Sage E-books on Linear Algebra and Calculus

Department of Mathematics
Sungkyunkwan University

Sang-Gu Lee* with Kyung-Won Kim
sglee@skku.edu, kwkim@skku.edu

JOINT MATHEMATICS MEETINGS
AMERICAN MATHEMATICAL SOCIETY
MATHEMATICAL ASSOCIATION OF AMERICA

2013 JOINT MATHEMATICS MEETINGS
Largest Mathematics Meeting in the World
JANUARY 9-12 (WED-SAT), 2013 | SAN DIEGO CONVENTION CENTER, SAN DIEGO, CA

Abstract

- Calculus and Linear Algebra Books.
- ‘E-books’ of Calculus and Linear Algebra.
- What’s new in it?

- My Answer : <Movie clip and CAS in it>

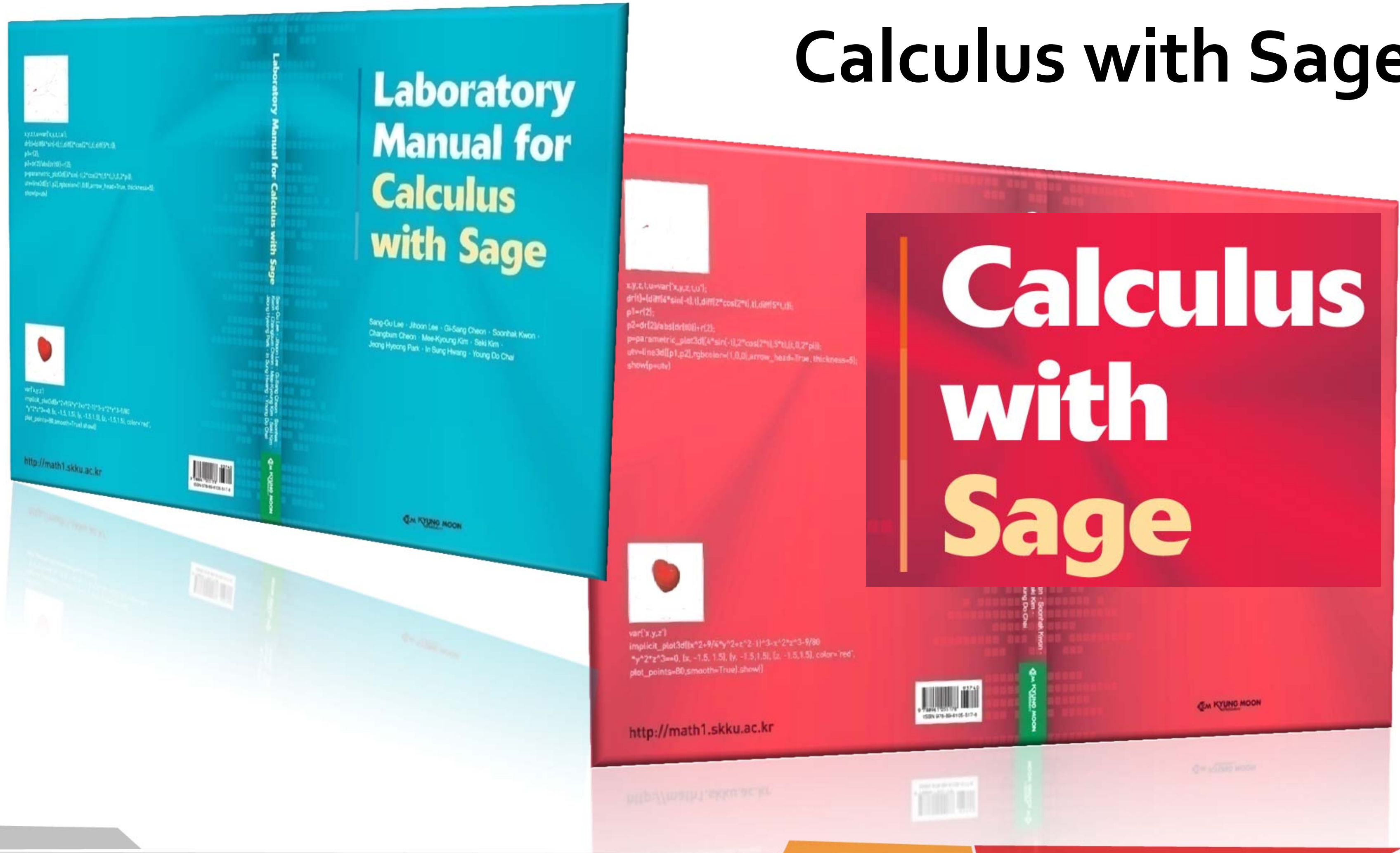
‘Sage’ (http://sagemath.org)

Laboratory Manual

for Calculus with Sage

ISBN: 978-89-6105-557-4, KyungMoon Pub. **Feb.2012**.

Calculus with Sage



Calculus with Sage

3.2 Derivatives of Polynomials, Exponential Functions, Trigonometric Functions, The Product Rule

In this section, we consider the derivatives of the basic functions, for example, polynomials, exponential functions, and trigonometric functions. Product rule and quotient rule are helpful to derive many derivatives of the functions which are expressed as the product and quotient. We also consider some applications of the derivative to the other sciences.

We introduce several differential rules that allow us to find the derivatives without the direct use of the limit definition.

Theorem 1 Derivative of a constant function

If f has the constant function, then

$$\frac{df}{dt} = \frac{d}{dx}(c) = 0$$

[Quick example 1] If f has the constant value $f(x) = 20$, then

$$\frac{df}{df} = \frac{d}{dx}(20) = 0$$

Similarly,

$$\frac{d}{dx}(\sqrt{3}) = 0 \quad \text{and} \quad \frac{d}{dx}(\pi) = 0$$

Note that the Constant Rule is equivalent to saying that the slope of horizontal line is zero.

The following rule tells how to differentiate x^n if n is a positive integer. to prove the following Rule we have to use the binomial expansion.

Theorem 1 The Power Rule for Positive Integer

If n is a positive integer, then $\frac{d}{dx}(x^n) = nx^{n-1}$.

molten rock cools by conduction of heat into surrounding rocks, and an engineer wants to know the rate at which water flows into or out of a reservoir.

CAS **Example 12**

Find an equation of the tangent line to the curve at the given point.

$$\begin{cases} x = \sin t \\ y = \cos 2t \end{cases}, \quad t = \frac{\pi}{4}$$

Solution

```
var('t, x, y, u')
x=sin(t)
y=cos(2*t)
dxdt=diff(x,t)
dydt=diff(y,t)
dydx=dydt/dxdt
yu=y(pi/4) + dydx(pi/4)*(u-x(pi/4))
p1=parametric_plot((x,y),(t,0,pi/2), color='blu
p2=plot(yu, u, 0, 1.5, color='red');
show(p1+p2)
```

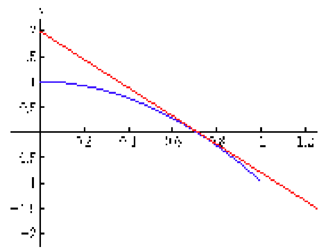


Figure 1

3.2 EXERCISES

CAS **7.** Find the equation of the tangent line to the curve $y = x^2\sqrt{x}$ at $(4,3)$.

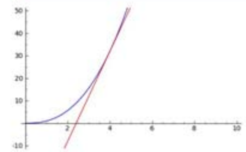
Sol $y' = \frac{5}{2}\sqrt{x^3} \Rightarrow y'(4) = 20$. So the slope of the tangent line is 20.

$y - 32 = 20(x-4) \Rightarrow y = 20x - 48$ ($\because y$ passes through $(4, 32)$).

```
f(x)=x^2*sqrt(x);
df(x)=diff(f(x),x);
y(x)=df(4)*(x-4)+32;
y(x)
```

20*x-48

```
p1=plot(f(x),x,0,10, color='blue');
p2=plot(y(x),x,0,10, color='red');
show(p1+p2,ymax=50,ymin=-10)
```



Laboratory Manual for Calculus with Sage

CAS

1. Introduction and Use of Sage-Math [1)]

Mathematical tools have long held an important place in classrooms. With the innovation of information and communication technologies, many tools have appeared and been adapted for educational purposes. Sage-Math is a popular mathematical software which was released in 2005. This software has efficient features which utilize the internet and can handle most mathematical problems, including linear algebra, algebra, combinatorics, numerical mathematics and calculus. In this book, we will introduce this powerful software and discuss how it can be used in classes.

Sage-Math is a mathematical CAS tool and is based on Internet Web environment. This tool was introduced in April 2008 at University of Washington, USA. It is free and has a powerful capability that can be compared with expensive commercial softwares such as Mathematica, Maple or Matlab, but can do more than that without requiring separate installations of the program. It is more like Web-Mathematica, but has some better features. When you connect to any Web browser, you can really solve almost all calculus problems in the book by using resources that we are offering. You can easily find pre-existing command to modify for your own problems.

Figure 1 We can use others existing codes as well in Sage

Korean Sage-Math model was developed and relevant experiments were done by BK 21 Mathematical Modelling HRD division at Sungkyunkwan University. We have built Korean servers that you can use as you see below. (Instructions will be given in your first class)

1) Mobile Sage-Math for Linear Algebra and its Application, Duk-Sun Kim, Sang-Gu Lee*, Greg Markowsky, Electronic Journal of Mathematics & Technology (eJMT) https://php.radford.edu/~ejmt/ContentIndex.php V.4, No. 3, pp. 1-13, Oct 2010. ISSN 1933-2823.

solving this gives,

$$n > 134.3 \text{ so that } n = 135.$$

8. (a) Determine the approximations and for T_{10} and S_{10} for $\int_0^1 e^{x^4}dx$ and the corresponding errors E_T and E_S.

(b) Compare the actual errors in part (a) with the error estimates given by [3] and [4].

(c) Determine how large do we have to choose so that the approximations T_n, M_n, and S_n to the integral in part (a) are accurate to within 0.00001?

Sol

(a) $T_{10} = 1.280262$, $S_{10} = 1.271599$, $E_T = -0.008975$, $E_S = -0.000312$.

(b) Since $K = 28e \approx 76.111891$ (3) and (4) gives

$$|E_T| \le \frac{76.111891}{12(10)^2} \approx 0.063426$$

$$|E_S| \le \frac{76.111891}{180(10)^4} \approx 0.000042$$

(c) For T_n, find n so that $\frac{K}{12n^2} < 0.00001$, $n = 797$.

For M_n, find n so that $\frac{K}{24n^2} < 0.00001$, $n = 564$.

For S_n, find n so that $\frac{K}{180n^4} < 0.00001$, $n = 15$.

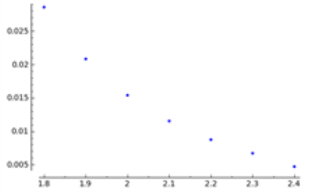
9. Given the function f at the following values,

x	1.8	1.9	2.0	2.1	2.2	2.3	2.4
$f(x)$	0.028561	0.020813	0.015384	0.011525	0.008742	0.006709	0.004079

approximate $\int_{1.8}^{2.4} f(x)dx$ using Simpson's Rule.

Sol

Plot $f(x)$



And Use Simpson's Rule

$\Delta x = (2.4-1.8)/6 = 0.1$

$$\Rightarrow \int_{1.8}^{2.4} f(x)dx \approx S_6$$

$$= \frac{0.1}{3}[f(1.8)+4f(1.9)+2f(2.0) + 4f(2.1)+2f(2.2)+4f(2.3)+f(2.4)]$$

$$\approx 0.007923$$

7.8 Improper Integrals

CAS **1.** $\int_e^{\infty} \frac{\ln x}{x}dx$

Sol

```
integral(ln(x)/x,x,e,infinity)
```

ValueError: Integral is divergent

CAS **2.** $\int_0^{\infty} xe^{-x}dx$

Sol

```
integral(x*e^(-x),x,0,infinity)
```

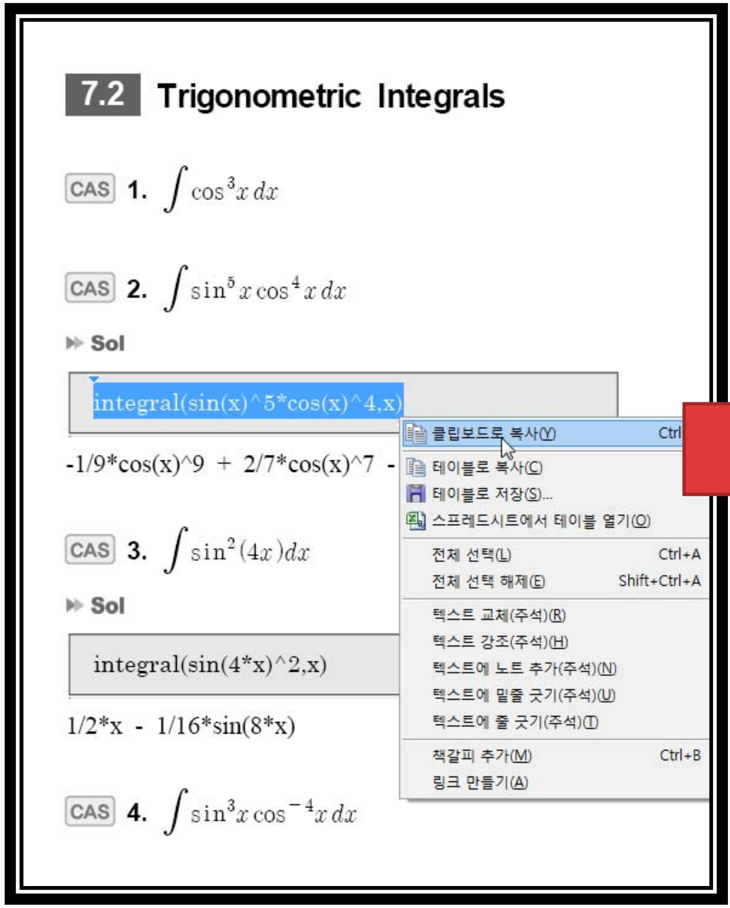
1

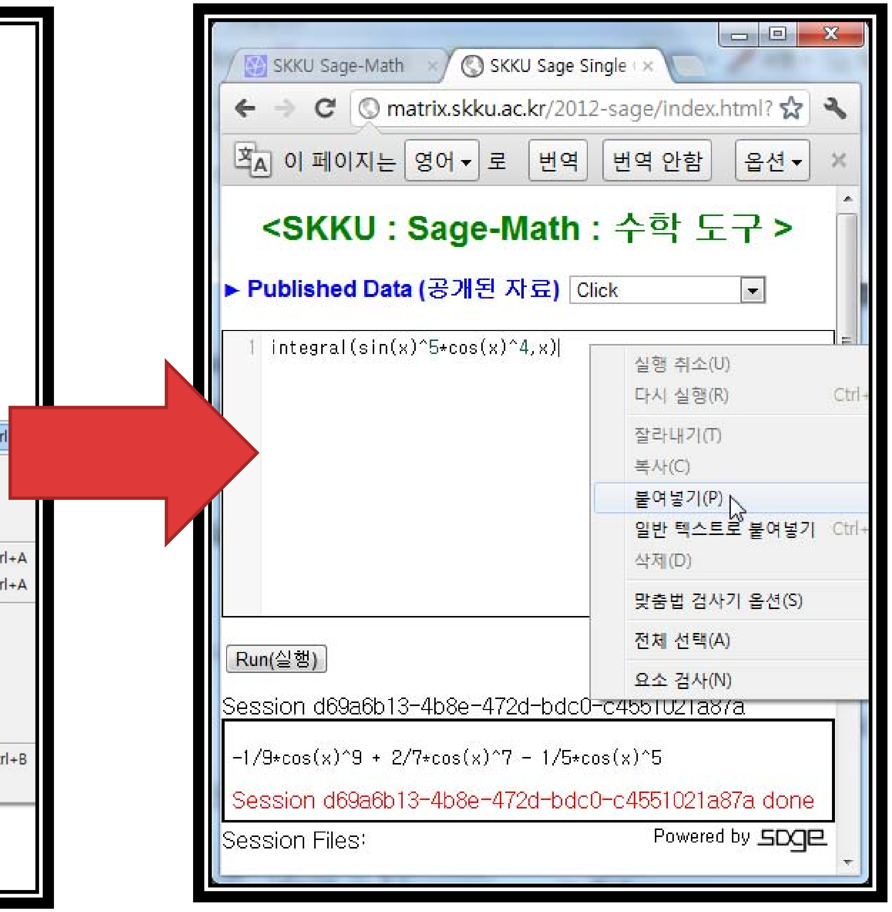
CAS **3.** $\int_{-\infty}^{\infty} \frac{1}{1+x^2}dx$

Sol

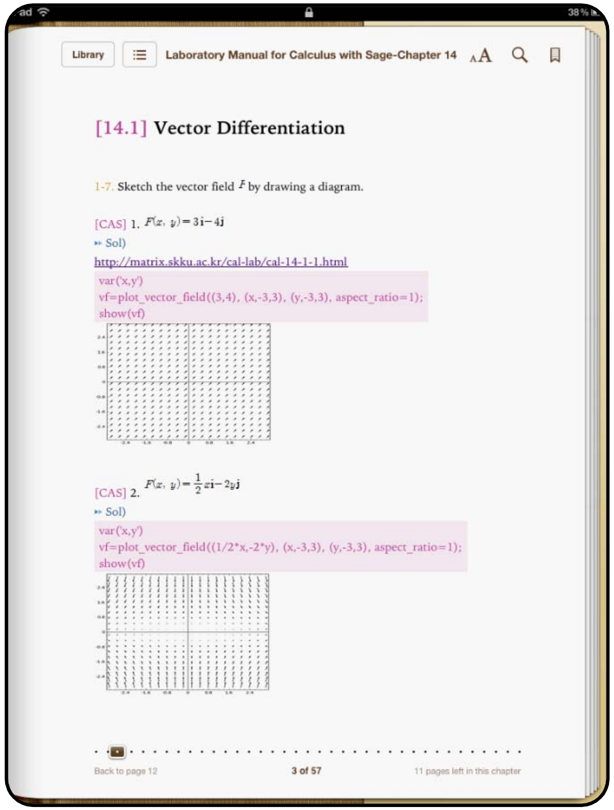
Calculus with SKKU Sage Cell Server

http://sage.skku.edu





What's more?



Library | Laboratory Manual for Calculus with Sage-Chapter 14

[14.1] Vector Differentiation

1-7. Sketch the vector field F by drawing a diagram.

[CAS] 1. $F(x, y) = 3\mathbf{i} - 4\mathbf{j}$

Sol)

http://matrix.skku.ac.kr/cal-lab/cal-14-1-1.html

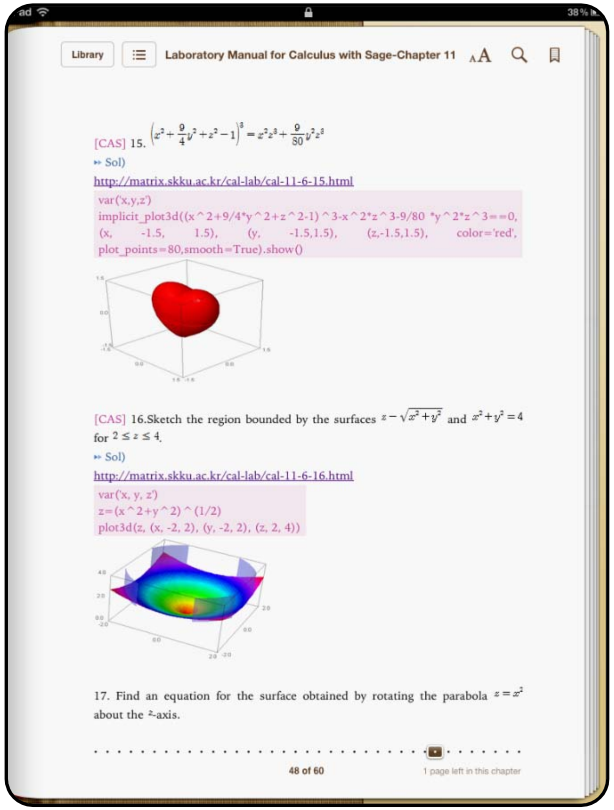
```
var('x,y')
vf=plot_vector_field((3,4), (x,-3,3), (y,-3,3), aspect_ratio=1);
show(vf)
```

[CAS] 2. $F(x, y) = \frac{1}{2}x\mathbf{i} - 2y\mathbf{j}$

Sol)

```
var('x,y')
vf=plot_vector_field((1/2*x,-2*y), (x,-3,3), (y,-3,3), aspect_ratio=1);
show(vf)
```

Back to page 12 | 3 of 57 | 11 pages left in this chapter



Library | Laboratory Manual for Calculus with Sage-Chapter 11

[CAS] 15. $\left(x^2 + \frac{9}{4}y^2 + z^2 - 1\right)^3 = x^2z^3 + \frac{9}{80}y^2z^3$

Sol)

http://matrix.skku.ac.kr/cal-lab/cal-11-6-15.html

```
var('x,y,z')
implicit_plot3d((x^2+9/4*y^2+z^2-1)^3-x^2*z^3-9/80 *y^2*z^3==0, (x, -1.5, 1.5), (y, -1.5,1.5), (z,-1.5,1.5), color='red', plot_points=80,smooth=True).show()
```

[CAS] 16.Sketch the region bounded by the surfaces $z = \sqrt{x^2+y^2}$ and $x^2+y^2=4$ for $2 \le z \le 4$.

Sol)

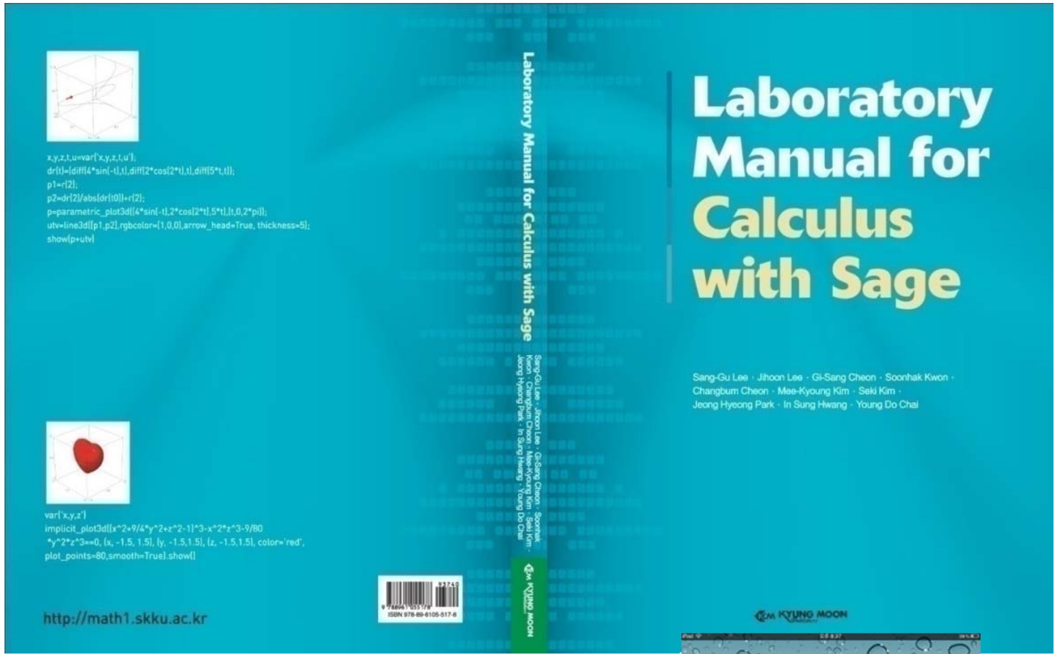
http://matrix.skku.ac.kr/cal-lab/cal-11-6-16.html

```
var('x, y, z')
z=(x^2+y^2)^(1/2)
plot3d(z, (x, -2, 2), (y, -2, 2), (z, 2, 4))
```

17. Find an equation for the surface obtained by rotating the parabola $z = x^2$ about the z-axis.

48 of 60 | 1 page left in this chapter

CAS Solutions

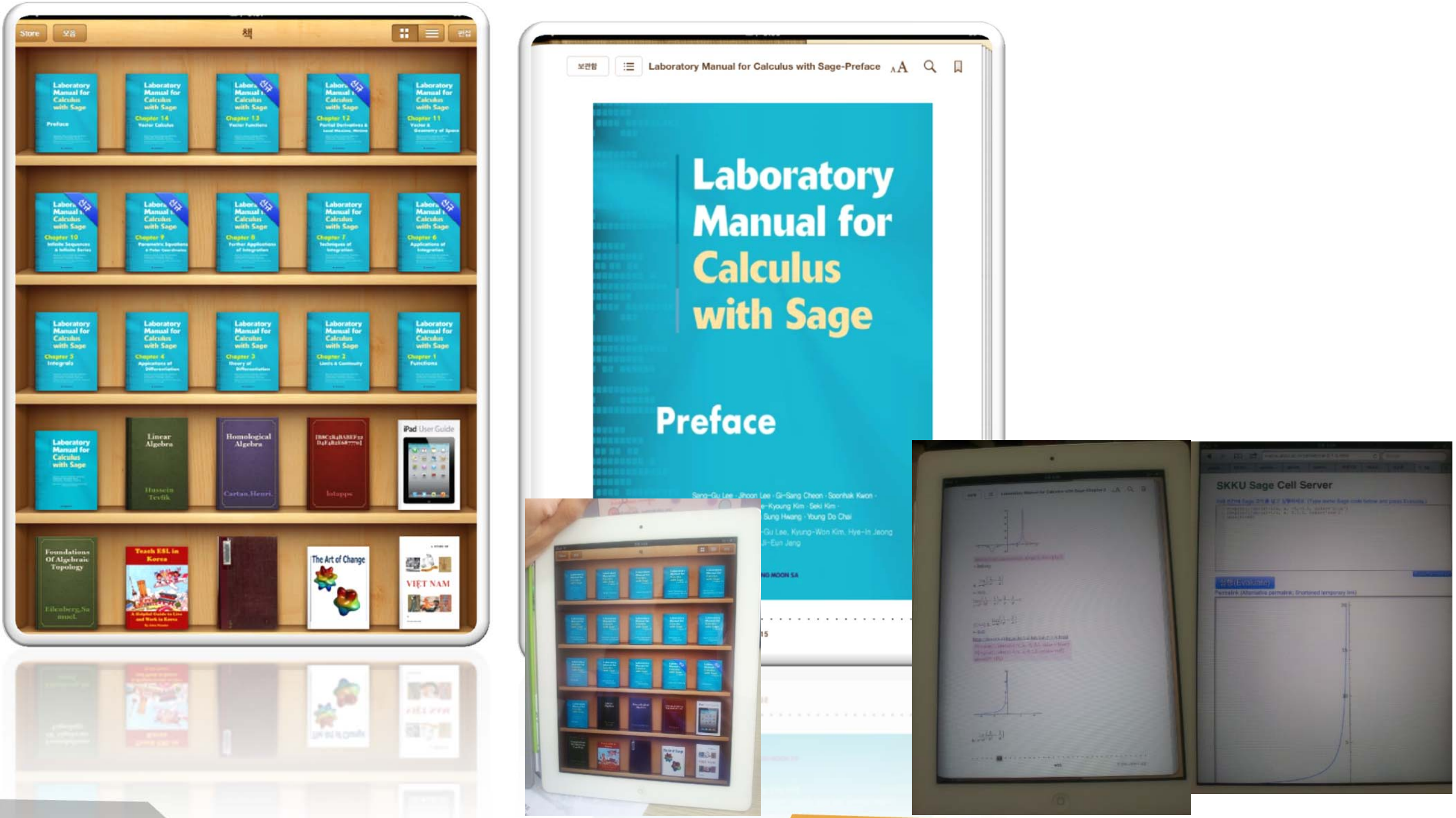


Session 1

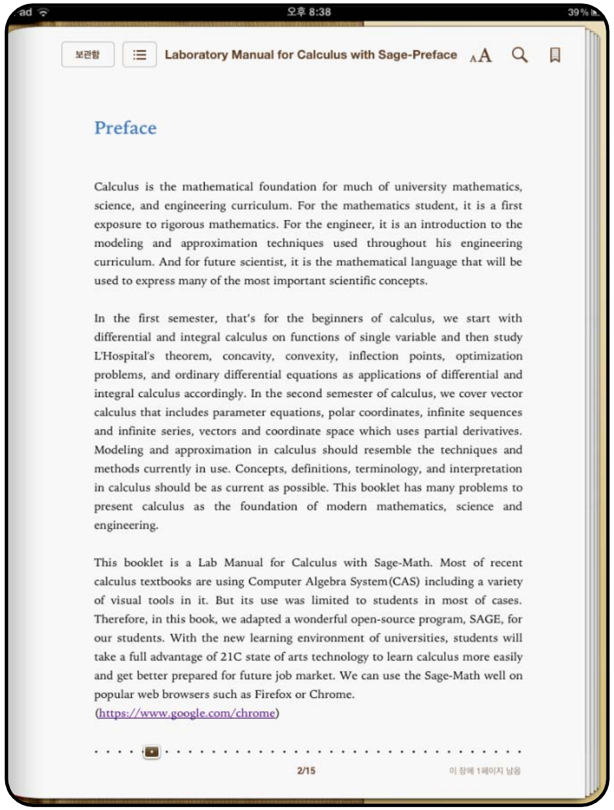
Laboratory Manual for Calculus with Sage

There are each Chapters 1 ~ 14 of the e-book for

Lab. Manual for Calculus with Sage



E-book of Lab. Manual for Calculus with Sage


Laboratory Manual for Calculus with Sage-Preface

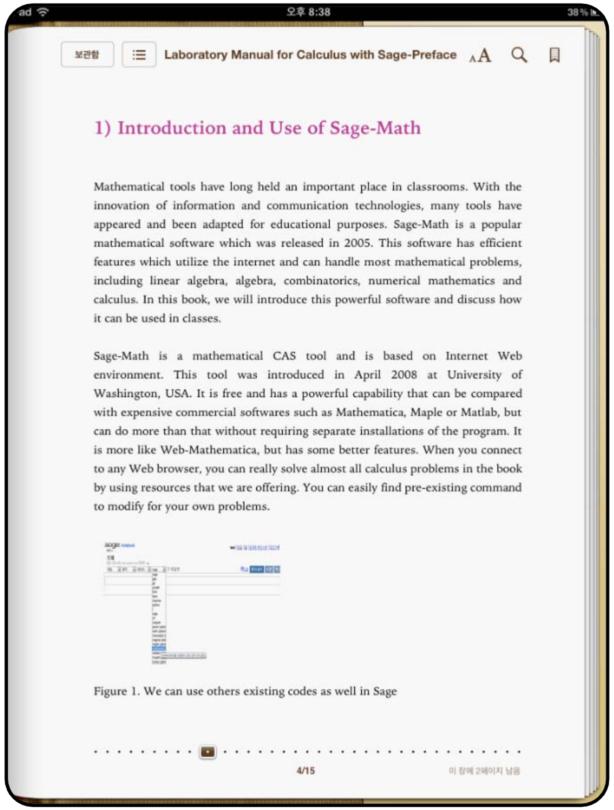
Preface

Calculus is the mathematical foundation for much of university mathematics, science, and engineering curriculum. For the mathematics student, it is a first exposure to rigorous mathematics. For the engineer, it is an introduction to the modeling and approximation techniques used throughout his engineering curriculum. And for future scientist, it is the mathematical language that will be used to express many of the most important scientific concepts.

In the first semester, that's for the beginners of calculus, we start with differential and integral calculus on functions of single variable and then study L'Hospital's theorem, concavity, convexity, inflection points, optimization problems, and ordinary differential equations as applications of differential and integral calculus accordingly. In the second semester of calculus, we cover vector calculus that includes parameter equations, polar coordinates, infinite sequences and infinite series, vectors and coordinate space which uses partial derivatives. Modeling and approximation in calculus should resemble the techniques and methods currently in use. Concepts, definitions, terminology, and interpretation in calculus should be as current as possible. This booklet has many problems to present calculus as the foundation of modern mathematics, science and engineering.

This booklet is a Lab Manual for Calculus with Sage-Math. Most of recent calculus textbooks are using Computer Algebra System(CAS) including a variety of visual tools in it. But its use was limited to students in most of cases. Therefore, in this book, we adapted a wonderful open-source program, SAGE, for our students. With the new learning environment of universities, students will take a full advantage of 21C state of arts technology to learn calculus more easily and get better prepared for future job market. We can use the Sage-Math well on popular web browsers such as Firefox or Chrome.
(https://www.google.com/chrome)

2/15


Laboratory Manual for Calculus with Sage-Preface

1) Introduction and Use of Sage-Math

Mathematical tools have long held an important place in classrooms. With the innovation of information and communication technologies, many tools have appeared and been adapted for educational purposes. Sage-Math is a popular mathematical software which was released in 2005. This software has efficient features which utilize the internet and can handle most mathematical problems, including linear algebra, algebra, combinatorics, numerical mathematics and calculus. In this book, we will introduce this powerful software and discuss how it can be used in classes.

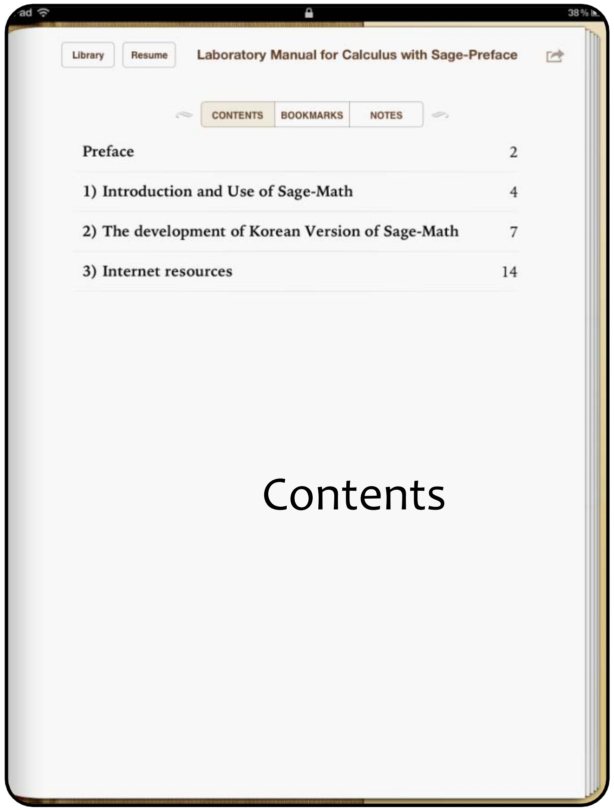
Sage-Math is a mathematical CAS tool and is based on Internet Web environment. This tool was introduced in April 2008 at University of Washington, USA. It is free and has a powerful capability that can be compared with expensive commercial softwares such as Mathematica, Maple or Matlab, but can do more than that without requiring separate installations of the program. It is more like Web-Mathematica, but has some better features. When you connect to any Web browser, you can really solve almost all calculus problems in the book by using resources that we are offering. You can easily find pre-existing command to modify for your own problems.

Figure 1. We can use others existing codes as well in Sage

4/15

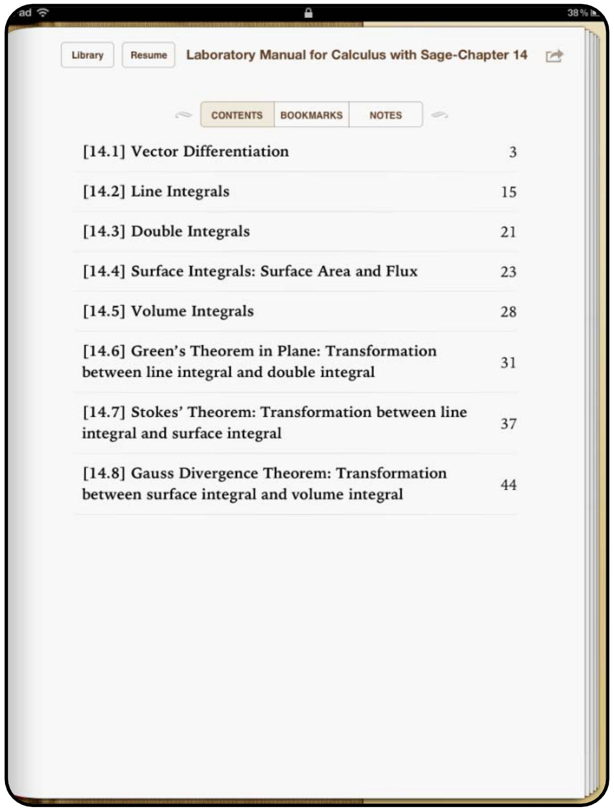
E-Paper

Contents for
E-book of Lab. Manual for Calculus with Sage



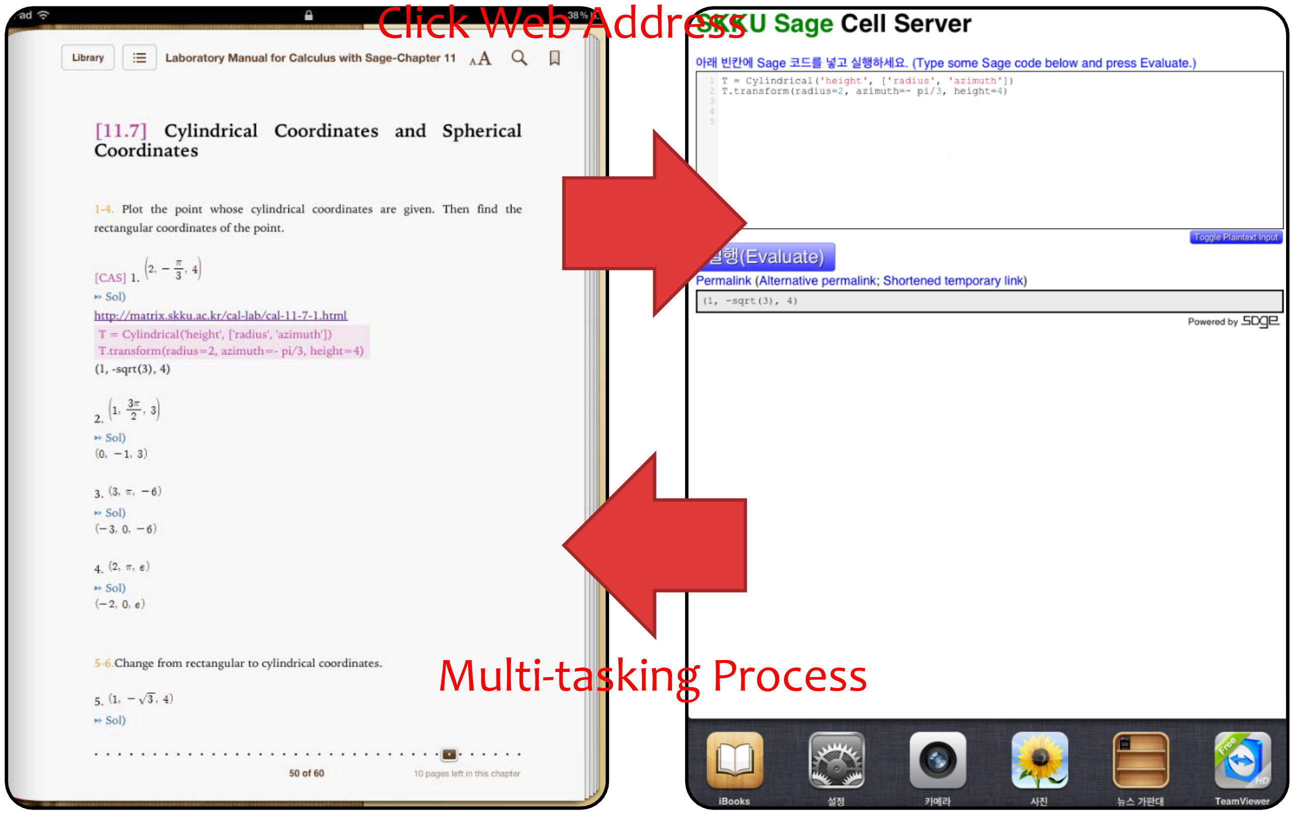
Contents

Part I Single Variable Calculus



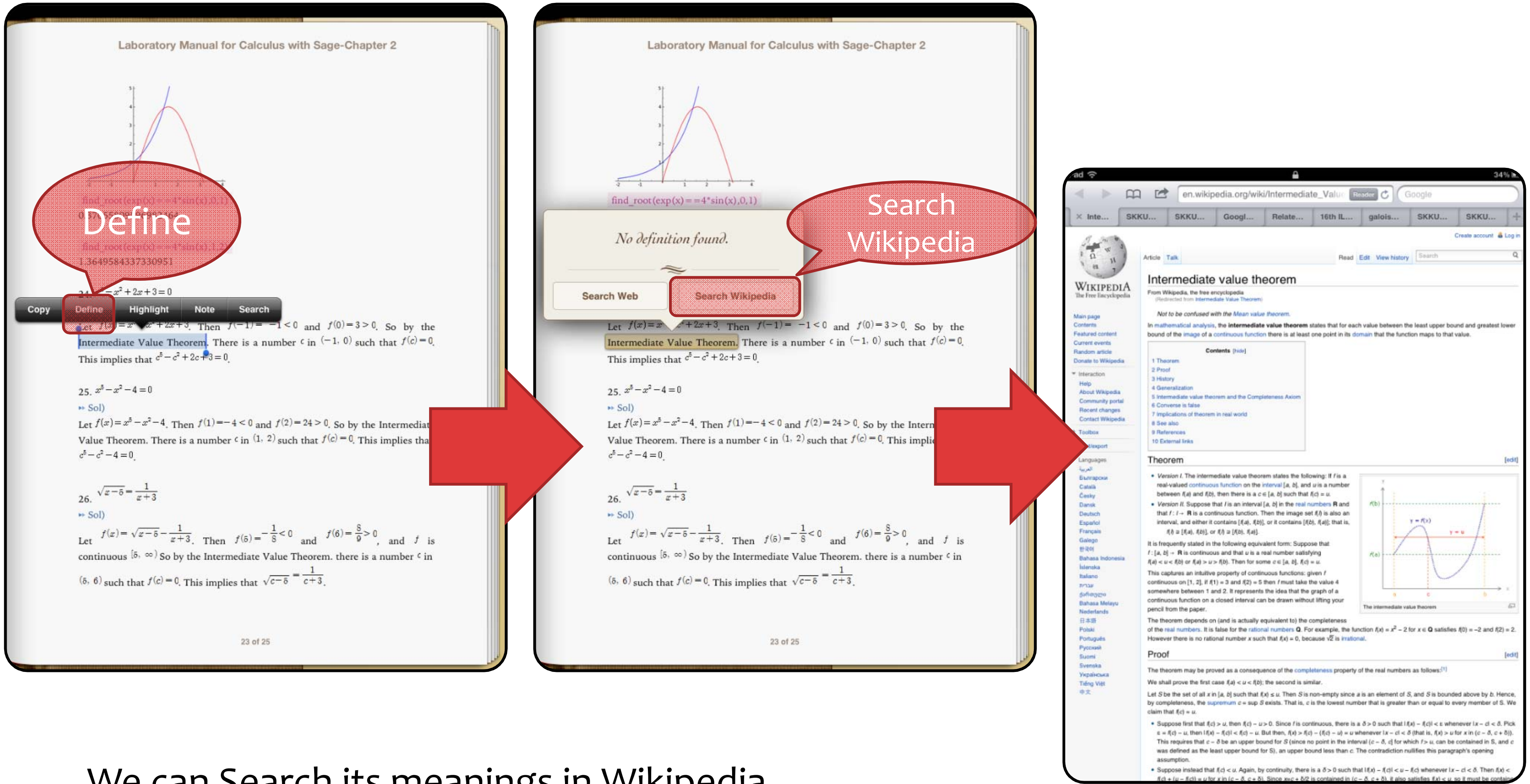
Part II Multivariate Calculus

In this E-book of Lab. Manual for Calculus with Sage



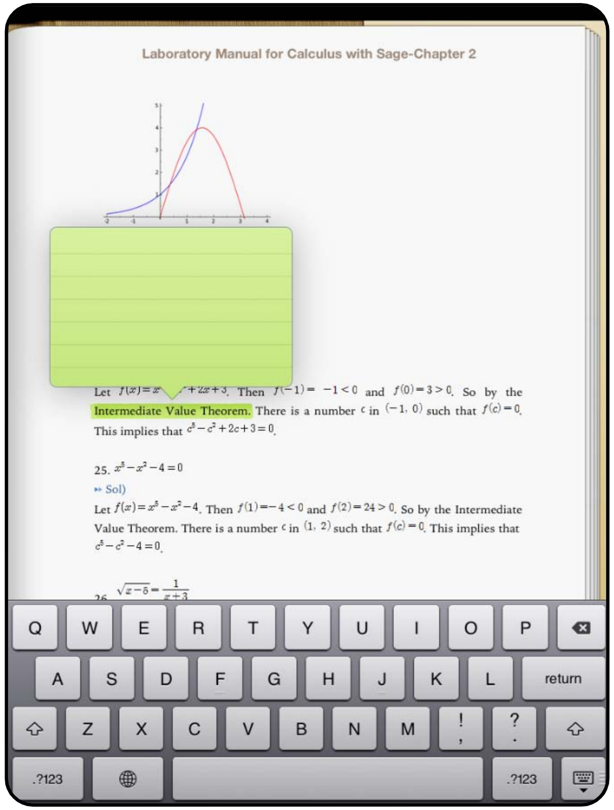
Using Sage Cell Server

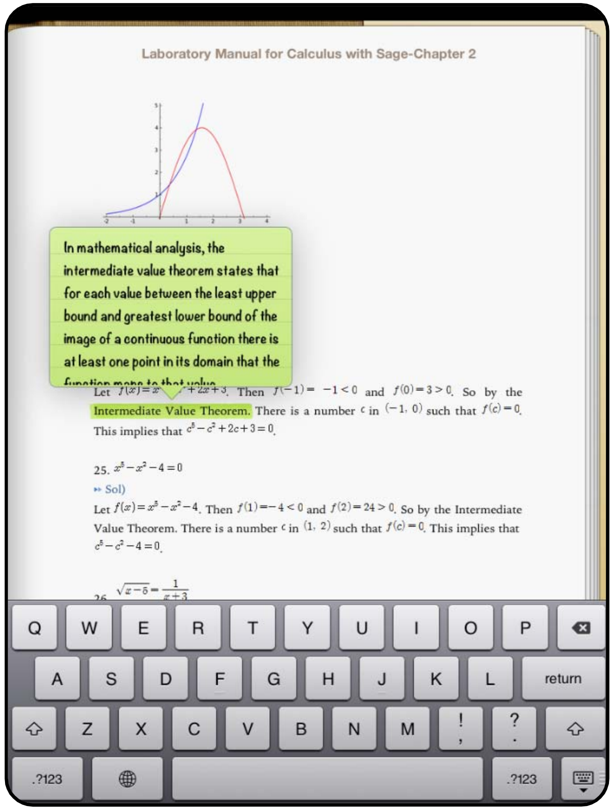
E-book of Lab. Manual for Calculus with Sage ○○○



We can Search its meanings in Wikipedia

E-book of Lab. Manual for Calculus with Sage

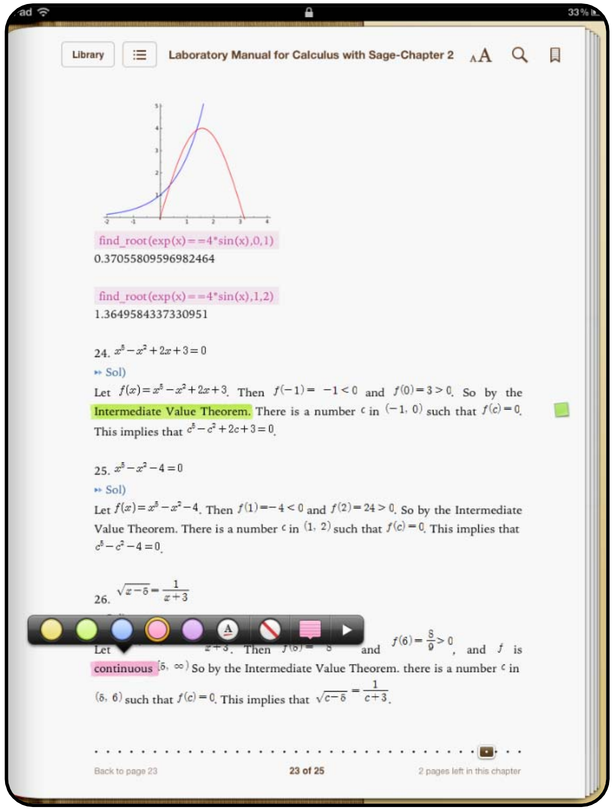


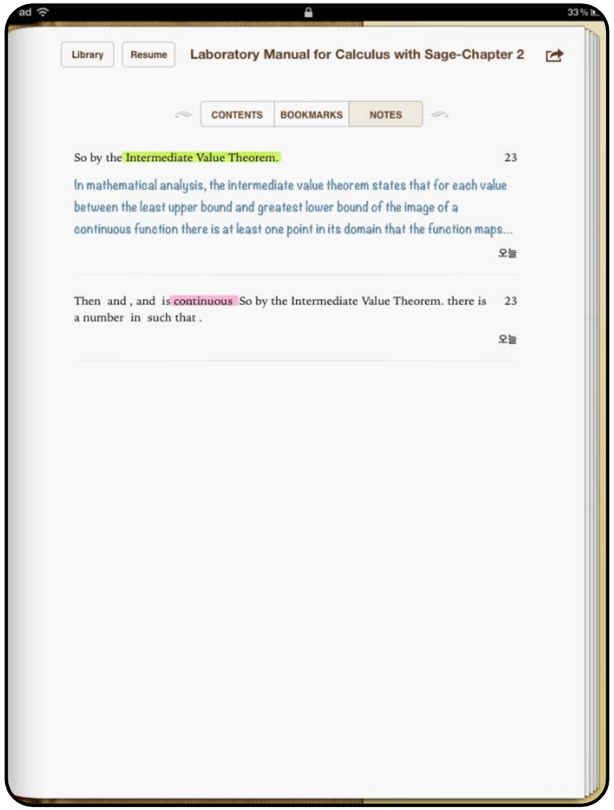


Note

We can add our notes in it.

E-book of Lab. Manual for Calculus with Sage

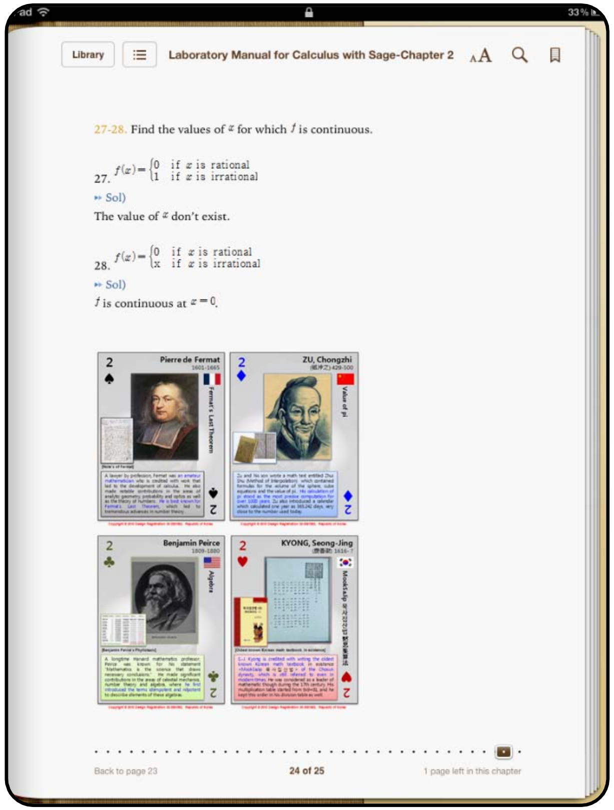




Marker

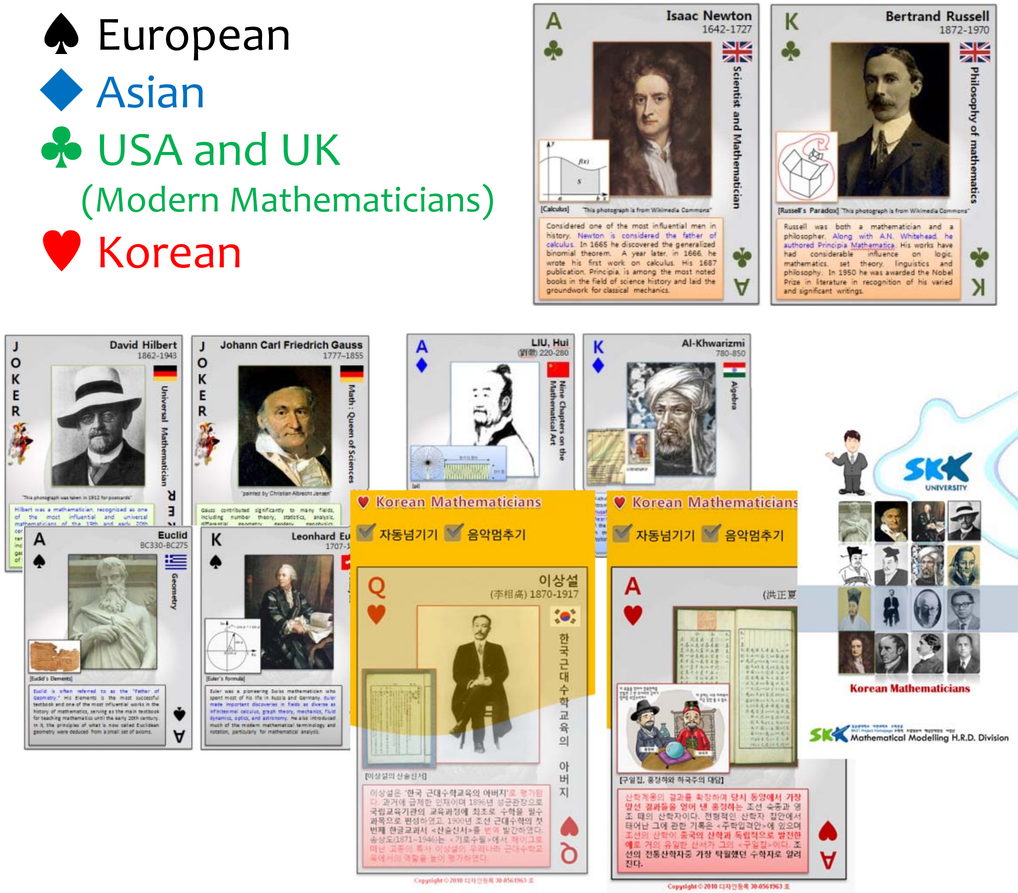
E-book of Lab. Manual for Calculus with Sage ○○○

http://matrix.skku.ac.kr/2011-Album/2011-Math-Appl/main.html



World Mathematicians Cards

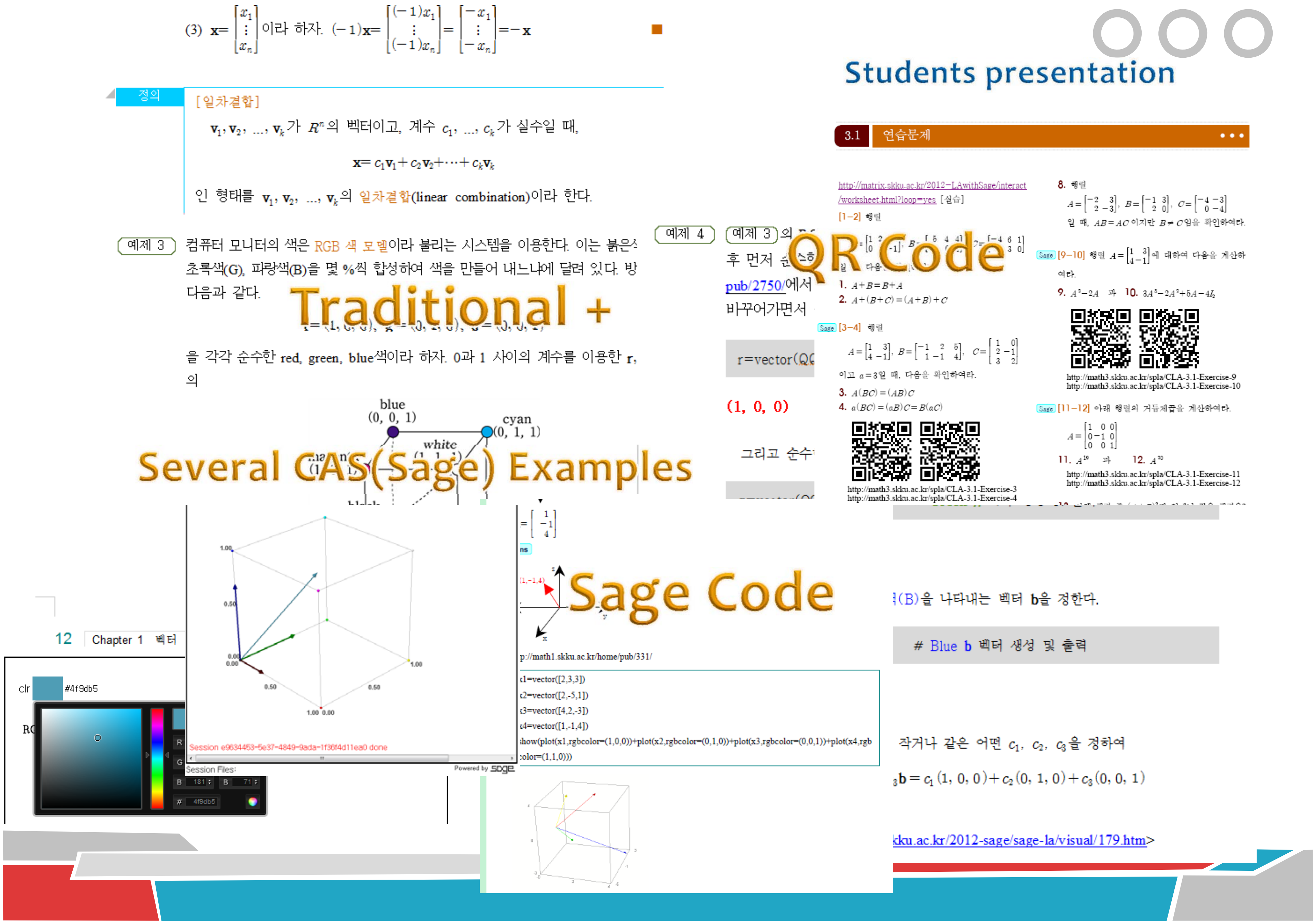
- ♠ European
- ◆ Asian
- ♣ USA and UK (Modern Mathematicians)
- ♥ Korean



Session 2 Linear Algebra

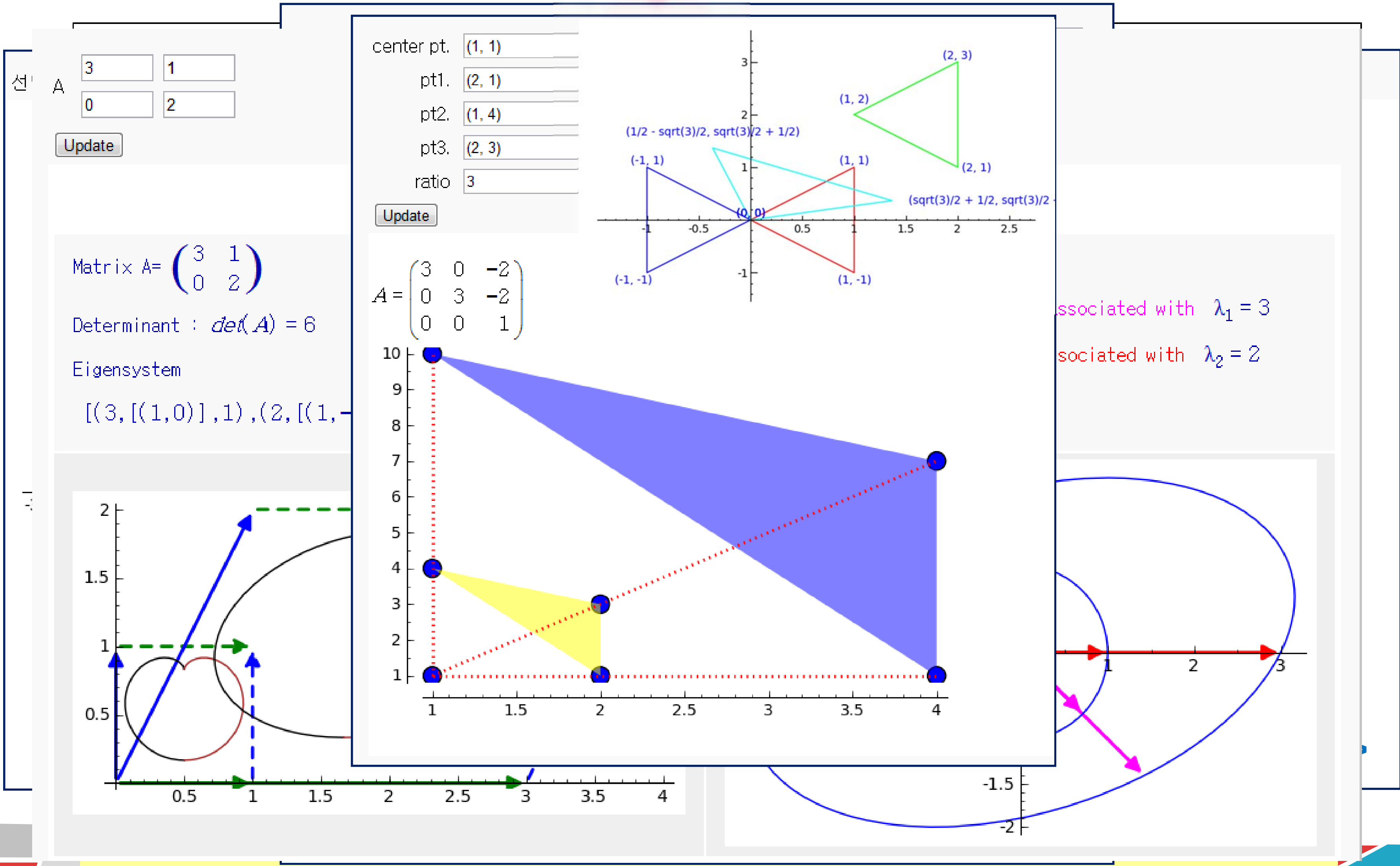
I made a preprint of Linear Algebra a year ago.

I had a classroom test over a year on New L. A. book.


Students presentation
Traditional +
Several CAS(Sage) Examples
QR Code
Sage Code
[일차결합]
예제 3
예제 4
3.1 연습문제
Blue b 벡터 생성 및 출력
12 Chapter 1 벡터

Linear Transformation

http://math1.skku.ac.kr/pub



Recorded Lecture

Our Linear Algebra Lec. in You-Tube (2012 Spring)

- SKKU Linear Algebra with Sage, 1. Preface, SGLee http://youtu.be/CbfJYPCkbm8
- SKKU Linear Algebra with Sage, 2. Introduction of CAS, SGLee http://youtu.be/0SQpiNe2LU8
- SKKU Linear Algebra with Sage, 3. Section 1-1, Vector SGLee http://youtu.be/85kGK6bJLns
- SKKU Linear Algebra with Sage, 4. Section 1-2, Norm, SGLee http://youtu.be/g55dfkmlTHE
- SKKU Linear Algebra with Sage, 5. Section 1-3, Vector Equations, SGLee http://youtu.be/YB976T1w0kE
- SKKU Linear Algebra with Sage, 6. Section 2-1, LSE, SGLee, http://youtu.be/AAUQvdjQ-qk
- SKKU Linear Algebra with Sage, 7. Section 2-2, RREF, SGLee, http://youtu.be/HSm69YigRr4
- SKKU Linear Algebra with Sage, 8. Section 2-3, Appl of LSE, SGLee, http://youtu.be/G790BLDSK5g
- SKKU Linear Algebra with Sage, 9. Section 3-1, Matrix, SGLee, http://youtu.be/JdNnHGdJBrQ
- SKKU Linear Algebra with Sage, 10. Section 3-2, Inverse Matrix, SGLee http://youtu.be/yeCUPdRx7Bk
- SKKU Linear Algebra with Sage, 11. Section 3-3, Elementary Matrix, SGLee http://youtu.be/oQ2m6SSSquc
- SKKU Linear Algebra with Sage, 12. Section 3-4, Subspace, SGLee http://youtu.be/UTTUg6JUFQM
- SKKU Linear Algebra with Sage, 13. Section 3-5, Solutions Set, SGLee http://youtu.be/OoTPCpKW_eY
- SKKU Linear Algebra with Sage, 14. Section 3-6, Special Matrices, SGLee http://youtu.be/jLh77sZOaM8
- SKKU Linear Algebra with Sage, 15. Section 3-7, LU-Factorization, SGLee http://youtu.be/lKJPnLCiAVU
- SKKU Linear Algebra with Sage, 16. Section 3-6, Theorem of Triangular matrix, SGLee http://youtu.be/UriXEI-xoRk
- SKKU Linear Algebra with Sage, 17. Section 4-1, Determinant, SGLee http://youtu.be/Vf8LlkKKHgg
- SKKU Linear Algebra with Sage, 17-2. Section 4-1, Determinant 2, SGLee http://youtu.be/_3WRlwDUU9Y
- SKKU Linear Algebra with Sage, 18. Section 4-2, Cofactor Expansion, SGLee http://youtu.be/m6l2my6pSwY
- SKKU Linear Algebra with Sage, 19. Section 4-3, Cramer's Law, SGLee http://youtu.be/m2NkOX7gE50
- SKKU Linear Algebra with Sage, 20. Section 4-4, Appl of Determinant, SGLee http://youtu.be/KtkOH5M3_Lc
- SKKU Linear Algebra with Sage, 21. Section 4-5, Eigenvalue & Eigenvector, SGLee http://youtu.be/96Brbkx1cQ4
- SKKU Linear Algebra with Sage, 22. Section 5-1, Power Method, SGLee http://youtu.be/CLxjkZuNJXw
- SKKU Linear Algebra with Sage, 23. Section 5-2, Encryption, SGLee http://youtu.be/umTIADxsEq8
- SKKU Linear Algebra with Sage, 24. Section 5-3, Blackout Game, SGLee http://youtu.be/_bS33lfa29s
- SKKU Linear Algebra with Sage, 25. Section 5-4, Markov Chains, SGLee http://youtu.be/156ezier6HQ
- SKKU Linear Algebra with Sage, 26. Section 5-5, Google Matrix, SGLee http://youtu.be/WNUoXLh8i_E
- SKKU Linear Algebra with Sage, 27. Section 5-6, Project, SGLee http://youtu.be/coNq48CW6Pg
- SKKU Linear Algebra with Sage, 28. Section 6-1, Linear Transformation, SGLee http://youtu.be/Yr23NRSpSoM
- SKKU Linear Algebra with Sage, 29. Section 6-2, Linear Operator, SGLee http://youtu.be/12WP-cb6Ymc

Student's Activities in Movies, You-Tube (2012 Spring Linear Algebra Class)

- Preview : http://matrix.skku.ac.kr/2012-Album/1-CLA-Preview.html

- SKKU Linear Algebra with Sage, S1. Section 1-1, Vector, Student http://youtu.be/fbCMyh-iDCQ
- SKKU Linear Algebra with Sage, S2. Section 1-2, Norm, Student http://youtu.be/sEFj_7t_bqc
- SKKU Linear Algebra with Sage, S3. Section 1-3, Vector Equations, Student http://youtu.be/avVJfeEoeVs
- SKKU Linear Algebra with Sage, S4. Chapter 1 Discuss, Student http://youtu.be/tys3taO5IHs
- SKKU Linear Algebra with Sage, S5. Section 2-1, LSE, Student http://youtu.be/N5ltl-bfdvkz
- SKKU Linear Algebra with Sage, S6. Section 2-2, RREF, Student http://youtu.be/sJeomjbRFmE
- SKKU Linear Algebra with Sage, S7. Section 2-3, Appl of LSE, Student http://youtu.be/vx_6rTJq5jk
- SKKU Linear Algebra with Sage, S8. Section 3-1, Matrix, Student http://youtu.be/LaAAruKbGyc
- SKKU Linear Algebra with Sage, S9. Section 3-2, Inverse Matrix, Student http://youtu.be/-MPszmMNvLE
- SKKU Linear Algebra with Sage, S10. Section 3-3, Elementary Matrix, Student http://youtu.be/ceI8oeXp6xU
- SKKU Linear Algebra with Sage, S11. Section 3-4, Subspace, Student http://youtu.be/s7jxVvVAel4
- SKKU Linear Algebra with Sage, S12. Section 3-5, Solutions Set, Student http://youtu.be/IygHFdWacds
- SKKU Linear Algebra with Sage, S13. Section 3-6, Special Matrices, Student http://youtu.be/rYBsPkeVhQo
- SKKU Linear Algebra with Sage, S14. Section 4-1, Determinant, Student http://youtu.be/Fne4gaZtE_Q
- SKKU Linear Algebra with Sage, S15. Section 4-2, Cofactor Expansion, Student http://youtu.be/nAabf3lpFU4
- SKKU Linear Algebra with Sage, S16. Section 4-3, Cramer's Law, Student http://youtu.be/Ygu4_7I4fGQ
- SKKU Linear Algebra with Sage, S17. Section 4-5, Eigenvalue & Eigenvector, Student http://youtu.be/WEyopW5iH6A
- SKKU Linear Algebra with Sage, S18. Section 6-1, Linear Transformation, Student http://youtu.be/dNt6NUDwc-Q
- SKKU Linear Algebra with Sage, S19. Section 6-2, Linear Operator, Student http://youtu.be/K241IVtU804

- . . .

In progress

What was Developed 1 !

Sage (embeded)

- http://math1.skku.ac.kr/home/pub/534 : SKKU-LA-1.1-(Vector)
- http://math1.skku.ac.kr/home/pub/535 : SKKU-LA-1.1-(Vector)
- http://math1.skku.ac.kr/home/pub/583 : LA-2.1-(Linear Equations)
- http://math1.skku.ac.kr/home/matrix/348 : **SKKU-LA-3.2-(inverse)**
- http://math1.skku.ac.kr/home/pub/730/ : **LA-4.4- Volume (Det)**
- http://math1.skku.ac.kr/home/pub/597 : **LA-4.4-Curve Fitting(Det)**
- http://math1.skku.ac.kr/home/pub/540 : LA-4.4-Equations(Det)
- http://math1.skku.ac.kr/home/pub/732 : LA-4.5-(Eigensystem)
- http://math1.skku.ac.kr/home/pub/733 : **SKKU-LA-5.2-Crypto**
- http://math1.skku.ac.kr/home/pub/734 : LA-5.2-Math Modeling

* http://math1.skku.ac.kr/home/pub/639
  : SKKU-LA-6.2- Flag **(Rotation)**
* http://math1.skku.ac.kr/home/pub/598
  : SKKU-LA-6.2- (Linear Transformation)
* http://math1.skku.ac.kr/home/pub/596
  : SKKU-LA-6.2- **(Linear Transformation)**
* http://math1.skku.ac.kr/home/pub/582
  : SKKU-LA-6.2- (Linear Transformation)
* http://math1.skku.ac.kr/home/pub/580
  : SKKU-LA-6.5- **(Dilation)**
* http://math1.skku.ac.kr/home/pub/729/
  : SKKU-LA-6.5-Tri (Dilation)
* http://math1.skku.ac.kr/home/pub/708
  : SKKU-LA-6.5- (Linear Transformation)
* http://math1.skku.ac.kr/home/pub/539
  : SKKU-LA-7.5- **(Projection)**

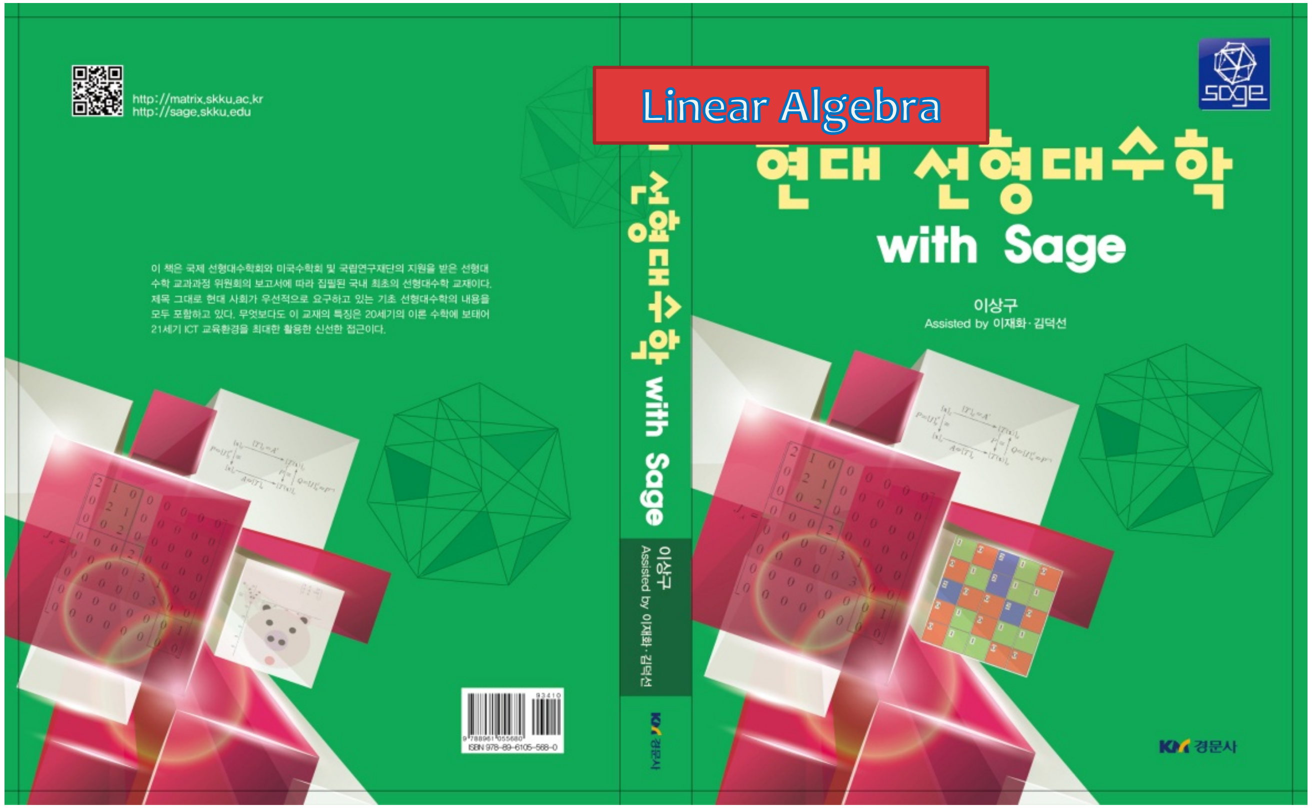
What was Developed 2 !

 # Sage with Geogebra

- Vector Sum : http://math1.skku.ac.kr/home/pub/534
- Scalar Multiplication : http://math1.skku.ac.kr/home/pub/535
- Linear System of Equations : http://math1.skku.ac.kr/home/pub/583
- Matrix Multiplication : http://math1.skku.ac.kr/home/pub/769
- Inverse Matrix : http://math1.skku.ac.kr/home/pub/731
- Area of Triangle : http://math1.skku.ac.kr/home/pub/730
- **Line(Plane) Equation** : http://math1.skku.ac.kr/home/pub/540
- **Curve Fitting** : http://math1.skku.ac.kr/home/pub/597
- Eigenvalue and Eigenvectors : http://math1.skku.ac.kr/home/pub/732
- Matrix Cryption : http://math1.skku.ac.kr/home/pub/733
- Light Out Game : http://matrix.skku.ac.kr/blackwhite2/blackwhite.html
- Rotation : http://math1.skku.ac.kr/home/pub/639
- Reflection and Projection : http://math1.skku.ac.kr/home/pub/598
  http://math1.skku.ac.kr/home/pub/582
- Shear : http://math1.skku.ac.kr/home/pub/596
- Dilation : http://math1.skku.ac.kr/home/pub/580
- Quadratic Form : http://math1.skku.ac.kr/home/pub/751
- Fourier Series : http://math1.skku.ac.kr/home/pub/752
- Jordan Blocks : http://math1.skku.ac.kr/home/pub/773

New LA textbook

Contemporary Linear Algebra with Sage



Published in September 1^{st}!

Linear algebra with Sage

E-book (Model 1)

10.1 점도표를 이용한 Jordan 표준형 구하기

YouTube 동영상 강의: http://youtu.be/NBLZPcWRHYI

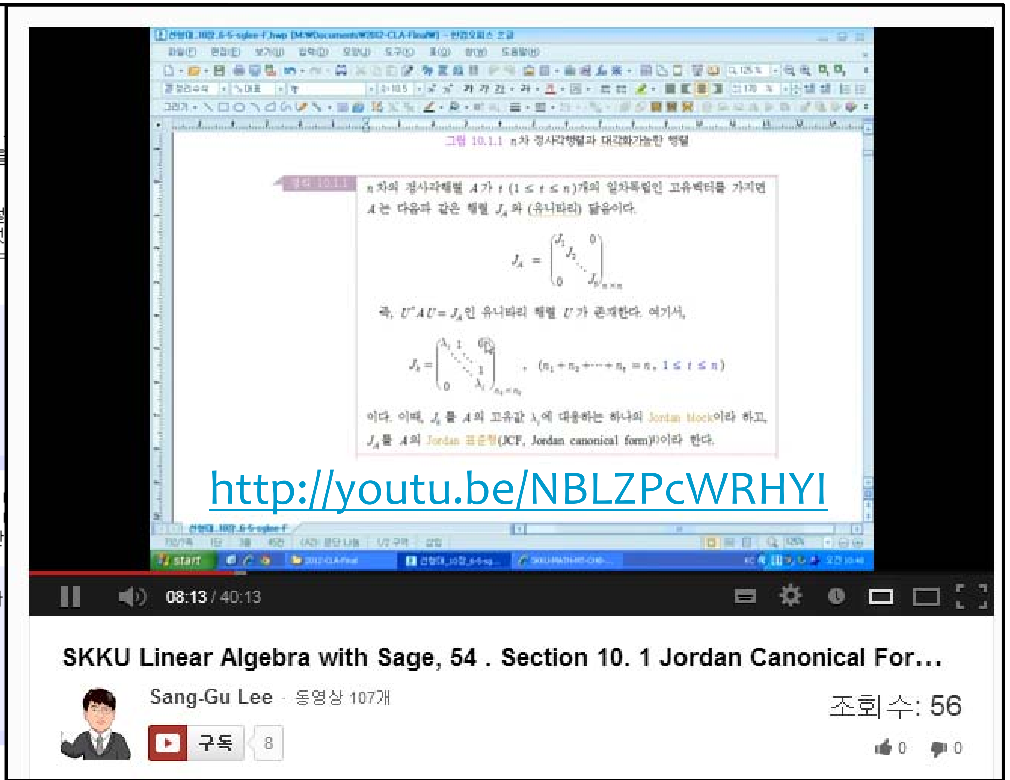
주어진 행렬이 대각화가능하다면 이 행렬과 관계된 대부분의 문제는 쉽게 다루어서 원하는 결론을 얻
행렬이 대각화가능한 것은 아니다. 이 절에서는 주어진 행렬과 닮음인 대각선 행렬과 거의 유사한 행
을 구하는 방법을 소개한다.

우리는 n차의 정사각행렬 A가 n개의 일차독립인 고유벡터들을 가지면 대각화가능하다는 것을 8.2절
8.8.5]에서 보았듯이 A가 유니타리 대각화가능일 필요충분조건은 A가 정규행렬(normal matrix)인 것
의 정규직교인 고유벡터를 갖고, 이 고유벡터들을 열로 갖는 행렬 U는 유니타리 행렬이며 $U^*AU =$
각선 성분으로 갖는 대각행렬이다.

> 행렬 $A \in M_n(C)$에 대하여 다음은 동치이다.
> (1) A는 유니타리 대각화가능하다.
> (2) A는 정규행렬이다.
> (3) A는 n개의 정규직교인 고유벡터를 갖는다.
>
> thm8-8-5.knowl

대각화가능한 행렬을 다루는 것은 이론적으로나 실제에 있어서 모두 대각행렬을 다루는 것과 같이 쉽
n차의 정사각행렬이 모두 n개의 일차독립인 고유벡터들을 갖지는 않으므로 대각화가능한 것은 아니
리]에 의하여 모든 행렬은 자신의 고유값을 대각선성분으로 갖는 상삼각행렬과 유니타리 닮음은 안

> 임의의 n차 정사각행렬은 자신의 고유값을 대각선성분으로 갖는 상삼각행렬과 유니타리 닮음이다
>
> 즉, $U^*AU = \begin{bmatrix} \lambda_1 & & * \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix}$.
>
> thm-schur.knowl



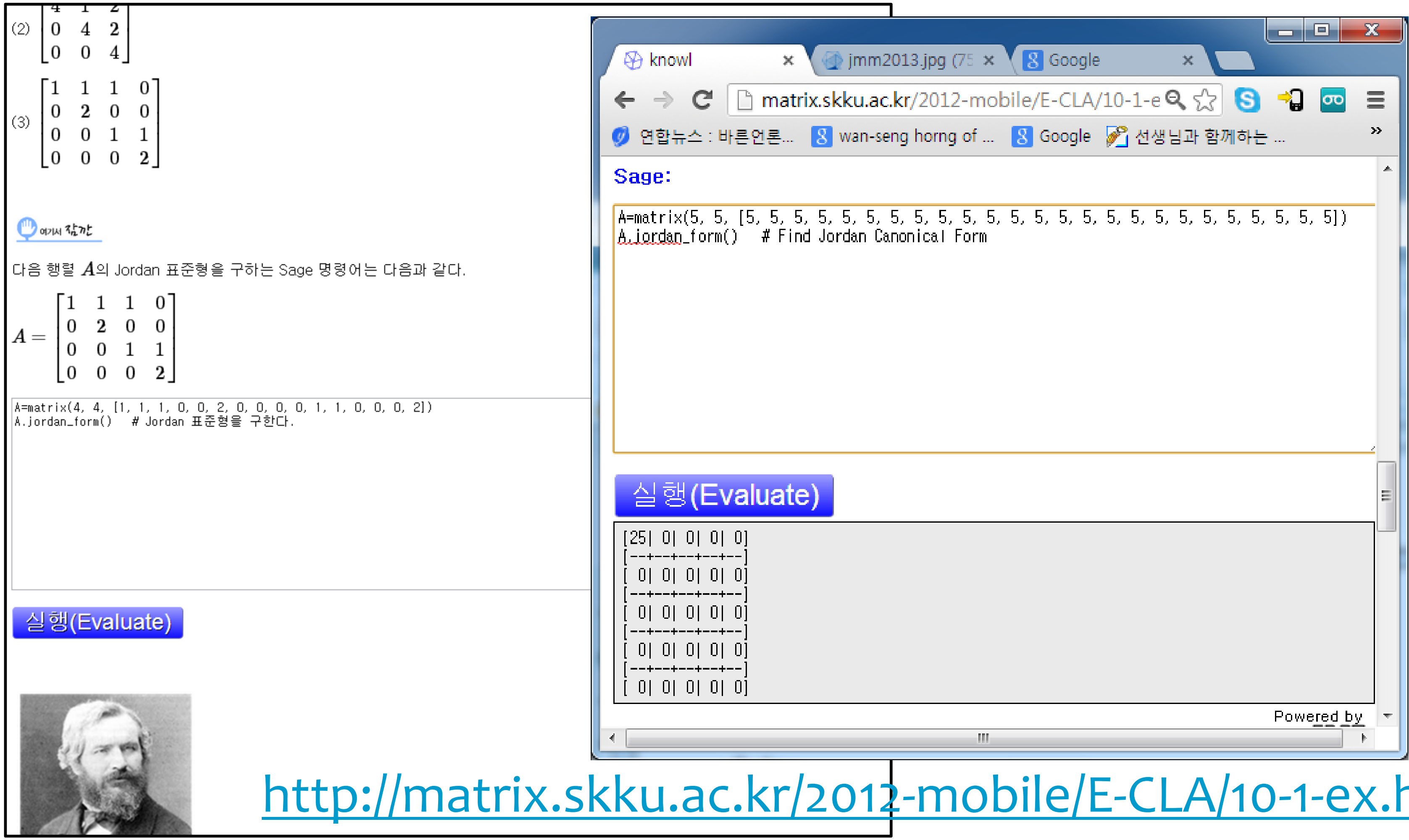
우선 대각화와 관련하여 지금까지 살펴본 내용을 요약하면 아래와 같다.

1. n차 정사각행렬 A가 대각화가능할 필요충분조건은 A가 n개의 일차독립인 고유벡터를 갖는다.
2. A가 정규행렬일 $(AA^* = A^*A)$ 필요충분조건은 A가 유니타리 대각화가능이다.
3. 그러나 정규행렬이 아니면서도 대각화가능한 행렬은 존재한다.
4. 행렬 A가 대각화가능이면, 각각의 고유값에 대한 고유공간 $\text{null}(\lambda I - A)$의 차원(기하적 중복도)이 그 고유값의 (대수적) 중복도와 같아야 한다. ($\Leftrightarrow$ n개의 일차독립인 고유벡터가 존재한다.)

http://matrix.skku.ac.kr/2012-mobile/e-cla/10-1.html

Linear algebra with Sage

E-book (Model 2)



http://matrix.skku.ac.kr/2012-mobile/E-CLA/10-1-ex.html

Linear algebra with Sage

E-book (Model 3)

10.1 연습문제

YouTube 문제풀이 동영상:

- http://youtu.be/9-G3Fd2xOW0
- http://www.youtube.com/watch?v=adWzUKKmO2k

1. 5차 정사각행렬 A가 중복도가 5인 고유값 λ만을 갖고, λ에 대응하는 일차독립 ... 형의 종류를 구하여라.

2. 다음 Jordan 표준형 J_A에 대하여 다음을 구하여라.

$$J_A = \begin{bmatrix} \lambda & 1 & 0 & 0 & 0 \\ 0 & \lambda & 1 & 0 & 0 \\ 0 & 0 & \lambda & 1 & 0 \\ 0 & 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & 0 & \lambda \end{bmatrix}$$

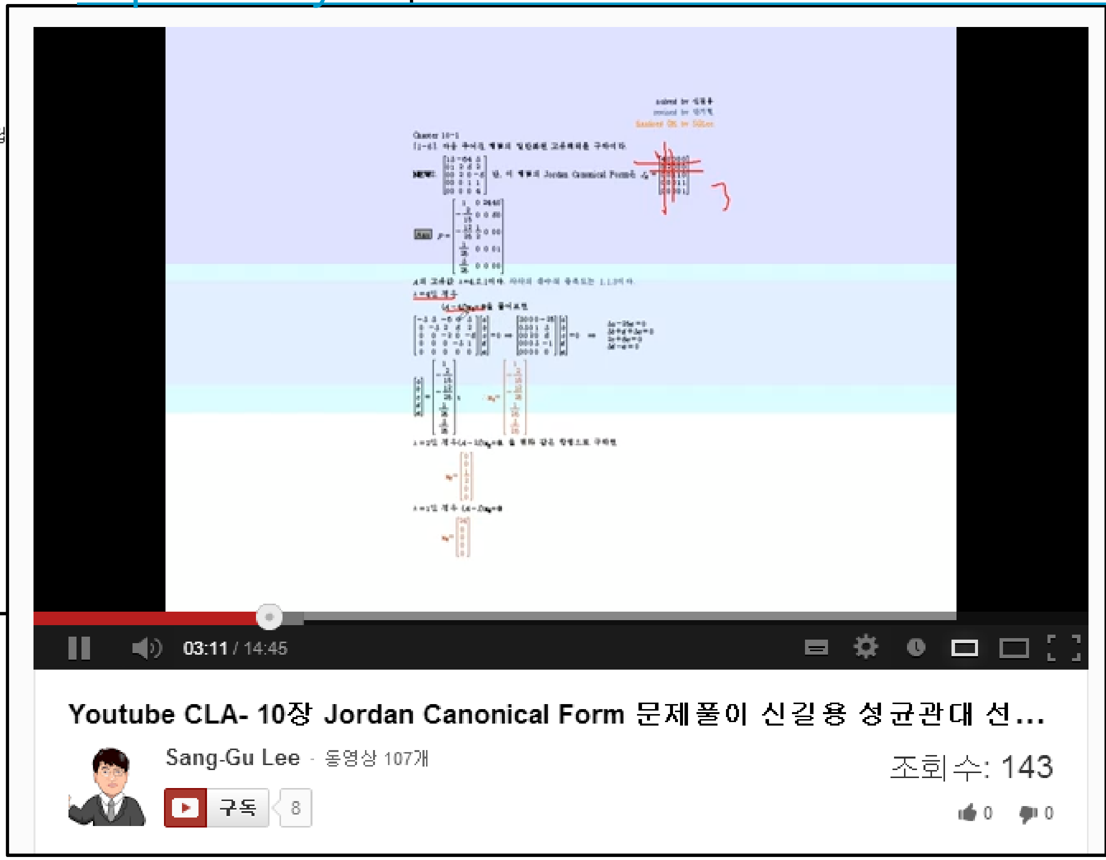
(1) $J_A - \lambda I$

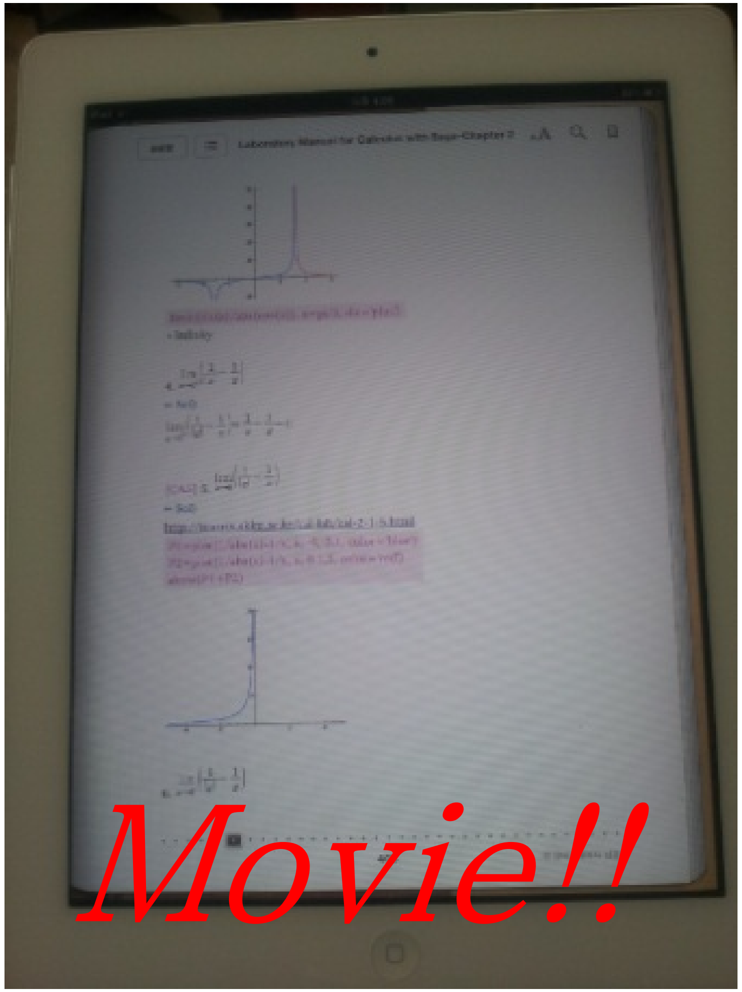
(2) $(J_A - \lambda I)^2$

(3) $(J_A - \lambda I)^3$

(4) $(J_A - \lambda I)^4$

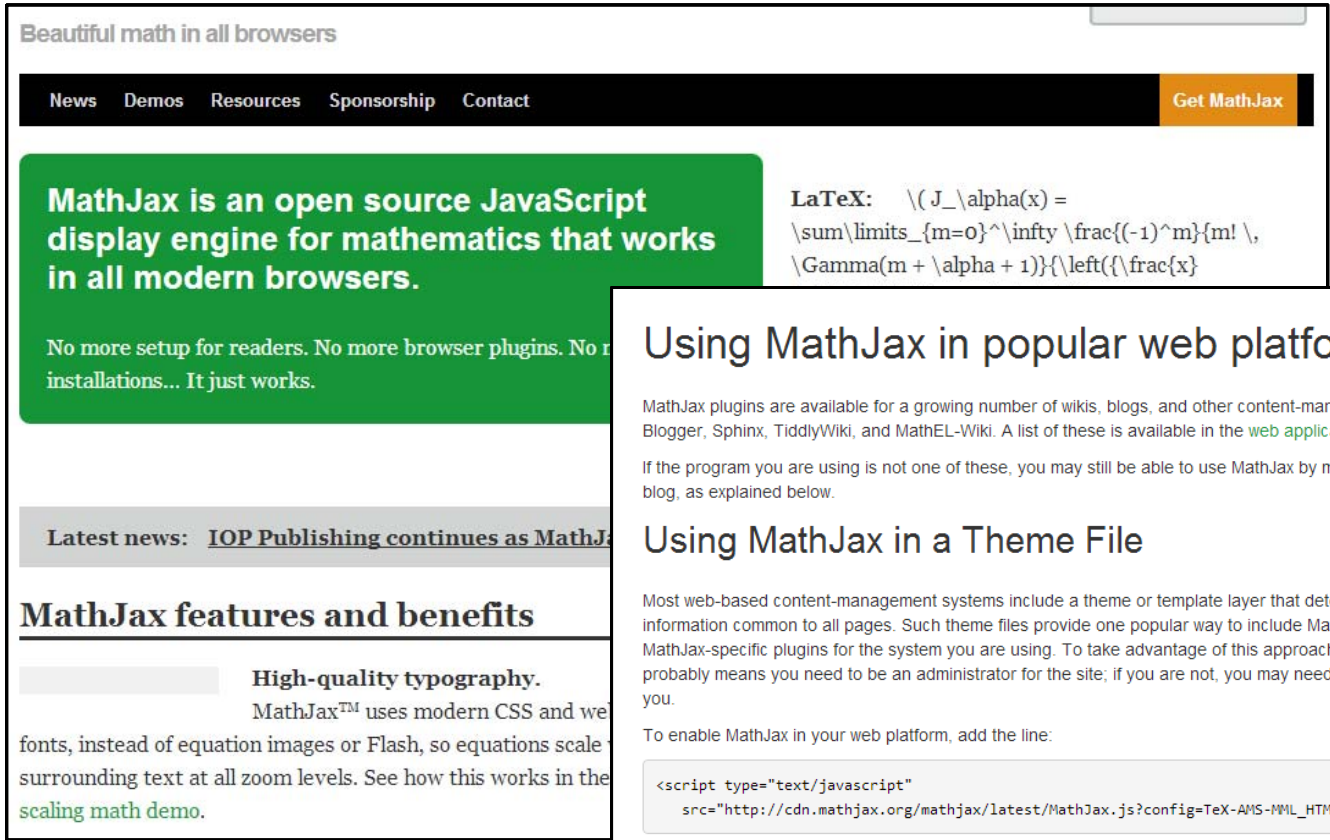
- http://youtu.be/9-G3Fd2xOW0
- http://www.youtube.com/watch?v=adWzUKKmO2k

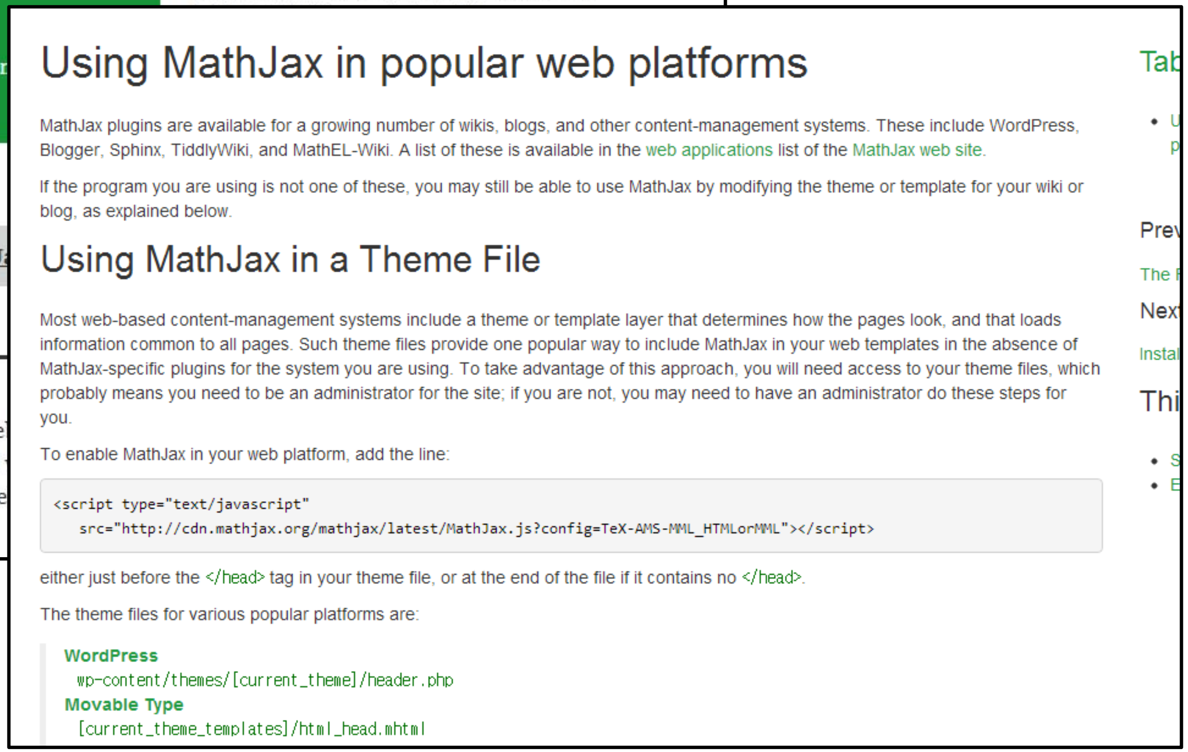




- Movie e-book : Calculus Lab book
- Movie e-book : Linear Algebra (knowls)

MathJax


Beautiful math in all browsers
News
Demos
Resources
Sponsorship
Contact
Get MathJax
MathJax is an open source JavaScript display engine for mathematics that works in all modern browsers.
No more setup for readers. No more browser plugins. No r
installations... It just works.
LaTeX: \(J_\alpha(x) = \sum\limits_{m=0}^\infty \frac{(-1)^m}{m! \, \Gamma(m + \alpha + 1)}{\left({\frac{x}
Latest news: IOP Publishing continues as MathJ
MathJax features and benefits
High-quality typography.
MathJax™ uses modern CSS and we
fonts, instead of equation images or Flash, so equations scale
surrounding text at all zoom levels. See how this works in the
scaling math demo.


Using MathJax in popular web platforms
MathJax plugins are available for a growing number of wikis, blogs, and other content-management systems. These include WordPress, Blogger, Sphinx, TiddlyWiki, and MathEL-Wiki. A list of these is available in the web applications list of the MathJax web site.
If the program you are using is not one of these, you may still be able to use MathJax by modifying the theme or template for your wiki or blog, as explained below.
Using MathJax in a Theme File
Most web-based content-management systems include a theme or template layer that determines how the pages look, and that loads information common to all pages. Such theme files provide one popular way to include MathJax in your web templates in the absence of MathJax-specific plugins for the system you are using. To take advantage of this approach, you will need access to your theme files, which probably means you need to be an administrator for the site; if you are not, you may need to have an administrator do these steps for you.
To enable MathJax in your web platform, add the line:
<script type="text/javascript"
src="http://cdn.mathjax.org/mathjax/latest/MathJax.js?config=TeX-AMS-MML_HTMLorMML"></script>
either just before the </head> tag in your theme file, or at the end of the file if it contains no </head>.
The theme files for various popular platforms are:
WordPress
wp-content/themes/[current_theme]/header.php
Movable Type
[current_theme_templates]/html_head.mhtml
Tab
Prev
The
Next
Insta
Thi

knowl

www.aimath.org/knowlepedia/

Introducing the knowl: better browsing on the Web

The American Institute of Mathematics (AIM) is pleased to introduce a new tool to the world of web pages—the knowl. L
referenced from within the body of a web page. But unlike the hyperlink, which simply takes you to a new web page, the
With a similar click, the knowl then disappears.

"The self-contained nature of knowls makes them useful in many different settings. They can be thought of as 'building
knowls, reliable and ready to be referenced anywhere." says David Farmer, Director of Programs at AIM.

Knowls were developed to support research projects in advanced mathematics, but they can be easily incorporated to
old, and the theoretical basis, known as transclusion, is more than 50 years old. Our contribution is to provide an interf

In addition to the examples in the paragraphs above, AIM has prepared a demon

The mathematics research projects which led to the development of knowls were

How to put knowls on your own website

The process is very similar to the way people currently make web pages and hyp

Step 1: Add the following three lines between the <head> and </head> tags at

```
<script type="text/javascript" src="http://code.jquery.com/jquery-latest.min.j
<link href="http://aimath.org/knowlstyle.css" rel="stylesheet" type="text/css"
<script type="text/javascript" src="http://aimath.org/knowl.js"></script>
```

www.aimath.org/knowlepedia/

Introducing the knowl: better browsing on the Web

The American Institute of Mathematics (AIM) is pleased to introduce a new tool to the world of web pages—the knowl. L
referenced from within the body of a web page. But unlike the hyperlink, which simply takes you to a new web page, the
With a similar click, the knowl then disappears.

> A Knowl is a piece of context-independent information. It is independent of the environment from which it was called,
> another knowl, forming a document which conveys information in a non-linear fashion.
>
> Knowls were created by Harald Schilly, with the first working version coming online on August 25, 2011. Schilly also co
> Sciences at the National Science Foundation.
>
> "Knowl" is pronounced *nohl*, as in "A knoll is a small hill."
>
> knowl.definition

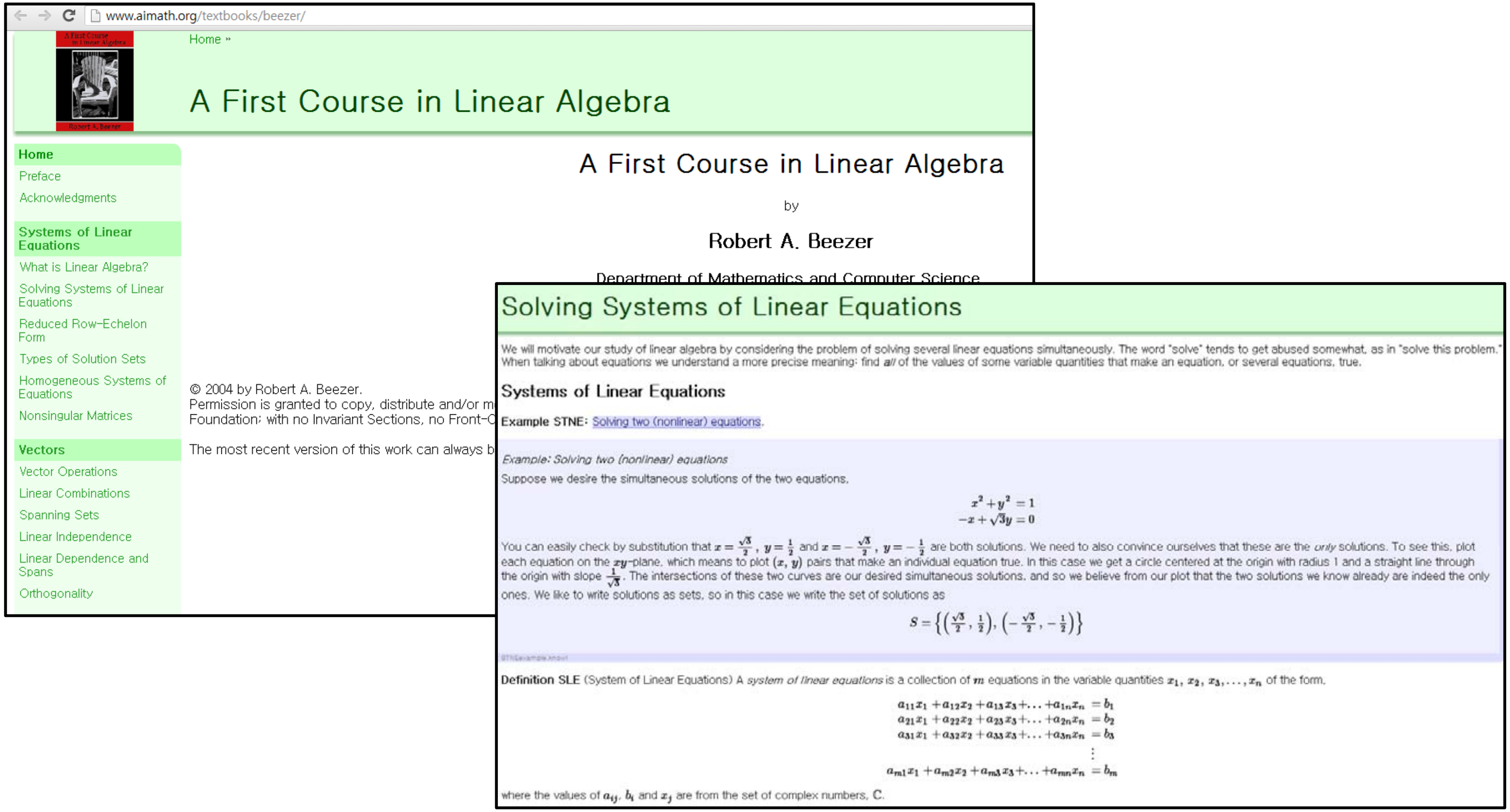
"The self-contained nature of knowls makes them useful in many different settings. They can be thought of as 'building
knowls, reliable and ready to be referenced anywhere." says David Farmer, Director of Programs at AIM.

Knowls were developed to support research projects in advanced mathematics, but they can be easily incorporated to
old, and the theoretical basis, known as transclusion, is more than 50 years old. Our contribution is to provide an interf

In addition to the examples in the paragraphs above, AIM has prepared a demonstration page with knowls for a section

The mathematics research projects which led to the development of knowls were funded by the Division of Mathematica

A First Course in Linear Algebra
written by Robert A. Beezer



References

•Duk-Sun Kim, Sang-Gu Lee*, Greg Markowsky, *Mobile Sage-Math for Linear Algebra and its Application*, Electronic Journal of Mathematics & Technology (eJMT) V.4, No. 3, p.1-13, (2010)

• Sang-Gu Lee, *Contemporary Linear Algebra (3rd Ed.)*, Kyungmoon Pub. (2009)

• Sang-Gu Lee, Duk-Sun Kim, *MathML and JAVA Implementation in Linear Algebra*, The Electronic Journal of Mathematics and Technology, Volume 3, Number 1, 1-11, ISSN 1933-2823 (2009)

• Sang-Gu Lee, Rae-Young Ko, Duk-Sun Kim, Jin-Yeong Bak, *Development of Mobile Sage-math and its use in Linear Algebra*, J. Korea Soc. Math. Ed. Ser. E: Communications of Mathematical Education, Vol. 23, No. 4, 483-506 (2009)

• Teresa Franklin and Li-Wei Peng, *Mobile math: math educators and students engage in mobile learning*, J. Comput. High. Educ. 20. (2008)

Conclusion

We believe you now have an affordable and inspiring CAS tool for Linear Algebra, and Calculus.

In this talk, we introduced how and what we have done on E-books with Sage on Linear Algebra and Calculus.

Now our students can talk more on Mathematics and concentrate on Mathematical concepts, and use very affordable CAS tool without spending time to learn and typing programing language.

http://icm2014.org/

SEOUL ICM 2014
INTERNATIONAL CONGRESS OF MATHEMATICIANS

JMM JOINT MATHEMATICS MEETINGS
AMERICAN MATHEMATICAL SOCIETY
MATHEMATICAL ASSOCIATION OF AMERICA

2013 JOINT MATHEMATICS MEETINGS
Largest Mathematics Meeting in the World
JANUARY 9-12 (WED-SAT), 2013 | SAN DIEGO CONVENTION CENTER, SAN DIEGO, CA