Sage E-books on Linear Algebra and Calculus

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Abstract

- Calculus and Linear Algebra Books.
- ‘E-books’ of Calculus and Linear Algebra.

- What’s new in it?
  - My Answer: <Movie clip and CAS in it>
    - ‘Sage’ (http://sagemath.org)
Laboratory Manual for Calculus with Sage

3.2 Derivatives of Polynomials, Exponential Functions, Trigonometric Functions, The Product Rule

In this section, we consider the derivatives of the basic functions, for example, polynomials, exponential functions, and trigonometric functions. The product rule and quotient rule are helpful to derive many derivatives of the functions which are expressed as the product and quotient. We also consider some applications of the derivative to other sciences.

We introduce several differential rules that allow us to find the derivatives without the direct use of the limit definition.

**Theorem 1** Derivative of a constant function

If $f$ has the constant function, then

$$\frac{df}{dx} = \frac{d}{dx}(c) = 0$$

[Quick example 1] If $f$ has the constant value $f(x) = 20$, then

$$\frac{df}{dx}(20) = 0$$

Similarly,

$$\frac{d}{dx}(√x) = 0$$

and

$$\frac{d}{dx}(e) = 0$$

Note that the Constant Rule is equivalent to saying that the slope of a horizontal line is zero.

The following rule tells how to differentiate $e^x$ if $n$ is a positive integer.

To prove the following rule we have to use the binomial expansion.

**Theorem 2** The Power Rule for Positive Integer

If $n$ is a positive integer, then

$$\frac{d}{dx}(e^n) = n e^{n-1}$$

melted rock cools by conduction of heat into surrounding rocks, and an
engineer wants to know the rate at which water flows into or out of a
reservoir.

**Example 2**

Find an equation of the tangent line to the curve at the given point.

$$\begin{align*}
\frac{dx}{dt} &= 3t^2 \\
\frac{dy}{dt} &= 6t
\end{align*}$$

Solution

$$\begin{align*}
\frac{dx}{dt} &= 3t^2 \\
\frac{dy}{dt} &= 6t
\end{align*}$$

Hence, the slope of the tangent line is $20$. To find the equation of the tangent line in $y = 20$.

$$y - 20 = 10(x - 20)$$

**Exercise 3.2**

1. Find the equation of the tangent line to the curve $y = x^2$ at $(4, 2)$.

2. Find the equation of the tangent line to the curve $y = 1/x$ at $(1, 1)$.

3. Find the equation of the tangent line to the curve $y = x^3$ at $(1, 1)$.

4. Find the equation of the tangent line to the curve $y = x^2$ at $(2, 4)$.
Laboratory Manual for Calculus with Sage

1. Introduction and Use of Sage-Math

Mathematical tools have long held an important place in classrooms. With the innovation of information and communication technologies, many tools have appeared and been adopted for educational purposes. Sage-Math is a popular mathematical software which was released in 2005. This software has efficient features which utilize the internet and can handle most mathematical problems, including linear algebra, algebra, combinatorics, numerical mathematics and calculus. In this book, we will introduce this powerful software and discuss how it can be used in classes.

Sage-Math is a mathematical CAS tool and is based on Internet Web environment. This tool was introduced in April 2001 at University of Washington, USA. It is free and has a powerful capability that can be compared with expensive commercial softwares such as Mathematica, Maple or Mathlab, but can do more than that without requiring separate installations of the program. It is more like Web-Mathematica, but has some better features. When you connect to any Web browser, you can easily solve almost all calculus problems in the book by using resources that we are offering. You can easily find pre-existing commands to modify for your own problems.

Figure 1. We can use other existing codes as well in Sage.

Korean Sage-Math model was developed and relevant experiments were done by BK 21 Mathematical Modelling HSED division at KangwonNational University. We have built Korean servers that you can use as you see below. (Instructions will be given in your first class)

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8. (a) Determine the approximations and for \( T_n \) and \( S_n \) for \( \int e^x \, dx \) and the corresponding errors \( E_T \) and \( E_S \).
(b) Compare the actual errors in part (a) with the error estimates given by 15 and 14.
(c) Determine how large do we have to choose so that the approximations \( T_n, M_n \) and \( S_n \) to the integral in part (a) are accurate to within 0.000017.

- \( T_8 = 1.200506 \), \( S_8 = 1.271109 \), \( E_T = 0.000475 \), \( E_S = 0.000126 \)
- Since \( K = 8 \approx 7.8 \), the integral is in part (a) given \( E_T \leq 0.311111 \cdot 0.001 \leq 0.000656 \)
- \( E_T = \frac{1}{8} \cdot 0.000475 \approx 0.000059 \)
- For \( T_n \) find \( \eta \) so that \( \int_{x=0}^{x=1} e^x \, dx \cdot e^{-0.000001} \), \( \eta = 0.07 \)
- For \( M_n \) find \( \eta \) so that \( \int_{x=0}^{x=1} e^x \, dx \cdot e^{-0.000001} \), \( \eta = 0.04 \)
- For \( S_n \) find \( \eta \) so that \( \int_{x=0}^{x=1} e^x \, dx \cdot e^{-0.000001} \), \( \eta = 0.13 \)

9. Given the function \( f \) at the following values,

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>1.08</td>
<td>1.09</td>
<td>1.10</td>
<td>1.11</td>
<td>1.12</td>
<td>1.13</td>
</tr>
</tbody>
</table>

approximate \( \int_{x=0}^{x=1} f(x) \, dx \) using Simpson’s Rule.

- \( \int_{x=0}^{x=1} f(x) \, dx \approx 0.5316 \)

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7.9 Improper Integrals

CAS 1. \( \int_0^{\infty} \frac{1}{x^2} \, dx \)

- \( \int_0^{\infty} \frac{1}{x^2} \, dx = \lim_{t \to \infty} \int_0^{t} \frac{1}{x^2} \, dx = \lim_{t \to \infty} [-\frac{1}{x}]_0^t = 1 \)

CAS 2. \( \int_{-\infty}^{\infty} e^{-x^2} \, dx \)

- \( \int_{-\infty}^{\infty} e^{-x^2} \, dx = 2 \int_{0}^{\infty} e^{-x^2} \, dx = \sqrt{\pi} \)

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S. 1. \( \int_0^{\infty} \frac{1}{x^2} \, dx \)

- \( \int_0^{\infty} \frac{1}{x^2} \, dx = \lim_{t \to \infty} \int_0^{t} \frac{1}{x^2} \, dx = \lim_{t \to \infty} [-\frac{1}{x}]_0^t = 1 \)

CAS 2. \( \int_{-\infty}^{\infty} e^{-x^2} \, dx \)

- \( \int_{-\infty}^{\infty} e^{-x^2} \, dx = 2 \int_{0}^{\infty} e^{-x^2} \, dx = \sqrt{\pi} \)
Calculus with SKKU Sage Cell Server

http://sage.skku.edu

7.2 Trigonometric Integrals

CAS 1. \( \int \cos^3 x \, dx \)

CAS 2. \( \int \sin^3 x \cos^4 x \, dx \)

CAS 3. \( \int \sin^2(4x) \, dx \)

CAS 4. \( \int \sin^4 x \cos^{-3} x \, dx \)
What’s more?

CAS Solutions
Session 1
Laboratory Manual for Calculus with Sage
There are each Chapters 1 ~ 14 of the e-book for

Lab. Manual for Calculus with Sage
Preface

Calculus is the mathematical foundation for much of university mathematics, science, and engineering curriculum. For the mathematics student, it is a first exposure to rigorous mathematics. For the engineer, it is an introduction to the modeling and approximation techniques used throughout the engineering curriculum. And for future scientists, it is the mathematical language that will be used to express many of the most important scientific concepts.

In the first semester, that's for the beginners of calculus, we start with differential and integral calculus on functions of single variable and then study limits of functions, continuity, derivative, Taylor series, optimization problems, and ordinary differential equations as applications of differential and integral calculus accordingly. In the second semester of calculus, we cover vector calculus that includes parameter equations, polar coordinates, infinite series and integral vector, vectors and coordinate which are partial derivatives.

Modeling and approximation in calculus should resemble the techniques and methods currently in use. Concepts, definitions, terminology, and interpretation in calculus should be as current as possible. This book has many problems to present calculus as the foundation of modern mathematics, science and engineering.

This book is a Lab Manual for Calculus with SageMath. Most of recent calculus textbooks are using Computer Algebra Systems (CAS) including a variety of typical tools in 3. But the use was limited to students in few of cases. Therefore, in this book, we adopted a wonderful open-source program, SageMath, for our students. With the new learning environment of calculators, students will take a full advantage of 21C state of art technology to learn calculus more easily and get better prepared for future job market. We can use the SageMath well on popular with browsers such as Firefox or Chrome.

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Figure 1. We can use code existing codes as well as Sage.
Contents for

Part I Single Variable Calculus

Part II Multivariate Calculus
In this E-book of Lab. Manual for Calculus with Sage

[11.7] Cylindrical Coordinates and Spherical Coordinates

1-4. Find the point whose cylindrical coordinates are given. Then find the rectangular coordinates of the point.

CAS: \( \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right] \)

\( \Rightarrow (0, 0, 0) \)

\( T = \text{CylindricalToRectangular} \left( \text{Height, \ radius, \ azimuth} \right) \)

\( T \text{.toJSON} \left( \right) \)

\( \text{result} \rightarrow \left( \right) \)

2. \( \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right] \)

\( \Rightarrow \left( \right) \)

3. \( \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right] \)

\( \Rightarrow \left( \right) \)

4. \( \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right] \)

\( \Rightarrow \left( \right) \)

3-4. Change from rectangular to cylindrical coordinates.

5. \( \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right] \)

\( \Rightarrow \left( \right) \)

Click Web Address

Using Sage Cell Server

Multi-tasking Process

We can Search its meanings in Wikipedia
Note

We can add our notes in it.
So by the Intermediate Value Theorem, there is a number \( c \) in its domain such that \( f(c) = 0 \).


World Mathematicians Cards

♣ European
◆ Asian
♣ USA and UK (Modern Mathematicians)
♥ Korean
Session 2  Linear Algebra

I made a preprint of Linear Algebra a year ago.
I had a classroom test over a year on **New L. A. book.**
(3) $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$이다 하자. $\begin{bmatrix} -1 & 0 \\ -1 & 1 \end{bmatrix} x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$이므로 $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$이다.

예제 3

 컴퓨터 모니터의 색은 RGB 색 모델이라 불리는 시스템을 이용한다. 이는 붉은 (red), 녹색 (green), 파란색 (blue)을 벡터의 합으로 나타내는 것이다. 여기서 각 벡터는 0과 1 사이의 값으로, 각 벡터는 0이나 1에 가깝다.

(1, 0, 0)은 각각 순수한 red, green, blue의 색이다. 0과 1 사이의 계수를 이용한 r, g, b이다.

QR Code

Students presentation

Traditional +

Several CAS(Sage) Examples

Sage Code

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} v = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$을 나타내는 벡터 $v$를 상속.

# Blue 벡터 생성 및 출력

```
# Blue 벡터 생성 및 출력

h = (1, 0, 0) + c1 (0, 1, 0) + c2 (0, 0, 1)
```
Matrix $A = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$
Determinant: $\det(A) = 6$
Eigenvalues
$\lambda_1 = 3, \lambda_2 = 2$

Linear Transformation
http://math1.skku.ac.kr/pub
Recorded Lecture
Our Linear Algebra Lec. in You-Tube (2012 Spring)

• SKKU Linear Algebra with Sage, 1. Preface, SGLee  http://youtu.be/CbfJPckkm8
• SKKU Linear Algebra with Sage, 2. Introduction of CAS, SGLee  http://youtu.be/oSQpiNezLU8
• SKKU Linear Algebra with Sage, 3. Section 1-1, Vector SGLee  http://youtu.be/8skGK6bJLns
• SKKU Linear Algebra with Sage, 4. Section 1-2, Norm, SGLee  http://youtu.be/gs5dfkmITHE
• SKKU Linear Algebra with Sage, 5. Section 1-3, Vector Equations, SGLee  http://youtu.be/YB976T1wokE
• SKKU Linear Algebra with Sage, 7. Section 2-2, RREF, SGLee,  http://youtu.be/HSm6yYigRr4
• SKKU Linear Algebra with Sage, 8. Section 2-3, Appl of LSE, SGLee,  http://youtu.be/G7s0BDlaK5g
• SKKU Linear Algebra with Sage, 9. Section 3-1, Matrix, SGLee,  http://youtu.be/JdNnHGDjBrQ
• SKKU Linear Algebra with Sage, 10. Section 3-2, Inverse Matrix, SGLee  http://youtu.be/yeCUPdRx7Bk
• SKKU Linear Algebra with Sage, 11. Section 3-3, Elementary Matrix, SGLee  http://youtu.be/oQzm6SSqsuc
• SKKU Linear Algebra with Sage, 12. Section 3-4, Subspace, SGLee  http://youtu.be/UTTUg6JUFQM
• SKKU Linear Algebra with Sage, 13. Section 3-5, Solutions Set, SGLee  http://youtu.be/OoTPpKw_eY
• SKKU Linear Algebra with Sage, 14. Section 3-6, Special Matrices, SGLee  http://youtu.be/JLh7GsZOaM8
• SKKU Linear Algebra with Sage, 15. Section 3-7, LU-Factorization, SGLee  http://youtu.be/IKJpNCiAYU
• SKKU Linear Algebra with Sage, 16. Section 3-6, Theorem of Triangular matrix, SGLee  http://youtu.be/UrIEJ-x0Rk
• SKKU Linear Algebra with Sage, 17. Section 4-1, Determinant, SGLee  http://youtu.be/Vf8LlkKKhgg
• SKKU Linear Algebra with Sage, 17. Section 4-1, Determinant 2, SGLee  http://youtu.be/_3WRlwDUuqY
• SKKU Linear Algebra with Sage, 18. Section 4-2, Cofactor Expansion, SGLee  http://youtu.be/m6l2my6pSwY
• SKKU Linear Algebra with Sage, 19. Section 4-3, Cramer's Law, SGLee  http://youtu.be/m2NkOX7EGs0
• SKKU Linear Algebra with Sage, 20. Section 4-4, Appl of Determinant, SGLee  http://youtu.be/KttOHsM3_Lc
• SKKU Linear Algebra with Sage, 21. Section 4-5, Eigenvalue & Eigenvector, SGLee  http://youtu.be/96Brbkx1cQ4
• SKKU Linear Algebra with Sage, 22. Section 5-1, Power Method, SGLee  http://youtu.be/CLxjkZuNjXw
• SKKU Linear Algebra with Sage, 23. Section 5-2, Encryption, SGLee  http://youtu.be/umTIAxU3Eq8
• SKKU Linear Algebra with Sage, 24. Section 5-3, Blackout Game, SGLee  http://youtu.be/_b5531fa29s
• SKKU Linear Algebra with Sage, 25. Section 5-4, Markov Chains, SGLee  http://youtu.be/1sS6ezeier6HQ
• SKKU Linear Algebra with Sage, 26. Section 5-5, Google Matrix, SGLee  http://youtu.be/WNUqXhl8i_E
• SKKU Linear Algebra with Sage, 27. Section 5-6, Project, SGLee  http://youtu.be/coNq48CW6Pg
• SKKU Linear Algebra with Sage, 28. Section 6-1, Linear Transformation, SGLee  http://youtu.be/Yr23NRSpSoM
• SKKU Linear Algebra with Sage, 29. Section 6-2, Linear Operator, SGLee  http://youtu.be/i2WP-cb6Ymc
Student’s Activities in Movies, You-Tube (2012 Spring Linear Algebra Class)

- Preview: http://matrix.skku.ac.kr/2012-Album/1-CLA-Preview.html

- SKKU Linear Algebra with Sage, S1. Section 1-1, Vector, Student http://youtu.be/fbCMyh-iDCQ
- SKKU Linear Algebra with Sage, S2. Section 1-2, Norm, Student http://youtu.be/sEFj_7t_bqc
- SKKU Linear Algebra with Sage, S3. Section 1-3, Vector Equations, Student http://youtu.be/avVJfeEoeVs
- SKKU Linear Algebra with Sage, S4. Chapter 1 Discuss, Student http://youtu.be/tys3taO5lHS
- SKKU Linear Algebra with Sage, S5. Section 2-1, LSE, Student http://youtu.be/N5ltl-bfdvkz
- SKKU Linear Algebra with Sage, S6. Section 2-2, RREF, Student http://youtu.be/sJeomjbRFmE
- SKKU Linear Algebra with Sage, S7. Section 2-3, Appl of LSE, Student http://youtu.be/vx_6rTJq5jk
- SKKU Linear Algebra with Sage, S8. Section 3-1, Matrix, Student http://youtu.be/LaAAruKbGyc
- SKKU Linear Algebra with Sage, S9. Section 3-2, Inverse Matrix, Student http://youtu.be/-MPszmMNvLE
- SKKU Linear Algebra with Sage, S10. Section 3-3, Elementary Matrix, Student http://youtu.be/cel8oeXp6xU
- SKKU Linear Algebra with Sage, S11. Section 3-4, Subspace, Student http://youtu.be/s7jxVvVAel4
- SKKU Linear Algebra with Sage, S12. Section 3-5, Solutions Set, Student http://youtu.be/lYgHFdWacds
- SKKU Linear Algebra with Sage, S13. Section 3-6, Special Matrices, Student http://youtu.be/rYBsPkeVhQo
- SKKU Linear Algebra with Sage, S14. Section 4-1, Determinant, Student http://youtu.be/Fne4gaZtE_Q
- SKKU Linear Algebra with Sage, S15. Section 4-2, Cofactor Expansion, Student http://youtu.be/nAabf3lpFU4
- SKKU Linear Algebra with Sage, S16. Section 4-3, Cramer's Law, Student http://youtu.be/Ygu4_7l4fGQ
- SKKU Linear Algebra with Sage, S17. Section 4-5, Eigenvalue & Eigenvector, Student http://youtu.be/WEyopWSiH6A
- SKKU Linear Algebra with Sage, S18. Section 6-1, Linear Transformation, Student http://youtu.be/dNt6NUDwc-Q

...
What was Developed 1!

# Sage (embedded)

- http://math1.skku.ac.kr/home/pub/534: SKKU-LA-1.1-(Vector)
- http://math1.skku.ac.kr/home/pub/583: LA-2.1-(Linear Equations)
- http://math1.skku.ac.kr/home/pub/583: LA-2.1-(Linear Equations)
- http://math1.skku.ac.kr/home/pub/730/: LA-4.4-Volume (Det)
- http://math1.skku.ac.kr/home/pub/597: LA-4.4-Curve Fitting (Det)
- http://math1.skku.ac.kr/home/pub/540: LA-4.4-Equations(Det)
- http://math1.skku.ac.kr/home/pub/733: SKKU-LA-5.2-Crypto
- http://math1.skku.ac.kr/home/pub/639: SKKU-LA-6.2-Flag (Rotation)
- http://math1.skku.ac.kr/home/pub/582: SKKU-LA-6.2-(Linear Transformation)
- http://math1.skku.ac.kr/home/pub/580: SKKU-LA-6.5-(Dilation)
- http://math1.skku.ac.kr/home/pub/729/: SKKU-LA-6.5-Tri (Dilation)
- http://math1.skku.ac.kr/home/pub/708: SKKU-LA-6.5-(Linear Transformation)
What was Developed 2!

# Sage with Geogebra

- Vector Sum: [http://math1.skku.ac.kr/home/pub/534](http://math1.skku.ac.kr/home/pub/534)
- Scalar Multiplication: [http://math1.skku.ac.kr/home/pub/535](http://math1.skku.ac.kr/home/pub/535)
- Linear System of Equations: [http://math1.skku.ac.kr/home/pub/583](http://math1.skku.ac.kr/home/pub/583)
- Matrix Multiplication: [http://math1.skku.ac.kr/home/pub/769](http://math1.skku.ac.kr/home/pub/769)
- Inverse Matrix: [http://math1.skku.ac.kr/home/pub/731](http://math1.skku.ac.kr/home/pub/731)
- Area of Triangle: [http://math1.skku.ac.kr/home/pub/730](http://math1.skku.ac.kr/home/pub/730)
- Curve Fitting: [http://math1.skku.ac.kr/home/pub/597](http://math1.skku.ac.kr/home/pub/597)
- Eigenvalue and Eigenvectors: [http://math1.skku.ac.kr/home/pub/732](http://math1.skku.ac.kr/home/pub/732)
- Matrix Cryption: [http://math1.skku.ac.kr/home/pub/733](http://math1.skku.ac.kr/home/pub/733)
- Rotation: [http://math1.skku.ac.kr/home/pub/639](http://math1.skku.ac.kr/home/pub/639)
- Reflection and Projection: [http://math1.skku.ac.kr/home/pub/598](http://math1.skku.ac.kr/home/pub/598)
  [http://math1.skku.ac.kr/home/pub/582](http://math1.skku.ac.kr/home/pub/582)
- Shear: [http://math1.skku.ac.kr/home/pub/596](http://math1.skku.ac.kr/home/pub/596)
- Dilation: [http://math1.skku.ac.kr/home/pub/580](http://math1.skku.ac.kr/home/pub/580)
- Quadratic Form: [http://math1.skku.ac.kr/home/pub/751](http://math1.skku.ac.kr/home/pub/751)
- Fourier Series: [http://math1.skku.ac.kr/home/pub/752](http://math1.skku.ac.kr/home/pub/752)
- Jordan Blocks: [http://math1.skku.ac.kr/home/pub/773](http://math1.skku.ac.kr/home/pub/773)
New LA textbook

Contemporary Linear Algebra with Sage

Published in September 1\textsuperscript{st}!
Linear algebra with Sage

E-book (Model 1)

10.1 Chapter 10.1 Jordan Canonical Form

1. Jordan canonical form of a matrix is a diagonal matrix with eigenvalues on the diagonal.
2. The matrix is in the form $A = P^{-1}JP$, where $J$ is the Jordan canonical form of $A$. The matrix $P$ is composed of eigenvectors of $A$.
3. The Jordan canonical form is unique up to the order of the eigenvalues on the diagonal.
4. The Jordan canonical form is particularly useful in the study of linear transformations and the analysis of systems of linear differential equations.
Linear algebra with Sage

E-book (Model 2)

http://matrix.skku.ac.kr/2012-mobile/E-CLA/10-1-ex.html
Linear algebra with Sage

E-book (Model 3)

10.1 연습문제

문제풀이 동영상:
- http://youtu.be/9-G3Fd2xOW0
- http://www.youtube.com/watch?v=adWzUKKmO2k

1. 다음 정사각행렬 $A$의 행렬 $A$의 고유값 $\lambda$을 구하고, $A$에 대응하는 고유벡터의 공간을 구하여라.

2. 다음 Jordan 표준형 $J_A$에 대하여 다음과 구하여라.

$$J_A = \begin{bmatrix} \lambda & 1 & 0 & 0 \\ 0 & \lambda & 1 & 0 \\ 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & \lambda \end{bmatrix}$$

(1) $J_A - \lambda I$
(2) $(J_A - \lambda I)^2$
(3) $(J_A - \lambda I)^3$
(4) $(J_A - \lambda I)^4$
Movie!!

- Movie e-book: Calculus Lab book
- Movie e-book: Linear Algebra (knowls)
MathJax

Beautiful math in all browsers

MathJax is an open source JavaScript display engine for mathematics that works in all modern browsers.

No more setup for readers. No more browser plugins. No more font installations... It just works.

Latest news: IOP Publishing continues as MathJax sponsor.

MathJax features and benefits

High-quality typography.

MathJax™ uses modern CSS and web fonts, instead of equation images or Flash, so equations scale with surrounding text at all zoom levels. See how this works in the scaling math demo.

LaTeX:
\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}

\Gamma(m + \alpha + 1) = \left(\frac{m \Gamma(\alpha)}{\Gamma(m)}\right) x^m$

Using MathJax in popular web platforms

MathJax plugins are available for a growing number of wiki, blogs, and other content-management systems. These include WordPress, Blogger, Sphinx, TiddlyWiki, and MathEL-Wiki. A list of these is available in the web applications list of the MathJax web site.

If the program you are using is not one of these, you may still be able to use MathJax by modifying the theme or template for your wiki or blog, as explained below.

Using MathJax in a Theme File

Most web-based content-management systems include a theme or template layer that determines how the pages look, and that loads information common to all pages. Such theme files provide one popular way to include MathJax in your web templates in the absence of MathJax-specific plugins for the system you are using. To take advantage of this approach, you will need access to your theme files, which probably means you need to be an administrator for the site. If you are not, you may need to have an administrator do these steps for you.

To enable MathJax in your web platform, add the line:

```html
<script type="text/javascript" src="http://cdn.mathjax.org/mathjax/latest/MathJax.js?config=TeX-AMS-MML_HTMLorMML"></script>
```

either just before the `</head>` tag in your theme file, or at the end of the file if it contains no `</head>`.

The theme files for various popular platforms are:

- WordPress
  - `wp-content/themes/{current_theme}/header.php`
- Movable Type
  - `{current_theme_templates}/html_head_whml`
Introducing the knowl: better browsing on the Web

The American Institute of Mathematics (AIM) is pleased to introduce a new tool to the world of web pages—the knowl, a referenced text from within the body of a web page. But unlike the hyperlink, which simply takes you to a new web page, the knowl with a similar click, the knowl then disappears.

The self-contained nature of knowls makes them useful in many different settings. They can be thought of as “building blocks, reliable and ready to be referenced anywhere,” says David Farmer, Director of Programs at AIM.

Knowls were developed to support research projects in advanced mathematics, but they can be easily incorporated to old, and the theoretical basis, known as transclusion, is more than 50 years old. Our contribution is to provide an interface.

In addition to the examples in the paragraphs above, AIM has prepared a demonstration page for mathematicians looking for a section of a math paper. The mathematics research projects which led to the development of knowls were funded by the Division of Mathematical Sciences at the National Science Foundation.

How to put knowls on your own website

The process is very similar to the way people currently make web pages and hyperlinks. With a single line of metadata, they can be inserted into a page. The knowl is displayed in primary color, with an additional link for further information.

Step 1: Add the following three lines between the <head> and </head> tags at the top of your page:

```html
<link id='knowl' href='http://aimath.org/knowlstyle.css' rel='stylesheet' type='text/css' />
<script type='text/javascript' src='http://aimath.org/knowlstyle.js'></script>
```

Step 2: Add the knowl in the body of your page (e.g., HTML, XML, LaTeX). Use the "knowl" template:

```
<knowl>
  <!-- the knowl content goes here -->
</knowl>
```

Step 3: Additional information can be added in the "knowl" template:

```html
<knowl id='exampleknowl' title='Example Knowl'>
  <!-- the knowl content goes here -->
</knowl>
```

Knowls are a way of embedding interrelated information on the Web. They can be thought of as “building blocks, reliable and ready to be referenced anywhere.”

Knowls were created by Harald Schilly, with the first working version coming online on August 25, 2011. Schilly also cofounded the National Science Foundation. Knowls are pronounced "nok," as in "A knowl is a small hill."
A First Course in Linear Algebra
written by Robert A. Beezer

Solving Systems of Linear Equations

We will motivate our study of linear algebra by considering the problem of solving several linear equations simultaneously. The word "solve" tends to get abused somewhat, as in "solve this problem," when talking about equations; we understand a more precise meaning: find set of the values of some variable quantities that make an equation, or several equations, true.

Example S1RE: Solving two linear equations

Suppose we desire the simultaneous solutions of the two equations.

\[ a_1 x + b_1 y = c_1 \]
\[ a_2 x + b_2 y = c_2 \]

You can easily check (by substitution) that \( (x_1, y_1) = (3, 1) \) and \( (x_2, y_2) = (-\frac{3}{2}, -\frac{1}{2}) \) are solutions. We want to add two linear equations that these are the only solutions. To save this, plot each equation on the \( xy \)-plane, which means to plot \( (x, y) \) pairs that make an individual equation true. In this case we get a circle centered at the origin with radius 1 and a straight line through the origin with slope \(-\frac{3}{2}\). The intersections of these two curves are our desired simultaneous solutions, and so we believe from our plot that the two solutions we know already are indeed the only ones. We like to write equations as sets, so in case we write the set of solutions as:

\[ S = \{ (3, 1), (-\frac{3}{2}, -\frac{1}{2}) \} \]

Definition SLE (System of Linear Equations): A system of linear equations is a collection of \( k \) equations in the \( n \) variable quantities \( x_1, x_2, \ldots, x_n \) of the form:

\[ a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n = b_1 \]
\[ a_{21} x_1 + a_{22} x_2 + \cdots + a_{2n} x_n = b_2 \]
\[ \vdots \]
\[ a_{k1} x_1 + a_{k2} x_2 + \cdots + a_{kn} x_n = b_k \]

where the values of \( a_{ij} \) and \( b_i \) are from the set of complex numbers, \( \mathbb{C} \).
References

We believe you now have an affordable and inspiring CAS tool for Linear Algebra, and Calculus.

In this talk, we introduced how and what we have done on E-books with Sage on Linear Algebra and Calculus.

Now our students can talk more on Mathematics and concentrate on Mathematical concepts, and use very affordable CAS tool without spending time to learn and typing programming language.
Thank you!