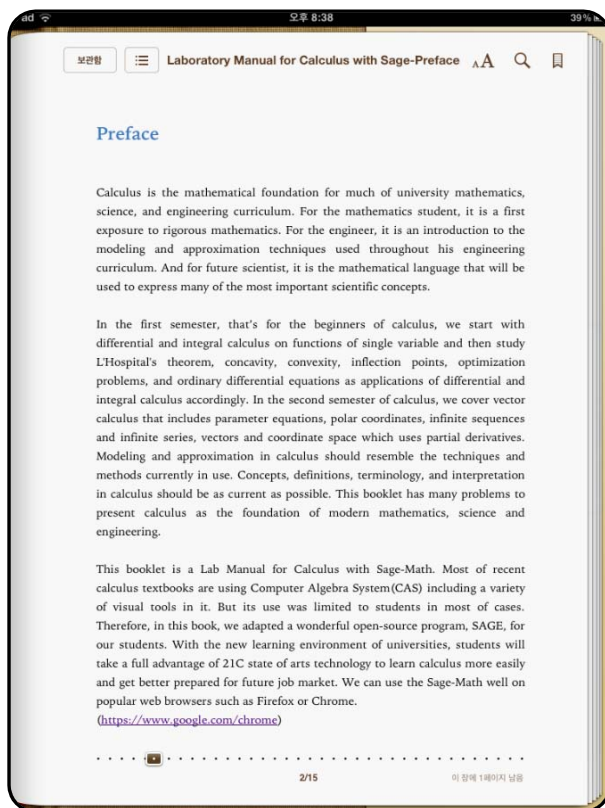


Lab. Manual for Calculus with Sage





E-book of Lab. Manual for Calculus with Sage



E-Paper



Contents for E-book of Lab. Manual for Calculus with Sage



Library Resume Laboratory Manual for Calculus with Sage-Preface

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Library Resume Laboratory Manual for Calculus with Sage-Chapter 14

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Part II Multivariate Calculus



In this E-book of Lab. Manual for Calculus with Sage

Click Web Address

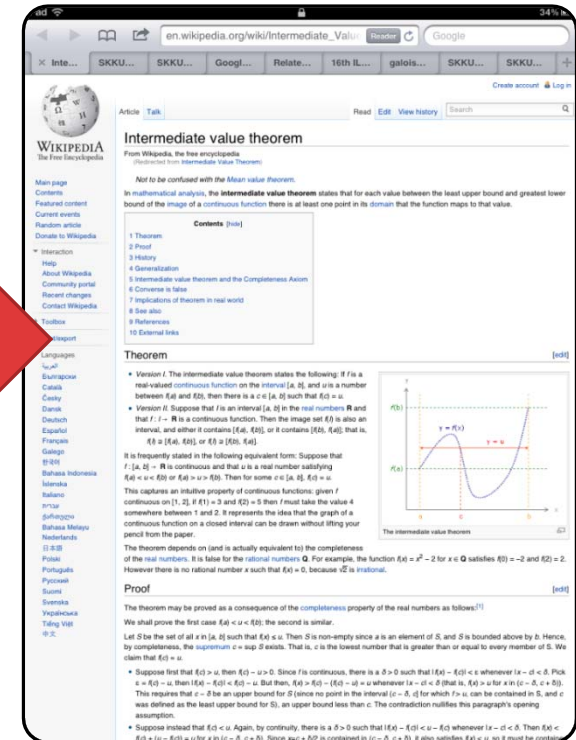
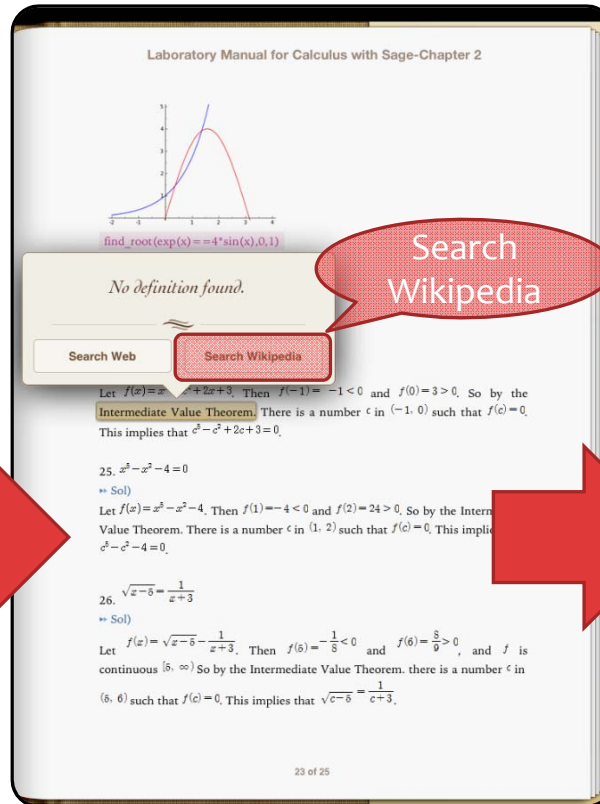
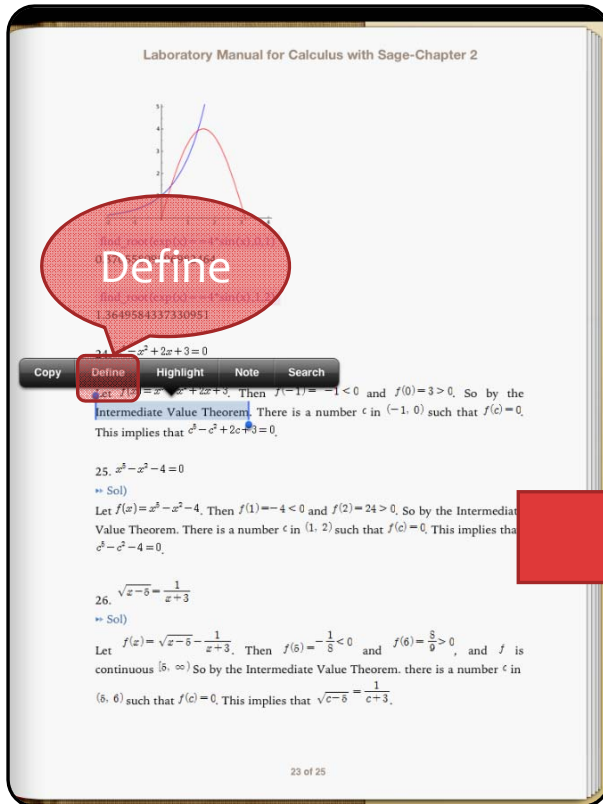
Multi-tasking Process

The diagram illustrates a multi-tasking process between two devices. On the left is a tablet displaying a page from the 'Laboratory Manual for Calculus with Sage-Chapter 11'. The page title is '[11.7] Cylindrical Coordinates and Spherical Coordinates'. It contains several problems (1-4 and 5-6) involving cylindrical coordinates. A red arrow points from the e-book to the right, where a smartphone displays the 'SKKU Sage Cell Server' interface. The server interface shows a code editor with Sage code: `T = Cylindrical('height', 'radius', 'azimuth')` and `T.transform(radius=2, azimuth=- pi/3, height=4)`. Below the code is an 'Evaluate' button. The output shows the result: `(1, -sqrt(3), 4)`. A second red arrow points from the server back to the e-book, indicating a return to the manual. The text 'Click Web Address' is positioned above the first arrow, and 'Multi-tasking Process' is positioned below the second arrow.

Using Sage Cell Server

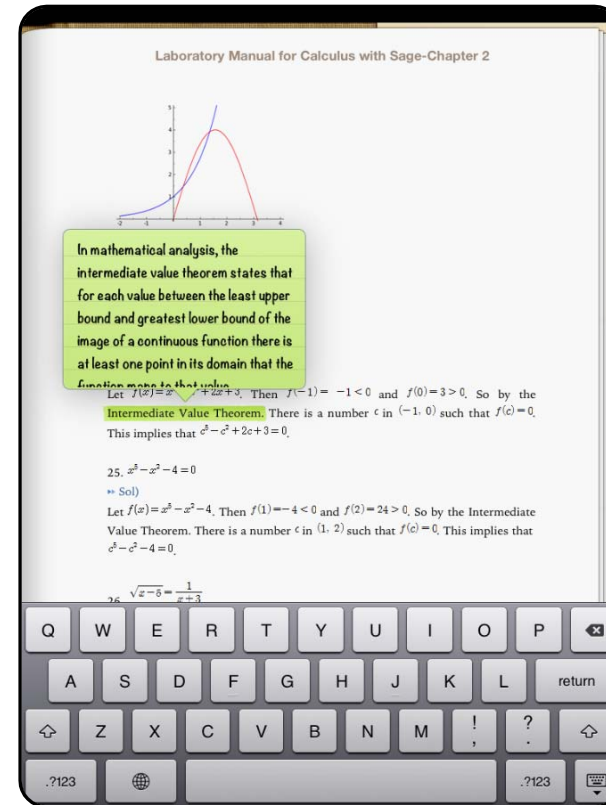
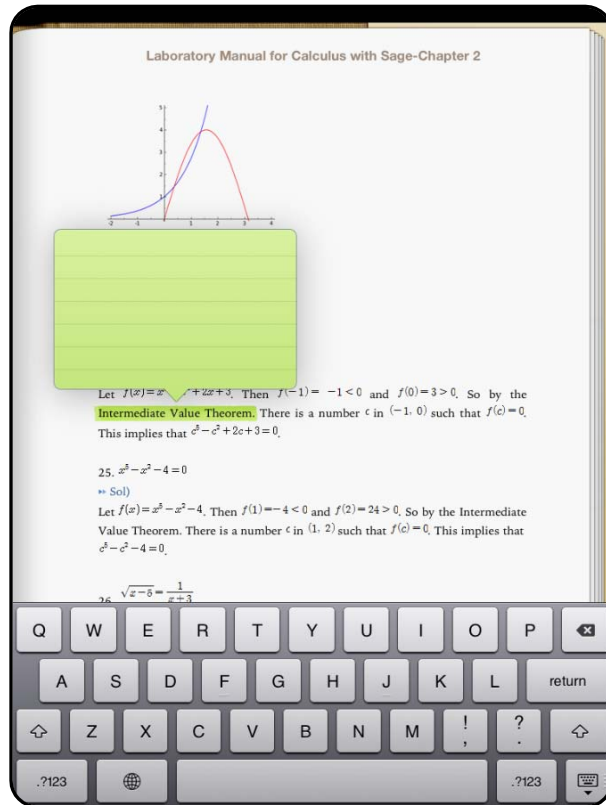


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We can Search its meanings in Wikipedia

E-book of Lab. Manual for Calculus with Sage

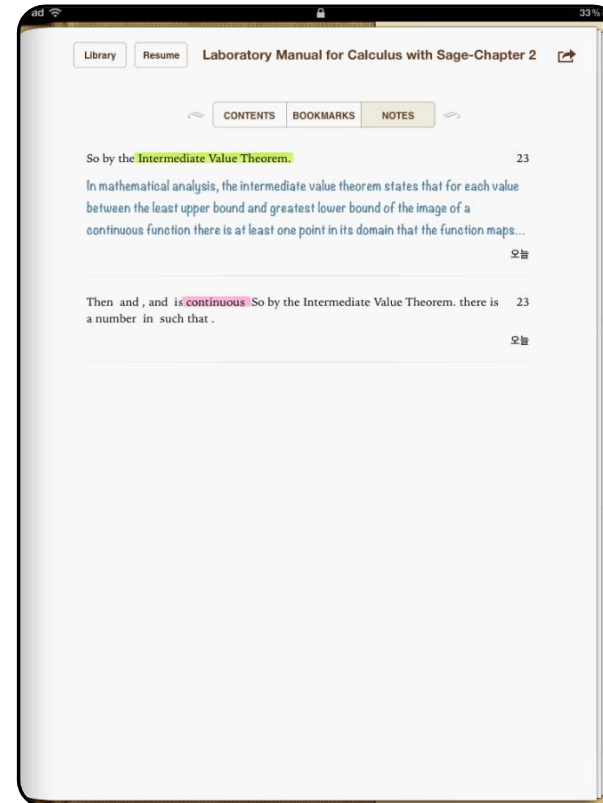
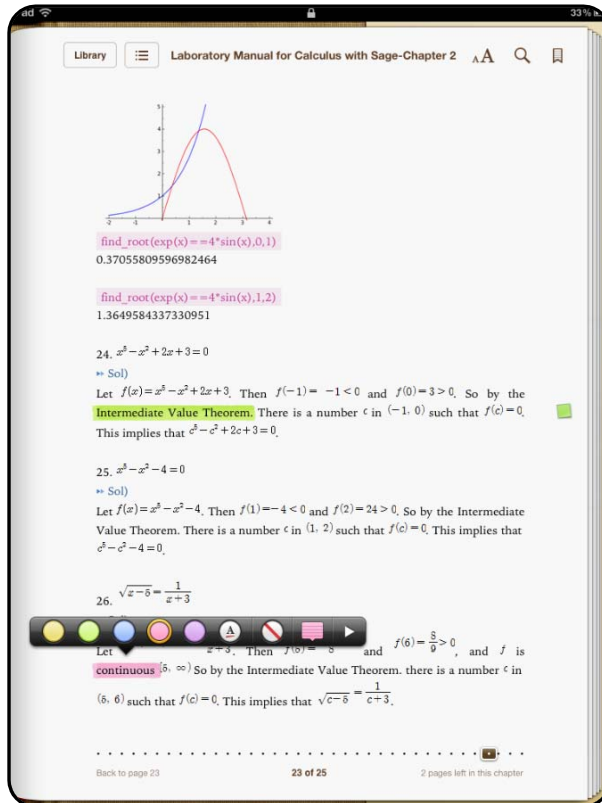


Note

We can add our notes in it.



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Marker



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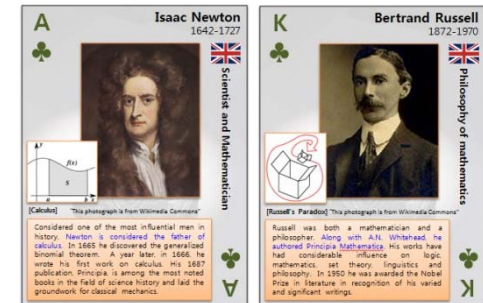


<http://matrix.skku.ac.kr/2011-Album/2011-Math-Appl/main.html>



World Mathematicians Cards

- ♠ European
- ♦ Asian
- ♣ USA and UK (Modern Mathematicians)
- ♥ Korean



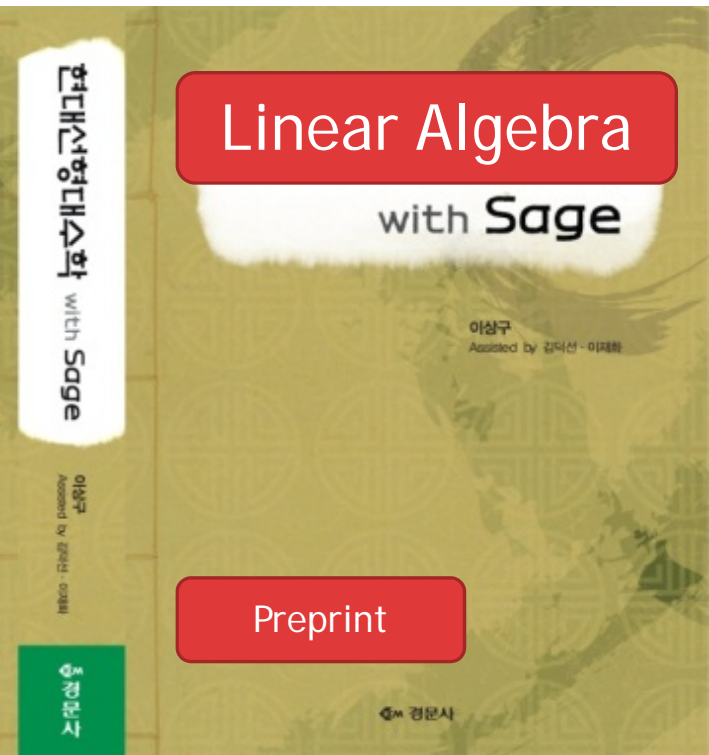
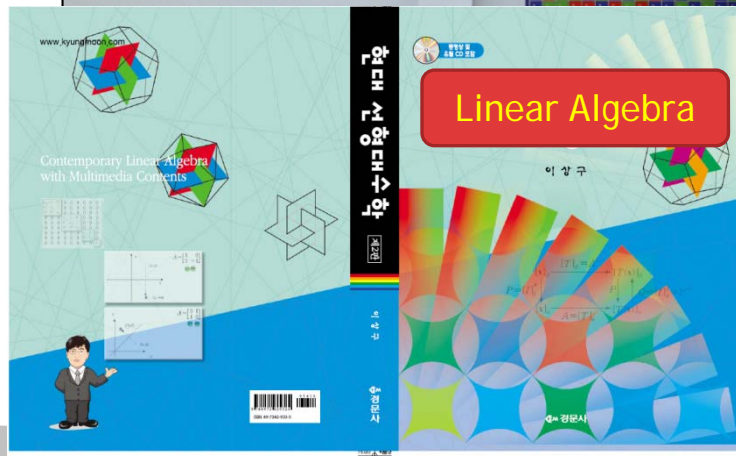
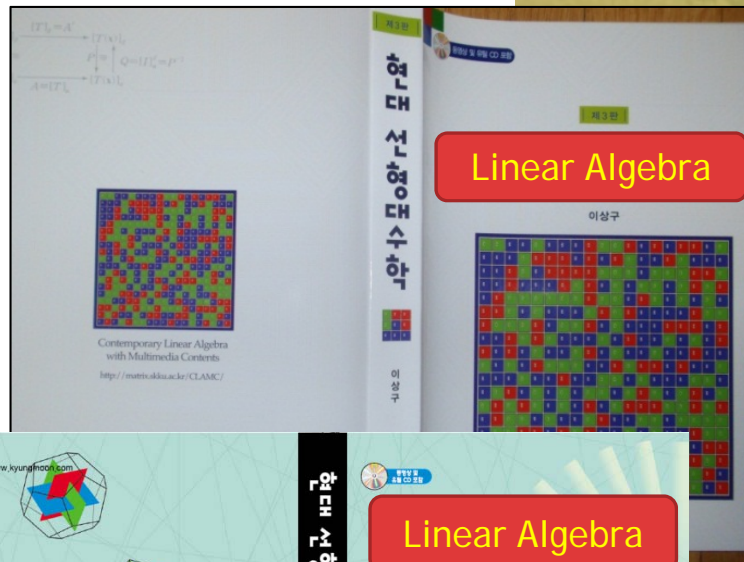
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Session 2 **Linear Algebra**



I made a preprint of Linear Algebra a year ago.



I had a classroom test over a year on **New L. A. book.**



Vector

1

- 1.1 공학과
- 1.2 내적과
- 1.3 직선과



선형대수학은 벡터(vector)와 행렬(matrix)이라는 두 개의 직한다. 우리 주위에는 온도, 시간, 길이, 높이 등과 같이 "크기"가 있는 "방향"을 나타내는 양도 있다. 벡터는 이러한 양에 이용되며, 고차원 벡터는 유전학이나 경제학, 생화학 등 다양한 중첩과 공간 물체의 분리를 표현하기 위하여 상대성 이론에 벡터에 대해서 정의하고 2차원 평면과 3차원 공간에서 벡터의 내적과 외적 및 정사영에 대해서 학습한다. 또 3차원 벡터를 이용하여 나타낸다. 벡터는 여러 개의 실수들의 순서로 표현될 수 있으며 행렬은 이러한 벡터들이 행이나 열로 있다.



한대선형대수학 with Sage
Linear Algebra

Linear Algebra
with Sage

이상구
Assisted by 김덕선 · 이재화

이상구
Assisted by 김덕선 · 이재화

경문사



(3) $\mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ 이라 하자. $(-1)\mathbf{x} = \begin{bmatrix} (-1)x_1 \\ \vdots \\ (-1)x_n \end{bmatrix} = \begin{bmatrix} -x_1 \\ \vdots \\ -x_n \end{bmatrix} = -\mathbf{x}$ ■

정의

[일차결합]

$\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k$ 가 R^n 의 벡터이고, 계수 c_1, \dots, c_k 가 실수일 때,

$$\mathbf{x} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \dots + c_k\mathbf{v}_k$$

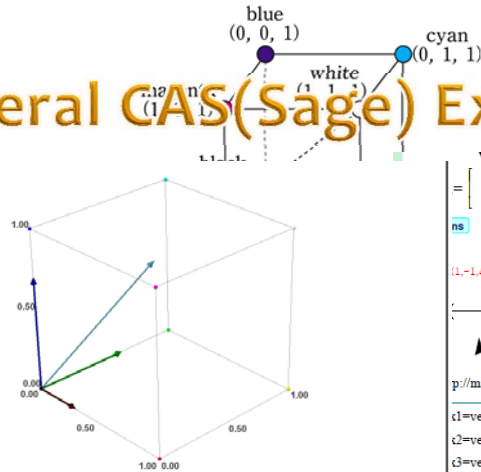
인 형태를 $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k$ 의 일차결합(linear combination)이라 한다.

예제 3 컴퓨터 모니터의 색은 RGB 색 모델이라 불리는 시스템을 이용한다. 이는 붉은색(R), 초록색(G), 파랑색(B)을 몇 %씩 합성하여 색을 만들어 내느냐에 달려 있다. 방

다음과 같다. **Traditional +**

를 각각 순수한 red, green, blue 색이라 하자. 0과 1 사이의 계수를 이용한 r, 의

Several CAS(Sage) Examples

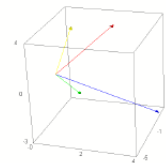


Sage Code

```

r = vector(QQ, (1, 0, 0))
g = vector(QQ, (0, 1, 0))
b = vector(QQ, (0, 0, 1))
w = vector(QQ, (1, 1, 1))

show(plot(x1.rgbcolor=(1,0,0))+plot(x2.rgbcolor=(0,1,0))+plot(x3.rgbcolor=(0,0,1))+plot(x4.rgbcolor=(1,1,0)))
    
```



Students presentation

3.1 연습문제

<http://matrix.skku.ac.kr/2012-LAwithSage/interact/worksheets.html?loop=yes> [실습]

[1-2] 행렬

$$A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 6 & 4 \\ 4 & 1 \end{bmatrix}, C = \begin{bmatrix} -4 & 6 \\ 3 & 0 \end{bmatrix}$$

- $A+B=B+A$
- $A+(B+C)=(A+B)+C$

[3-4] 행렬

$$A = \begin{bmatrix} 1 & 3 \\ 4 & -1 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 5 \\ 1 & -1 & 4 \end{bmatrix}, C = \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}$$

이고 $a=3$ 일 때, 다음을 확인하여라.

- $A(BC)=(AB)C$
- $a(BC)=(aB)C=B(aC)$

(1, 0, 0)

그리고 순수



<http://math3.skku.ac.kr/spla/CLA-3.1-Exercise-3>
<http://math3.skku.ac.kr/spla/CLA-3.1-Exercise-4>

8. 행렬

$$A = \begin{bmatrix} -2 & 3 \\ 2 & -3 \end{bmatrix}, B = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}, C = \begin{bmatrix} -4 & -3 \\ 0 & -4 \end{bmatrix}$$

일 때, $AB=AC$ 이지만 $B=C$ 임을 확인하여라.

[9-10] 행렬 $A = \begin{bmatrix} 1 & 3 \\ 4 & -1 \end{bmatrix}$ 에 대하여 다음을 계산하여라.

- A^2-2A 과
- $3A^3-2A^2+5A-4E_2$



<http://math3.skku.ac.kr/spla/CLA-3.1-Exercise-9>
<http://math3.skku.ac.kr/spla/CLA-3.1-Exercise-10>

[11-12] 아래 행렬의 거듭제곱을 계산하여라.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- A^{10} 과
- A^{20}

<http://math3.skku.ac.kr/spla/CLA-3.1-Exercise-11>
<http://math3.skku.ac.kr/spla/CLA-3.1-Exercise-12>

(B)을 나타내는 벡터 \mathbf{b} 을 정한다.

Blue \mathbf{b} 벡터 생성 및 출력

작거나 같은 어떤 c_1, c_2, c_3 을 정하여

$$\mathbf{3b} = c_1(1, 0, 0) + c_2(0, 1, 0) + c_3(0, 0, 1)$$

ku.ac.kr/2012-sage/sage-la/visual/179.htm

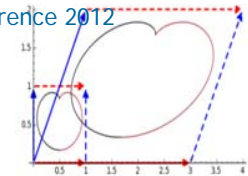
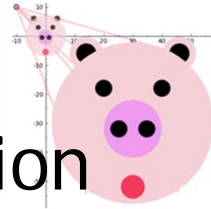
12 Chapter 1 벡터

clr #419db5



Linear Transformation

<http://math1.skku.ac.kr/pub>



Matrix A = $\begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$

Determinant : $\det(A) = 6$

Eigensystem
 $[(3, [(1,0)], 1), (2, [(1,$

center pt. (1, 1)

pt1. (2, 1)

pt2. (1, 4)

pt3. (2, 3)

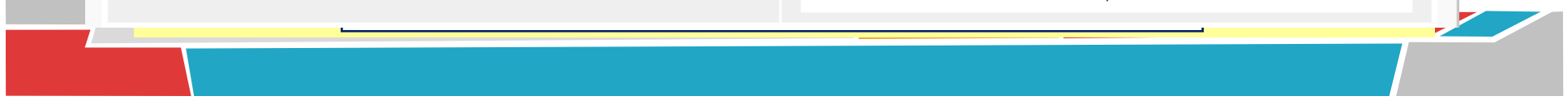
ratio 3

Update

$A = \begin{pmatrix} 3 & 0 & -2 \\ 0 & 3 & -2 \\ 0 & 0 & 1 \end{pmatrix}$

associated with $\lambda_1 = 3$

associated with $\lambda_2 = 2$



Recorded Lecture

Our Linear Algebra Lec. in **You-Tube** (2012 Spring)

- SKKU Linear Algebra with Sage, 1. Preface, SGLee <http://youtu.be/CbfJYPCkbn8>
- SKKU Linear Algebra with Sage, 2. Introduction of CAS, SGLee <http://youtu.be/oSQpiNe2LU8>
- SKKU Linear Algebra with Sage, 3. Section 1-1, Vector SGLee <http://youtu.be/85kGK6bJLns>
- SKKU Linear Algebra with Sage, 4. Section 1-2, Norm, SGLee <http://youtu.be/g55dfkmlTHE>
- SKKU Linear Algebra with Sage, 5. Section 1-3, Vector Equations, SGLee <http://youtu.be/YB976T1wokE>
- SKKU Linear Algebra with Sage, 6. Section 2-1, LSE, SGLee, <http://youtu.be/AAUQvdjQ-qk>
- SKKU Linear Algebra with Sage, 7. Section 2-2, RREF, SGLee, <http://youtu.be/HSm69YigRr4>
- SKKU Linear Algebra with Sage, 8. Section 2-3, Appl of LSE, SGLee, <http://youtu.be/G790BLDSK5g>
- SKKU Linear Algebra with Sage, 9. Section 3-1, Matrix, SGLee, <http://youtu.be/JdNnHGdJBrQ>
- SKKU Linear Algebra with Sage, 10. Section 3-2, Inverse Matrix, SGLee <http://youtu.be/yeCUPdRx7Bk>
- SKKU Linear Algebra with Sage, 11. Section 3-3, Elementary Matrix, SGLee <http://youtu.be/oQ2m6SSSquc>
- SKKU Linear Algebra with Sage, 12. Section 3-4, Subspace, SGLee <http://youtu.be/UTTUG6JUFQM>
- SKKU Linear Algebra with Sage, 13. Section 3-5, Solutions Set, SGLee http://youtu.be/OoTPCpKW_eY
- SKKU Linear Algebra with Sage, 14. Section 3-6, Special Matrices, SGLee <http://youtu.be/jLh77sZOaM8>
- SKKU Linear Algebra with Sage, 15. Section 3-7, LU-Factorization, SGLee <http://youtu.be/lKJPnLCiAVU>
- SKKU Linear Algebra with Sage, 16. Section 3-6, Theorem of Triangular matrix, SGLee <http://youtu.be/UriXEI-xoRk>
- SKKU Linear Algebra with Sage, 17. Section 4-1, Determinant, SGLee <http://youtu.be/Vf8LlkKKHgg>
- SKKU Linear Algebra with Sage, 17-2. Section 4-1, Determinant 2, SGLee http://youtu.be/_3WRIwDUUgY
- SKKU Linear Algebra with Sage, 18. Section 4-2, Cofactor Expansion, SGLee <http://youtu.be/m6l2my6pSwY>
- SKKU Linear Algebra with Sage, 19. Section 4-3, Cramer's Law, SGLee <http://youtu.be/m2NkOX7gE5o>
- SKKU Linear Algebra with Sage, 20. Section 4-4, Appl of Determinant, SGLee http://youtu.be/KtkOH5M3_Lc
- SKKU Linear Algebra with Sage, 21. Section 4-5, Eigenvalue & Eigenvector, SGLee <http://youtu.be/96BrbKx1cQ4>
- SKKU Linear Algebra with Sage, 22. Section 5-1, Power Method, SGLee <http://youtu.be/CLxjkZuNJXw>
- SKKU Linear Algebra with Sage, 23. Section 5-2, Encryption, SGLee <http://youtu.be/umTIADxsEq8>
- SKKU Linear Algebra with Sage, 24. Section 5-3, Blackout Game, SGLee http://youtu.be/_bS33lfa29s
- SKKU Linear Algebra with Sage, 25. Section 5-4, Markov Chains, SGLee <http://youtu.be/156ezier6HQ>
- SKKU Linear Algebra with Sage, 26. Section 5-5, Google Matrix, SGLee http://youtu.be/WNUoXLh8i_E
- SKKU Linear Algebra with Sage, 27. Section 5-6, Project, SGLee <http://youtu.be/coNq48CW6Pg>
- SKKU Linear Algebra with Sage, 28. Section 6-1, Linear Transformation, SGLee <http://youtu.be/Yr23NRSpSoM>
- SKKU Linear Algebra with Sage, 29. Section 6-2, Linear Operator, SGLee <http://youtu.be/12WP-cb6Ymc>

Student's Activities in Movies, **You-Tube** (2012 Spring Linear Algebra Class)



- Preview : <http://matrix.skku.ac.kr/2012-Album/1-CLA-Preview.html>
- SKKU Linear Algebra with Sage, S1. Section 1-1, Vector, Student <http://youtu.be/fbCMYh-iDCQ>
- SKKU Linear Algebra with Sage, S2. Section 1-2, Norm, Student http://youtu.be/sEFj_7t_bqc
- SKKU Linear Algebra with Sage, S3. Section 1-3, Vector Equations, Student <http://youtu.be/avVJfeEoeVs>
- SKKU Linear Algebra with Sage, S4. Chapter 1 Discuss, Student <http://youtu.be/tys3taO5IHs>
- SKKU Linear Algebra with Sage, S5. Section 2-1, LSE, Student <http://youtu.be/N5ltl-bfdvkz>
- SKKU Linear Algebra with Sage, S6. Section 2-2, RREF, Student <http://youtu.be/sJeomjbRFmE>
- SKKU Linear Algebra with Sage, S7. Section 2-3, Appl of LSE, Student http://youtu.be/vx_6rTJq5jk
- SKKU Linear Algebra with Sage, S8. Section 3-1, Matrix, Student <http://youtu.be/LaAAruKbGyc>
- SKKU Linear Algebra with Sage, S9. Section 3-2, Inverse Matrix, Student <http://youtu.be/-MPszmMNVLE>
- SKKU Linear Algebra with Sage, S10. Section 3-3, Elementary Matrix, Student <http://youtu.be/cel8oeXp6xU>
- SKKU Linear Algebra with Sage, S11. Section 3-4, Subspace, Student <http://youtu.be/s7jxVvVAel4>
- SKKU Linear Algebra with Sage, S12. Section 3-5, Solutions Set, Student <http://youtu.be/lygHFdWacds>
- SKKU Linear Algebra with Sage, S13. Section 3-6, Special Matrices, Student <http://youtu.be/rYBsPkeVhQo>
- SKKU Linear Algebra with Sage, S14. Section 4-1, Determinant, Student http://youtu.be/Fne4gaZtE_Q
- SKKU Linear Algebra with Sage, S15. Section 4-2, Cofactor Expansion, Student <http://youtu.be/nAabf3lpFU4>
- SKKU Linear Algebra with Sage, S16. Section 4-3, Cramer's Law, Student http://youtu.be/Ygu4_7l4fGQ
- SKKU Linear Algebra with Sage, S17. Section 4-5, Eigenvalue & Eigenvector, Student <http://youtu.be/WEyopW5iH6A>
- SKKU Linear Algebra with Sage, S18. Section 6-1, Linear Transformation, Student <http://youtu.be/dNt6NUDwc-Q>
- SKKU Linear Algebra with Sage, S19. Section 6-2, Linear Operator, Student <http://youtu.be/K241VtU8o4>



What was Developed 1 !



Sage (embedded)

- ▶ <http://math1.skku.ac.kr/home/pub/534> : SKKU-LA-1.1-(Vector)
- ▶ <http://math1.skku.ac.kr/home/pub/535> : SKKU-LA-1.1-(Vector)
- ▶ <http://math1.skku.ac.kr/home/pub/583> : LA-2.1-(Linear Equations)
- ▶ <http://math1.skku.ac.kr/home/matrix/348> : **SKKU-LA-3.2-(inverse)**
- ▶ <http://math1.skku.ac.kr/home/pub/730/> : **LA-4.4- Volume (Det)**
- ▶ <http://math1.skku.ac.kr/home/pub/597> : **LA-4.4-Curve Fitting(Det)**
- ▶ <http://math1.skku.ac.kr/home/pub/540> : LA-4.4-Equations(Det)
- ▶ <http://math1.skku.ac.kr/home/pub/732> : LA-4.5-(Eigensystem)
- ▶ <http://math1.skku.ac.kr/home/pub/733> : **SKKU-LA-5.2-Crypto**
- ▶ <http://math1.skku.ac.kr/home/pub/734> : LA-5.2-Math Modeling
- * <http://math1.skku.ac.kr/home/pub/639>
: SKKU-LA-6.2- Flag (**Rotation**)
- * <http://math1.skku.ac.kr/home/pub/598>
: SKKU-LA-6.2- (Linear Transformation)
- * <http://math1.skku.ac.kr/home/pub/596>
: SKKU-LA-6.2- (**Linear Transformation**)
- * <http://math1.skku.ac.kr/home/pub/582>
: SKKU-LA-6.2- (Linear Transformation)
- * <http://math1.skku.ac.kr/home/pub/580>
: SKKU-LA-6.5- (**Dilation**)
- * <http://math1.skku.ac.kr/home/pub/729/>
: SKKU-LA-6.5-Tri (Dilation)
- * <http://math1.skku.ac.kr/home/pub/708>
: SKKU-LA-6.5- (Linear Transformation)
- * <http://math1.skku.ac.kr/home/pub/539>
: SKKU-LA-7.5- (**Projection**)



What was Developed 2 !



Sage with Geogebra

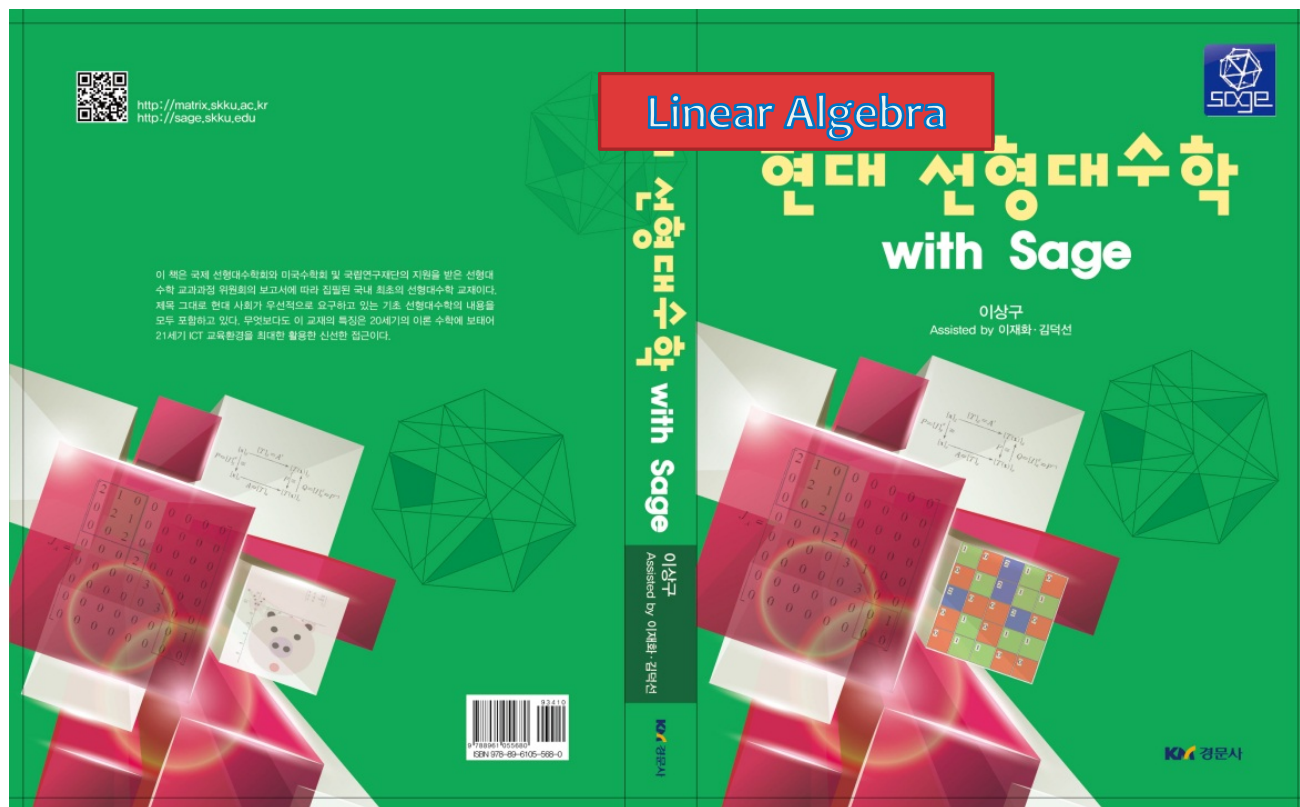
- ❖ Vector Sum : <http://math1.skku.ac.kr/home/pub/534>
- ❖ Scalar Multiplication : <http://math1.skku.ac.kr/home/pub/535>
- ❖ Linear System of Equations : <http://math1.skku.ac.kr/home/pub/583>
- ❖ Matrix Multiplication : <http://math1.skku.ac.kr/home/pub/769>
- ❖ Inverse Matrix : <http://math1.skku.ac.kr/home/pub/731>
- ❖ Area of Triangle : <http://math1.skku.ac.kr/home/pub/730>
- ❖ **Line(Plane) Equation** : <http://math1.skku.ac.kr/home/pub/540>
- ❖ **Curve Fitting** : <http://math1.skku.ac.kr/home/pub/597>
- ❖ Eigenvalue and Eigenvectors : <http://math1.skku.ac.kr/home/pub/732>
- ❖ Matrix Crypton : <http://math1.skku.ac.kr/home/pub/733>
- ❖ Light Out Game : <http://matrix.skku.ac.kr/blackwhite2/blackwhite.html>
- ❖ Rotation : <http://math1.skku.ac.kr/home/pub/639>
- ❖ Reflection and Projection : <http://math1.skku.ac.kr/home/pub/598>
<http://math1.skku.ac.kr/home/pub/582>
- ❖ Shear : <http://math1.skku.ac.kr/home/pub/596>
- ❖ Dilation : <http://math1.skku.ac.kr/home/pub/580>
- ❖ Quadratic Form : <http://math1.skku.ac.kr/home/pub/751>
- ❖ Fourier Series : <http://math1.skku.ac.kr/home/pub/752>
- ❖ Jordan Blocks : <http://math1.skku.ac.kr/home/pub/773>



New LA textbook



Contemporary Linear Algebra with Sage



Published in
September 1st!



Linear algebra with Sage

E-book (Model 1)

10.1 점도표를 이용한 Jordan 표준형 구하기

YouTube 동영상 강의: <http://youtu.be/NBLZPcWRHYI>

주어진 행렬이 대각화가능하다면 이 행렬과 관계된 대부분의 문제는 쉽게 다루어서 원하는 결론을 얻 행렬이 대각화가능한 것은 아니다. 이 절에서는 주어진 행렬과 닮음인 대각선 행렬과 거의 유사한 행렬을 구하는 방법을 소개한다.

우리는 n 차의 정사각행렬 A 가 n 개의 일차독립인 고유벡터를 가지면 대각화가능하다는 것을 8.2절 8.8.5에서 보았듯이 A 가 유니타리 대각화가능 필요충분조건은 A 가 정규행렬(normal matrix)인 것의 정규직교인 고유벡터를 갖고, 이 고유벡터를 일로 갖는 행렬 U 는 유니타리 행렬이며 $U^*AU = \Lambda$ 각선 성분으로 갖는 대각행렬이다.

행렬 $A \in M_n(C)$ 에 대하여 다음은 동치이다.

- (1) A 는 유니타리 대각화가능하다.
- (2) A 는 정규행렬이다.
- (3) A 는 n 개의 정규직교인 고유벡터를 갖는다.

대각화가능한 행렬을 다루는 것은 이론적으로나 실제에 있어서 모두 대각행렬을 다루는 것과 같이 쉽 n 차의 정사각행렬이 모두 n 개의 일차독립인 고유벡터를 갖지는 않으므로 대각화가능한 것은 아니 리]에 의하여 모든 행렬은 자신의 고유값을 대각선성분으로 갖는 상삼각행렬과 유니타리 닮음임은 안

임의의 n 차 정사각행렬은 자신의 고유값을 대각선성분으로 갖는 상삼각행렬과 유니타리 닮음이다

$$\text{즉, } U^*AU = \begin{bmatrix} \lambda_1 & & * \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix}$$

우선 대각화와 관련하여 지금까지 살펴본 내용을 요약하면 아래와 같다.

1. n 차 정사각행렬 A 가 대각화가능할 필요충분조건은 A 가 n 개의 일차독립인 고유벡터를 갖는다.
2. A 가 정규행렬일 ($AA^* = A^*A$) 필요충분조건은 A 가 유니타리 대각화가능하다.
3. 그러나 정규행렬이 아니면서도 대각화가능한 행렬은 존재한다.
4. 행렬 A 가 대각화가능이면, 각각의 고유값에 대한 고유공간 $\text{null}(\lambda I - A)$ 의 차원(기하적 중복도)이 그 고유값의 (대수적) 중복도와 같아야 한다. ($\Leftrightarrow n$ 개의 일차독립인 고유벡터가 존재한다.)

10.1.1 n 차 정사각행렬 A 가 $(1 \leq i \leq n)$ 개의 일차독립인 고유벡터를 가지면 A 는 다음과 같은 행렬 J_i 와 (유니타리) 닮음이다.

$$J_i = \begin{pmatrix} \lambda_i & 1 & & 0 \\ & \lambda_i & \ddots & \\ & & \ddots & 1 \\ 0 & & & \lambda_i \end{pmatrix}$$

즉, $U^*AU = J_i$ 인 유니타리 행렬 U 가 존재한다. 여기서,

$$J_i = \begin{pmatrix} \lambda_i & 1 & & 0 \\ & \lambda_i & \ddots & \\ & & \ddots & 1 \\ 0 & & & \lambda_i \end{pmatrix}, \quad (n_1 + n_2 + \dots + n_r = n, 1 \leq i \leq r)$$

이다. 이때, J_i 를 A 의 고유값 λ_i 에 대응하는 하나의 Jordan block이라 하고, J_i 를 A 의 Jordan 표준형(JCF, Jordan canonical form)이라 한다.

<http://youtu.be/NBLZPcWRHYI>

SKKU Linear Algebra with Sage, 54 . Section 10. 1 Jordan Canonical For...

Sang-Gu Lee · 동영상 107개

조회수: 56

구독 8

<http://matrix.skku.ac.kr/2012-mobile/e-cla/10-1.html>

Linear algebra with Sage

E-book (Model 2)

(2) $\begin{bmatrix} 4 & 1 & 2 \\ 0 & 4 & 2 \\ 0 & 0 & 4 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

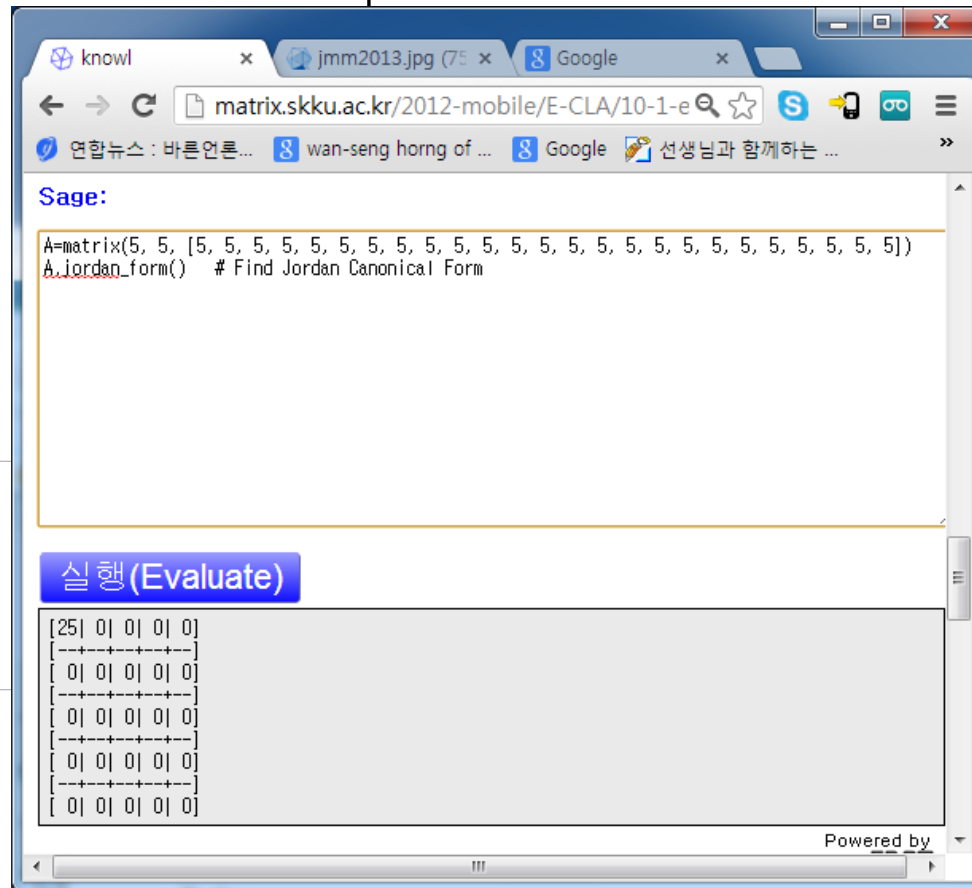

이제 시작

다음 행렬 A 의 Jordan 표준형을 구하는 Sage 명령어는 다음과 같다.

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

```
A=matrix(4, 4, [1, 1, 1, 0, 2, 0, 0, 0, 0, 1, 1, 0, 0, 0, 2])
A.Jordan_form() # Jordan 표준형을 구한다.
```

실행(Evaluate)



```
A=matrix(5, 5, [5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5])
A.Jordan_form() # Find Jordan Canonical Form
```

실행(Evaluate)

```
[25] mi mi mi mi
[---+---+---+---]
[ mi mi mi mi mi ]
[---+---+---+---]
[ mi mi mi mi mi ]
[---+---+---+---]
[ mi mi mi mi mi ]
[---+---+---+---]
[ mi mi mi mi mi ]
[---+---+---+---]
[ mi mi mi mi mi ]
```

Powered by

<http://matrix.skku.ac.kr/2012-mobile/E-CLA/10-1-ex.html>

Linear algebra with Sage

E-book (Model 3)

10.1 연습문제

YouTube 문제풀이 동영상:

- <http://youtu.be/9-G3Fd2xOW0>
- <http://www.youtube.com/watch?v=adWzUKKmO2k>

1. 5차 정사각행렬 A 가 중복도가 5인 고유값 λ 만을 갖고, λ 에 대응하는 일차 독립형의 종류를 구하여라.

2. 다음 Jordan 표준형 J_A 에 대하여 다음을 구하여라.

$$J_A = \begin{bmatrix} \lambda & 1 & 0 & 0 & 0 \\ 0 & \lambda & 1 & 0 & 0 \\ 0 & 0 & \lambda & 1 & 0 \\ 0 & 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & 0 & \lambda \end{bmatrix}$$

- (1) $J_A - \lambda I$
- (2) $(J_A - \lambda I)^2$
- (3) $(J_A - \lambda I)^3$
- (4) $(J_A - \lambda I)^4$

• <http://youtu.be/9-G3Fd2xOW0>

• <http://www.youtube.com/watch?v=adWzUKKmO2k>

Chapter 10-1
[1-4] 다음 행렬의 행렬의 일차 독립 고유벡터를 구하여라.
[답] $J = \begin{bmatrix} \lambda & 1 & 0 & 0 & 0 \\ 0 & \lambda & 1 & 0 & 0 \\ 0 & 0 & \lambda & 1 & 0 \\ 0 & 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & 0 & \lambda \end{bmatrix}$

4의 고유값 $\lambda = 4$ 에 대해, 각자의 중첩의 종류는 1, 1, 1이다.
→ 1차 독립형
→ 2차 독립형
→ 3차 독립형

Youtube CLA- 10장 Jordan Canonical Form 문제풀이 신길용 성균관대 선...
Sang-Gu Lee · 동영상 107개
구독 8
조회수: 143

오후 6:19

Sage Matrix

Sage Calculator for Matrix

Published Data ▼

- Free Worksheet -

Matrices(행렬생성)

Matrix Operations(행렬연산)

Matrix Spaces(행렬공간)

Row Operations(행연산)

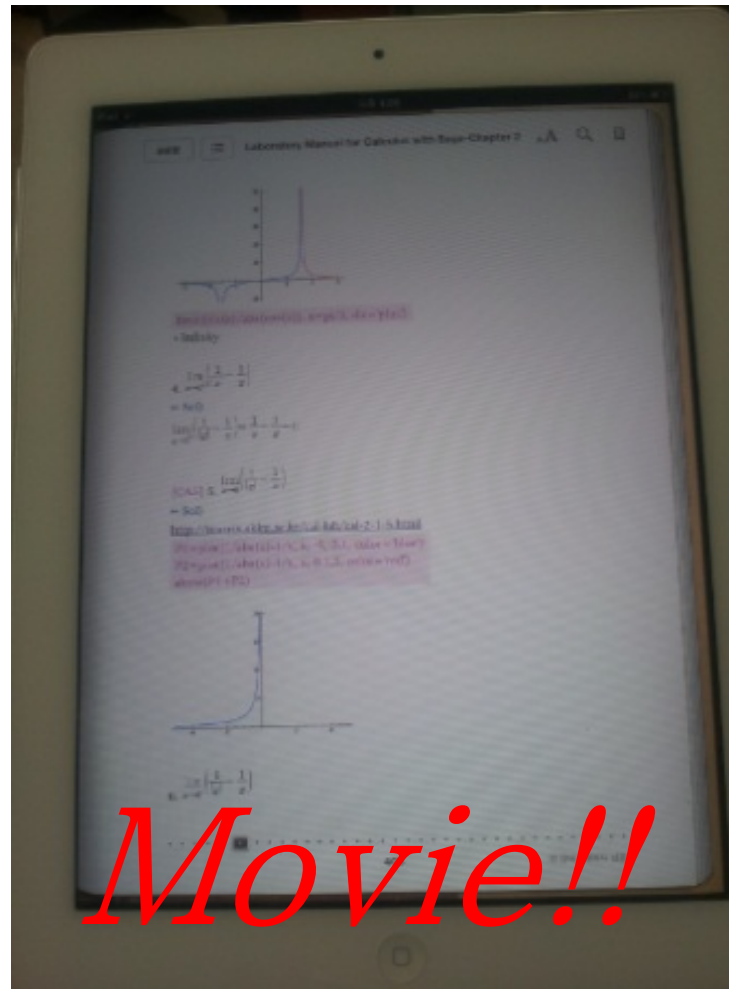
Echelon Form(행사다리꼴)

Constructing Submatrices(부분행렬)

Combining Matrices(행렬결합)

Matrix Decomposition(행렬분해)

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Sage C.

Problems

Sage

Q/A

SKKU Mathematical Modelling H.R.D. Division

Made by Prof. Sang-Gu Lee with SKKU Students (sglee@skku.edu)

- ▶ Movie e-book : [Calculus](#) Lab book
- ▶ Movie e-book : [Linear Algebra](#) (knowls)

MathJax

Beautiful math in all browsers

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Get MathJax

MathJax is an open source JavaScript display engine for mathematics that works in all modern browsers.

No more setup for readers. No more browser plugins. No more installations... It just works.

LaTeX:
$$\left(J_{\alpha}(x) = \sum \lim_{m \rightarrow \infty} \frac{(-1)^m}{m!} \Gamma(m + \alpha + 1) \left(\frac{x}{\dots} \right) \right)$$

Latest news: [IOP Publishing continues as MathJax](#)

MathJax features and benefits

High-quality typography.

MathJax™ uses modern CSS and web fonts, instead of equation images or Flash, so equations scale surrounding text at all zoom levels. See how this works in the [scaling math demo](#).

Using MathJax in popular web platforms

MathJax plugins are available for a growing number of wikis, blogs, and other content-management systems. These include WordPress, Blogger, Sphinx, TiddlyWiki, and MathEL-Wiki. A list of these is available in the [web applications](#) list of the [MathJax web site](#).

If the program you are using is not one of these, you may still be able to use MathJax by modifying the theme or template for your wiki or blog, as explained below.

Using MathJax in a Theme File

Most web-based content-management systems include a theme or template layer that determines how the pages look, and that loads information common to all pages. Such theme files provide one popular way to include MathJax in your web templates in the absence of MathJax-specific plugins for the system you are using. To take advantage of this approach, you will need access to your theme files, which probably means you need to be an administrator for the site; if you are not, you may need to have an administrator do these steps for you.

To enable MathJax in your web platform, add the line:

```
<script type="text/javascript"
  src="http://cdn.mathjax.org/mathjax/latest/MathJax.js?config=TeX-AMS-MML_HTMLorMML"></script>
```

either just before the `</head>` tag in your theme file, or at the end of the file if it contains no `</head>`.

The theme files for various popular platforms are:

WordPress

```
wp-content/themes/[current_theme]/header.php
```

Movable Type

```
[current_theme_templates]/html_head.mhtml
```

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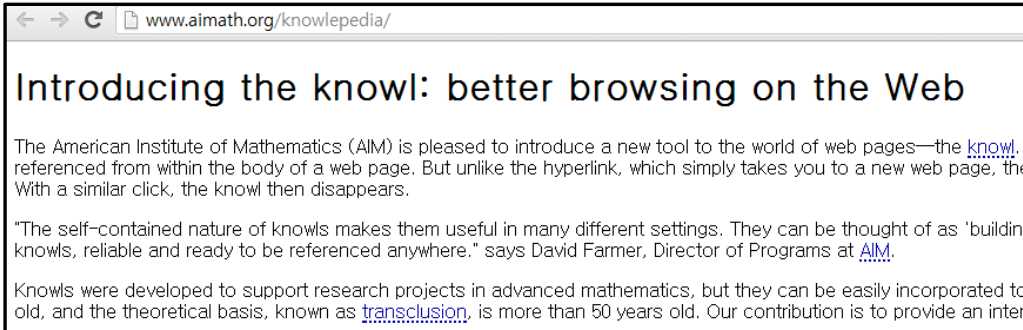
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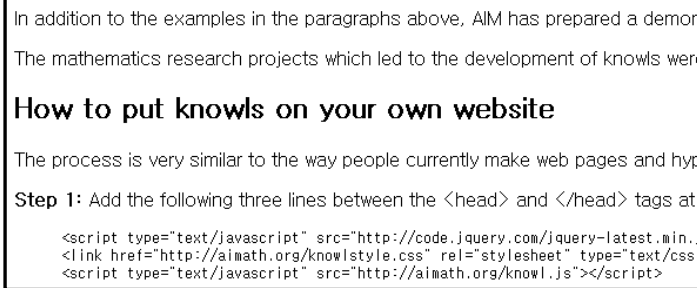
← → ↻ www.aimath.org/knowledgepedia/

Introducing the knowl: better browsing on the Web

The American Institute of Mathematics (AIM) is pleased to introduce a new tool to the world of web pages—the [knowl](#). It is referenced from within the body of a web page. But unlike the hyperlink, which simply takes you to a new web page, the knowl remains on the page. With a similar click, the knowl then disappears.

"The self-contained nature of knowls makes them useful in many different settings. They can be thought of as 'building blocks' of information, reliable and ready to be referenced anywhere," says David Farmer, Director of Programs at [AIM](#).

Knowls were developed to support research projects in advanced mathematics, but they can be easily incorporated to any website, and the theoretical basis, known as [transclusion](#), is more than 50 years old. Our contribution is to provide an interface for using knowls.



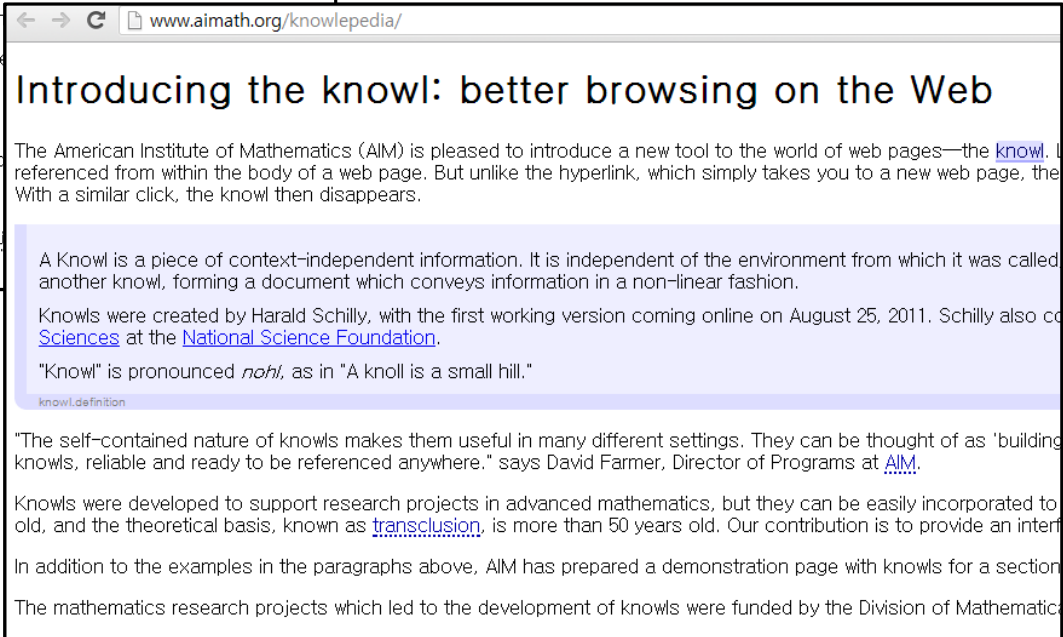
← → ↻ www.aimath.org/knowledgepedia/

How to put knowls on your own website

The process is very similar to the way people currently make web pages and hyperlinks. You simply add a few lines of code to your page.

Step 1: Add the following three lines between the <head> and </head> tags at the top of your page:

```
<script type="text/javascript" src="http://code.jquery.com/jquery-latest.min.js"></script>
<link href="http://aimath.org/knowlstyle.css" rel="stylesheet" type="text/css">
<script type="text/javascript" src="http://aimath.org/knowl.js"></script>
```



← → ↻ www.aimath.org/knowledgepedia/

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A Knowl is a piece of context-independent information. It is independent of the environment from which it was called, forming a document which conveys information in a non-linear fashion.

Knowls were created by Harald Schilly, with the first working version coming online on August 25, 2011. Schilly also co-edited the book [Mathematical Sciences](#) at the [National Science Foundation](#).

"Knowl" is pronounced *nohl*, as in "A knoll is a small hill."

knowldefinition

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In addition to the examples in the paragraphs above, AIM has prepared a demonstration page with knowls for a section of the [AIM website](#).

The mathematics research projects which led to the development of knowls were funded by the Division of Mathematical Sciences at the National Science Foundation.



A First Course in Linear Algebra

written by Robert A. Beezer

www.aimath.org/textbooks/beezer/

Home »

A First Course in Linear Algebra

Home

- Preface
- Acknowledgments

Systems of Linear Equations

- What is Linear Algebra?
- Solving Systems of Linear Equations
- Reduced Row-Echelon Form
- Types of Solution Sets
- Homogeneous Systems of Equations
- Nonsingular Matrices

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Vectors

- Vector Operations
- Linear Combinations
- Spanning Sets
- Linear Independence
- Linear Dependence and Spans
- Orthogonality

Solving Systems of Linear Equations

We will motivate our study of linear algebra by considering the problem of solving several linear equations simultaneously. The word "solve" tends to get abused somewhat, as in "solve this problem." When talking about equations we understand a more precise meaning: find *a!* of the values of some variable quantities that make an equation, or several equations, true.

Systems of Linear Equations

Example STNE: Solving two (nonlinear) equations.

Example: Solving two (nonlinear) equations
 Suppose we desire the simultaneous solutions of the two equations.

$$\begin{aligned} x^2 + y^2 &= 1 \\ -x + \sqrt{3}y &= 0 \end{aligned}$$

You can easily check by substitution that $x = \frac{\sqrt{3}}{2}, y = \frac{1}{2}$ and $x = -\frac{\sqrt{3}}{2}, y = -\frac{1}{2}$ are both solutions. We need to also convince ourselves that these are the *only* solutions. To see this, plot each equation on the xy -plane, which means to plot (x, y) pairs that make an individual equation true. In this case we get a circle centered at the origin with radius 1 and a straight line through the origin with slope $\frac{1}{\sqrt{3}}$. The intersections of these two curves are our desired simultaneous solutions, and so we believe from our plot that the two solutions we know already are indeed the only ones. We like to write solutions as sets, so in this case we write the set of solutions as

$$S = \left\{ \left(\frac{\sqrt{3}}{2}, \frac{1}{2} \right), \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2} \right) \right\}$$

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Definition SLE (System of Linear Equations) A *system of linear equations* is a collection of m equations in the variable quantities $x_1, x_2, x_3, \dots, x_n$ of the form.

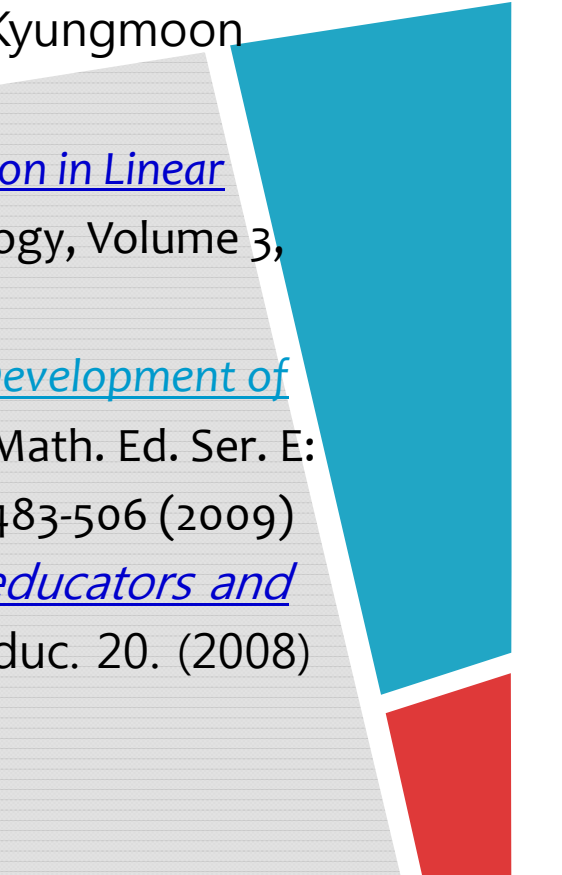
$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2n}x_n &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \dots + a_{3n}x_n &= b_3 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \dots + a_{mn}x_n &= b_m \end{aligned}$$

where the values of a_{ij}, b_i and x_j are from the set of complex numbers, \mathbb{C} .





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- 




Conclusion

We believe you now have an affordable and inspiring CAS tool for Linear Algebra, and Calculus.

In this talk, we introduced how and what we have done on E-books with Sage on Linear Algebra and Calculus.

Now our students can talk more on Mathematics and concentrate on Mathematical concepts, and use very affordable CAS tool without spending time to learn and typing programming language.





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