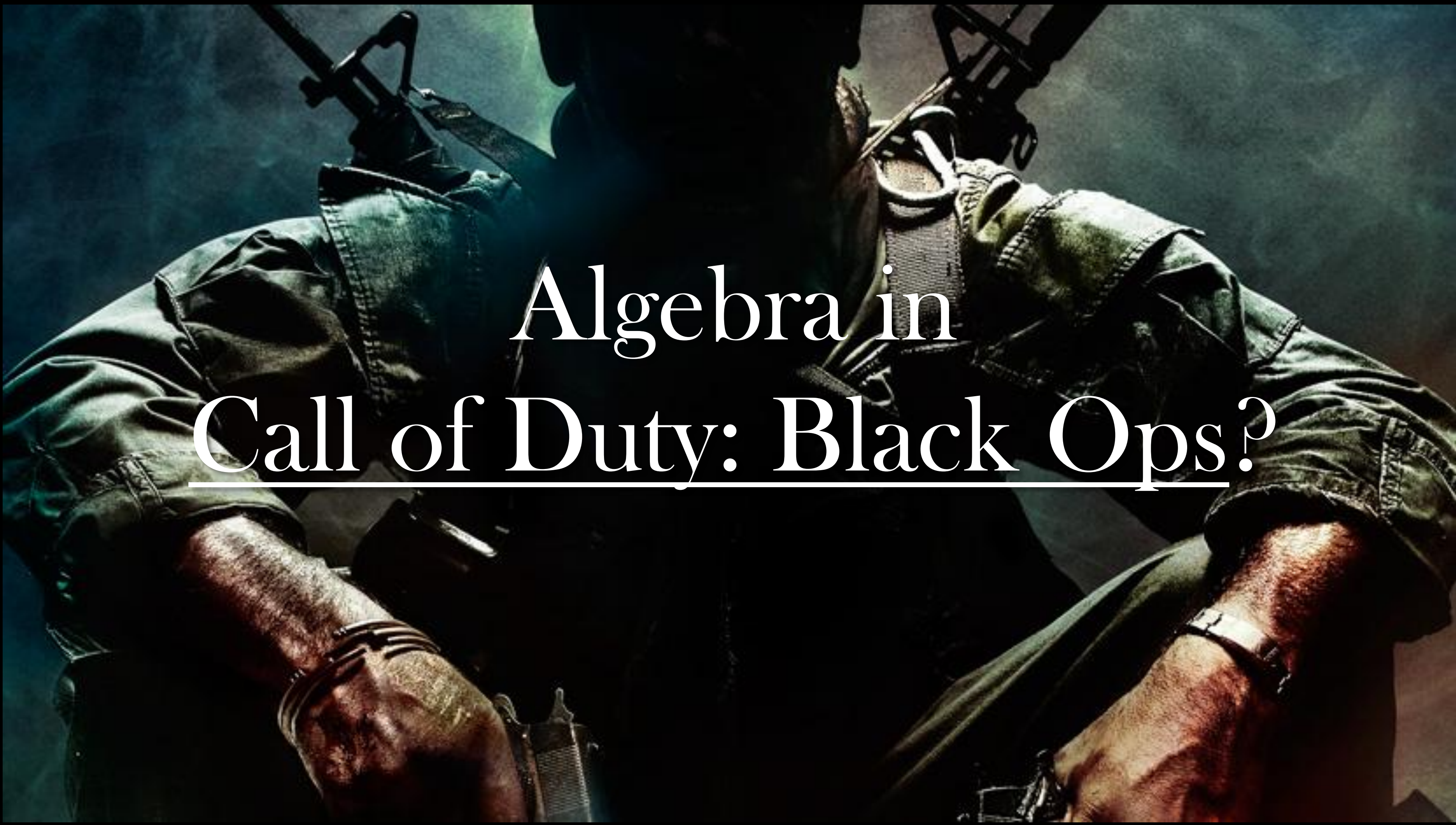
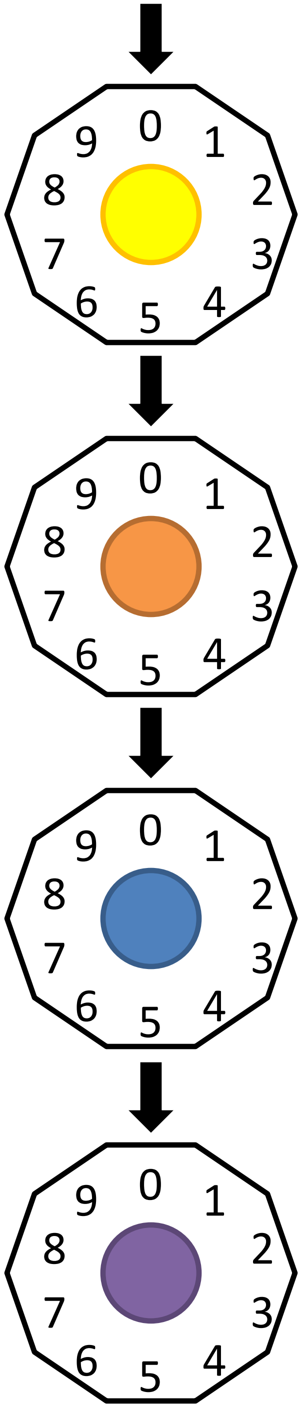


Algebra in Call of Duty: Black Ops?

Heidi Hulsizer, Hampden-Sydney College

Call of the Dead: The Lighthouse

http://www.youtube.com/watch?v=TUbwbhcdsUo


0 1 2 3 4 5 6 7 8 9
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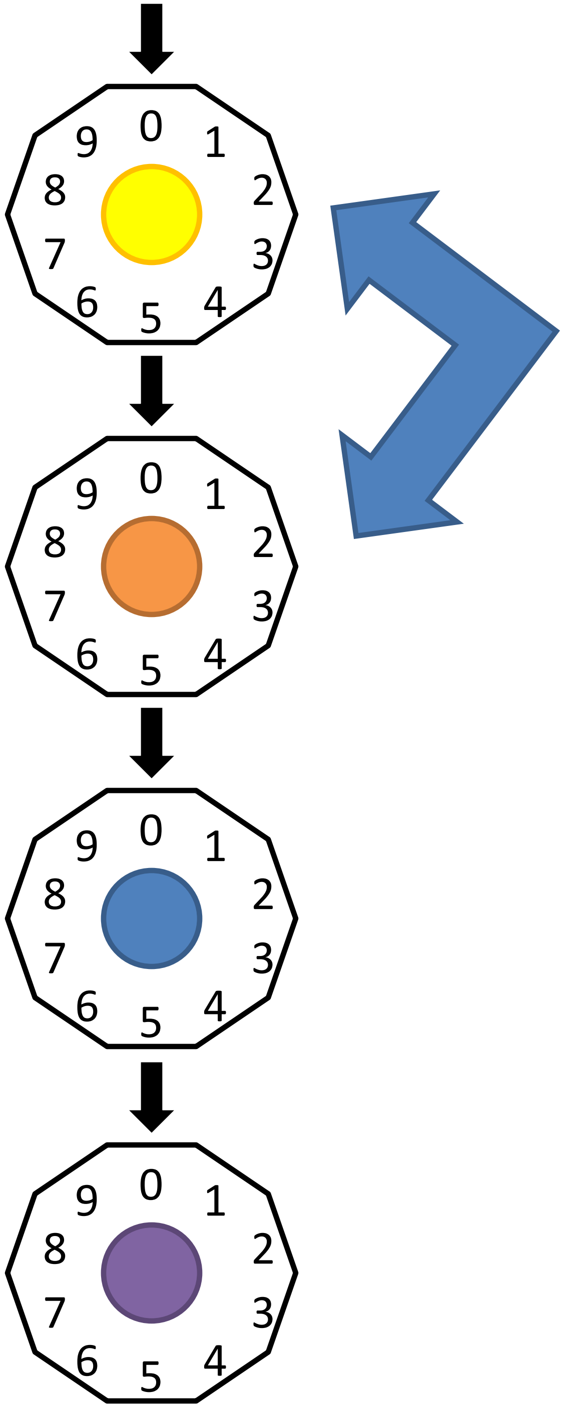
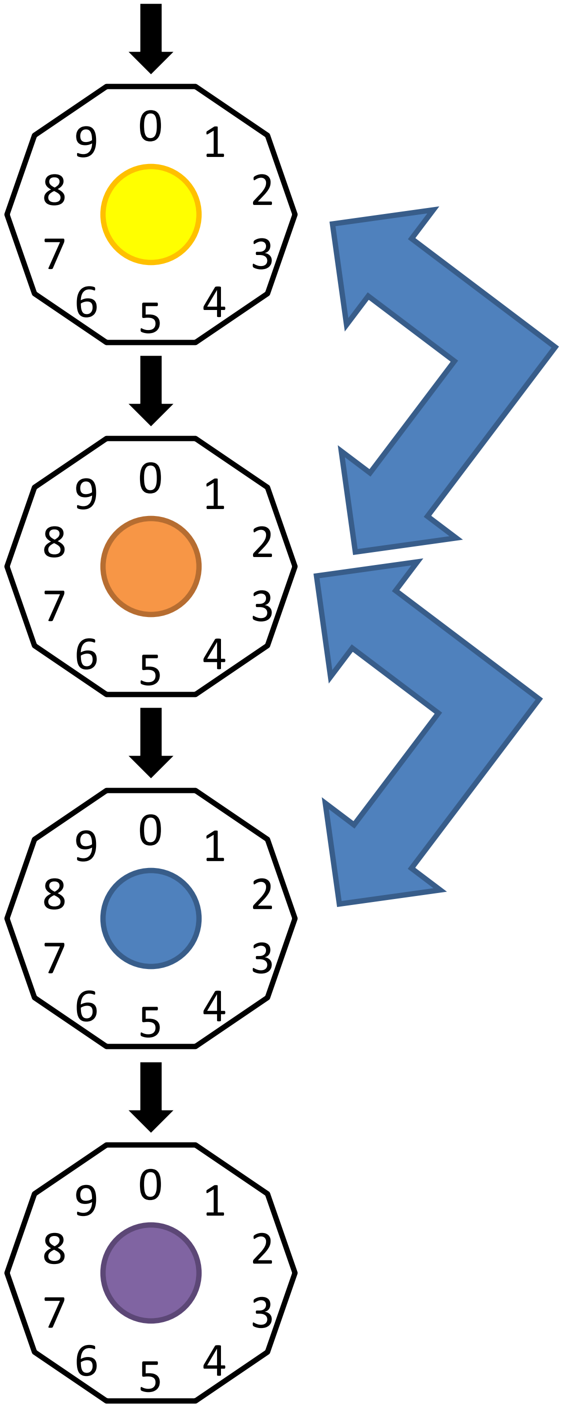
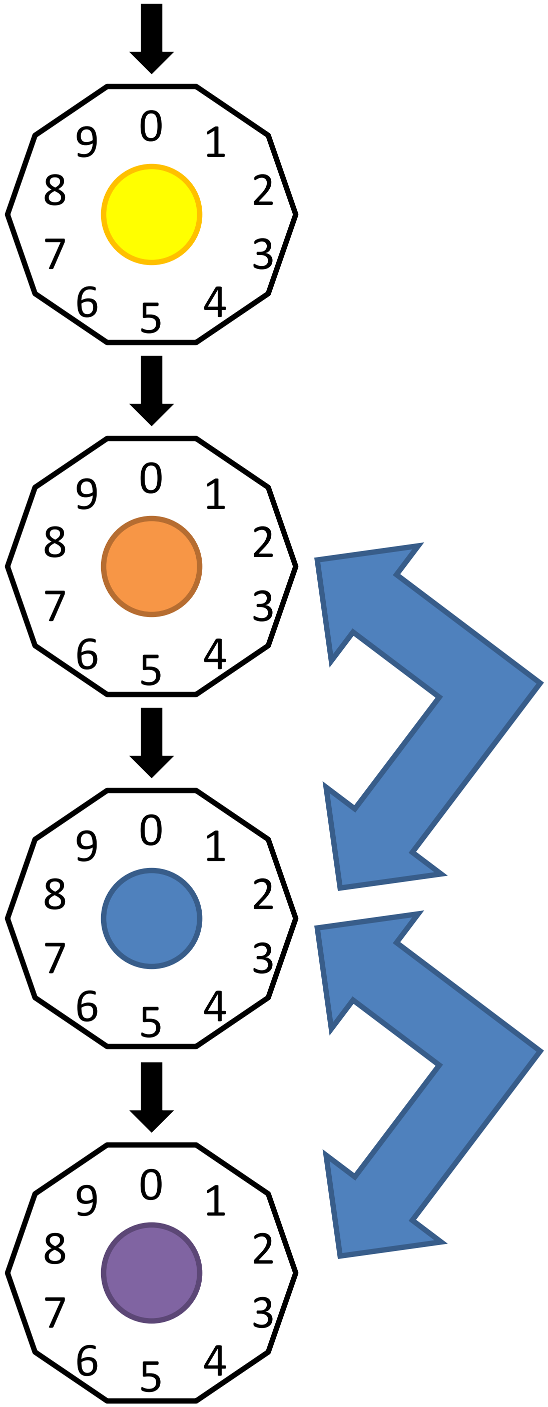


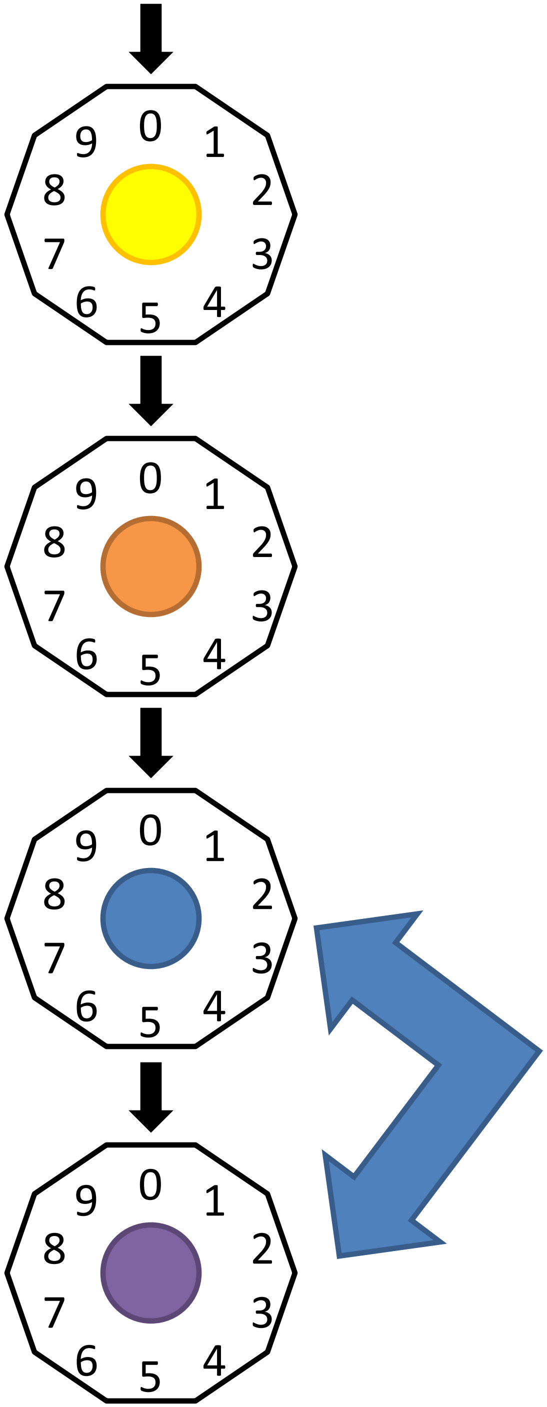
Turn Yellow, it turns Orange



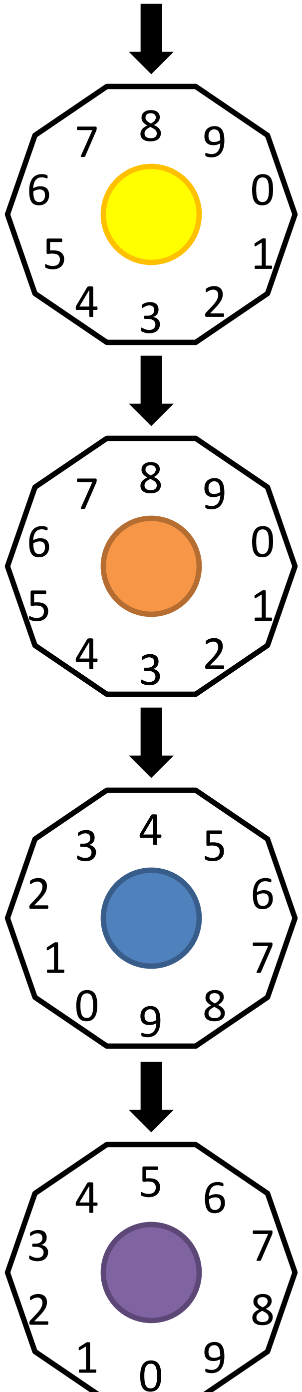
Turn Orange, it turns Yellow and Blue



Turn Blue, it turns Orange and Purple



Turn Purple, it turns Blue

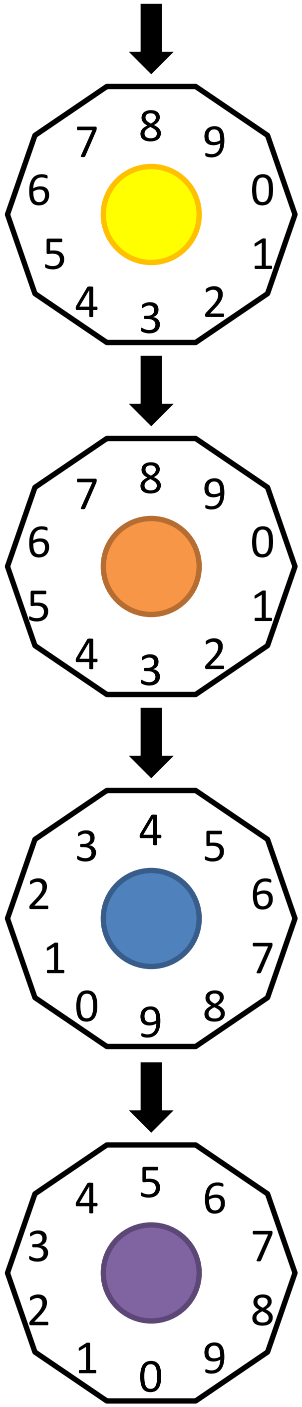


Yellow starts at 8

Orange starts at 8

Blue starts at 4

Purple starts at 5



	We want:
Yellow starts at 8	2
Orange starts at 8	7
Blue starts at 4	4
Purple starts at 5	6

Now to the math…

Method 1:

Definition: Let $m \neq 0$ be an integer. We say that two integers a and b are *congruent modulo m* if there is an integer k such that $a - b = km$, and in this case we write

$$a \equiv b \bmod m.$$

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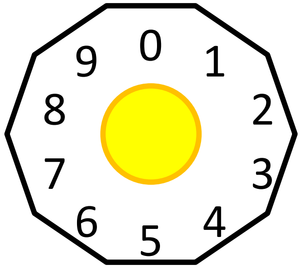
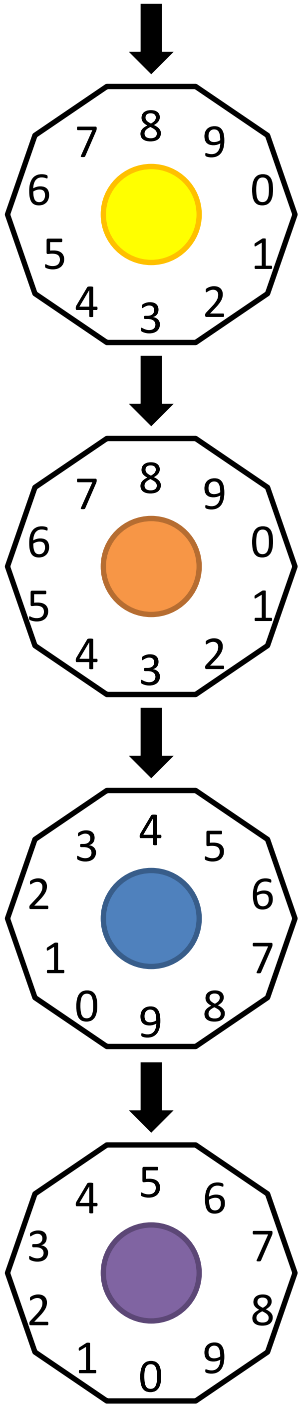


Let x = number of times turn the top dial



We want:

Yellow starts at 8 — **2**

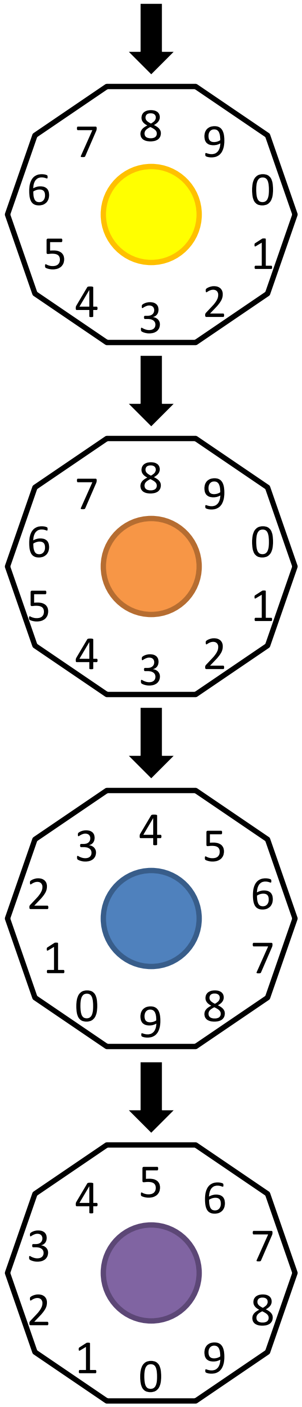
Orange starts at 8 — **7**

Blue starts at 4 — **4**

Purple starts at 5 — **6**

Let x = number of times turn the top dial

Let y = number of times turn the orange dial



We want:

Yellow starts at 8	**2**
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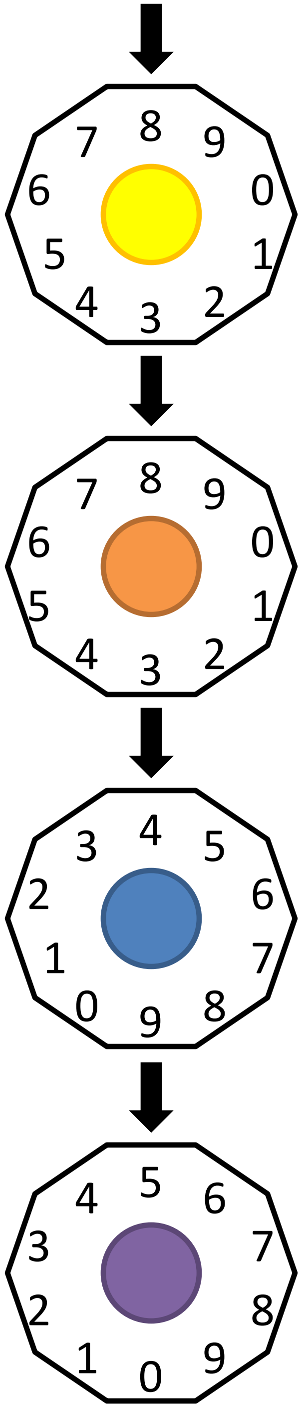
Let x = number of times turn the top dial

Let y = number of times turn the orange dial

Let z = number of times turn the blue dial



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Yellow starts at 8	**2**
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We want:

Let x = number of times turn the top dial

Yellow starts at 8 — 2

Let y = number of times turn the orange dial

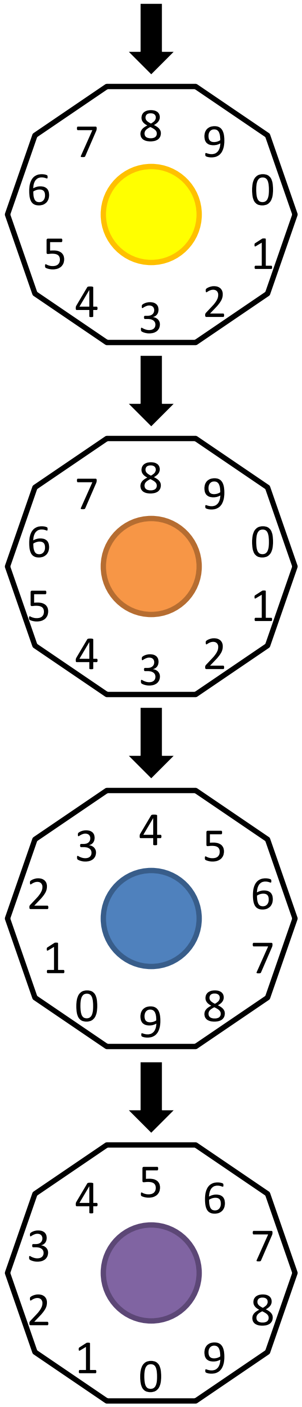
Orange starts at 8 — 7

Let z = number of times turn the blue dial

Blue starts at 4 — 4

Let w = number of times turn the bottom dial

Purple starts at 5 — 6



Yellow starts at 8 **2**

Orange starts at 8 **7**

Blue starts at 4 **4**

Purple starts at 5 **6**

Yellow starts at 8 **2**

Orange starts at 8 **7**

Blue starts at 4 **4**

Purple starts at 5 **6**

Equations:

- $8 + x + y \equiv 2 \mod 10$

Yellow starts at 8 2

Orange starts at 8 7

Blue starts at 4 4

Purple starts at 5 6

Equations:

- $8 + x + y \equiv 2 \mod 10$
- $8 + x + y + z \equiv 7 \mod 10$
- $4 + y + z + w \equiv 4 \mod 10$
- $5 + z + w \equiv 6 \mod 10$

Combine:

- $8 + x + y \equiv 2 \bmod 10$
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- $2 + z \equiv 7 \mod 10$

$z \equiv 5 \mod 10$

Equations:

- $8 + x + y \equiv 2 \mod 10$
- $8 + x + y + z \equiv 7 \mod 10$
- $4 + y + z + w \equiv 4 \mod 10$
- $5 + z + w \equiv 6 \mod 10$

$z \equiv 5 \mod 10$

$w \equiv 6 \mod 10$

$y \equiv 9 \mod 10$

$x \equiv 5 \mod 10$

- $z \equiv 5 \bmod 10$
- $w \equiv 6 \bmod 10$
- $y \equiv 9 \bmod 10$
- $x \equiv 5 \bmod 10$

Turn the blue dial 5 times, the purple dial 6 times, the orange dial 9 times and the yellow dial 5 times.

General Method:

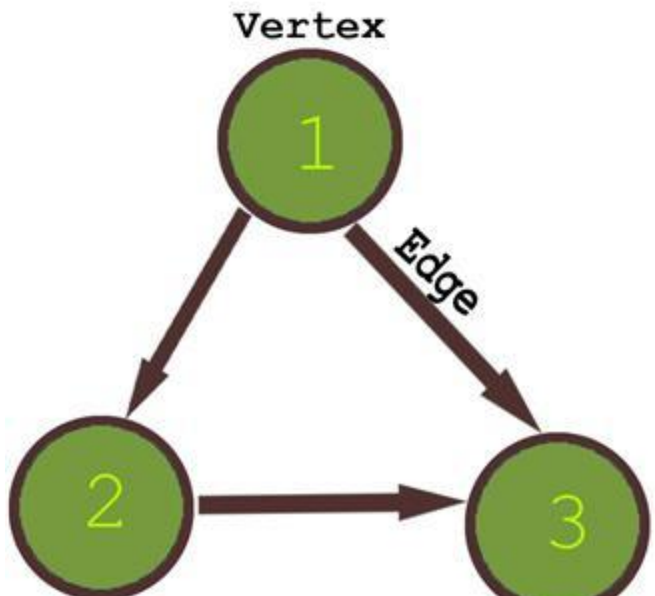
If you turn the dials without counting at first, insert the starting values into the equations:

- $? + x + y \equiv 2 \mod 10$
- $? + x + y + z \equiv 7 \mod 10$
- $? + y + z + w \equiv 4 \mod 10$
- $? + z + w \equiv 6 \mod 10$

Solve a system of equations modulo 10 and you are on your way to an achievement in Call of Duty!!

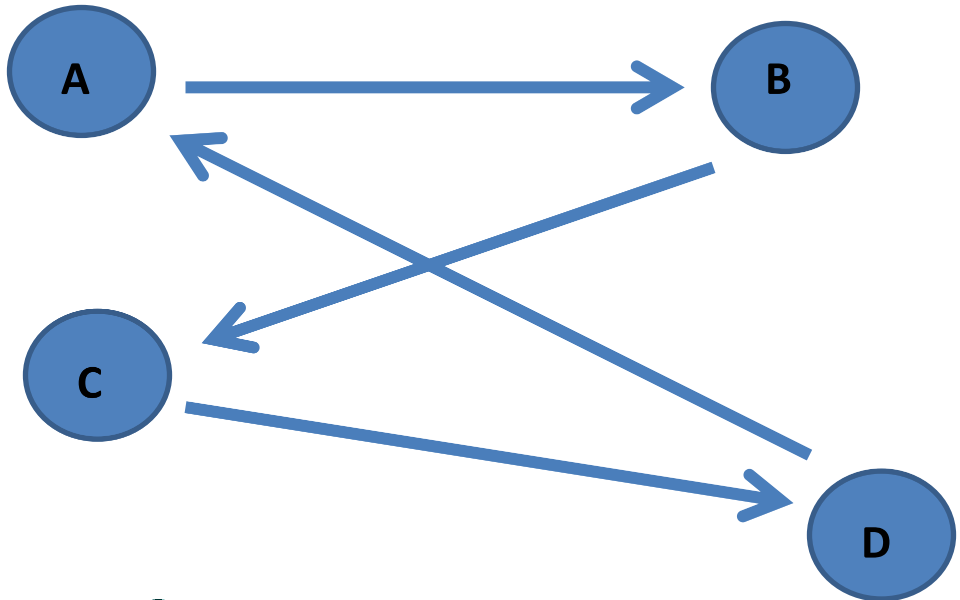
Method 2:

<u>Definition</u>: A *directed graph* is a set of nodes connected by edges where the edges have a direction associated to them.



http://www.google.com/imgres?q=directed+graph&hl=en&sa=X&biw=1680&bih=922&tbm=isch&prmd=imvns&tbnid=-iVW0er91QKSCM:&imgrefurl=http://www.aspfree.com/c/a/code-examples/an-insight-into-graphs/&docid=Q9zjzkTVeoWfQM&imgurl=http://images.devshed.com/af/stories/an_insight/2.jpg&w=325&h=311&ei=akqJUPz5HYOW8gT3yIDICA&zoom=1&iact=hc&vpx=487&vpy=589&dur=88&hovh=220&hovw=230&tx=117&ty=144&sig=105301745323159958231&page=2&tbnh=152&tbnw=159&start=38&ndsp=47&ved=1t:429,r:20,s:20,i:222

Example:



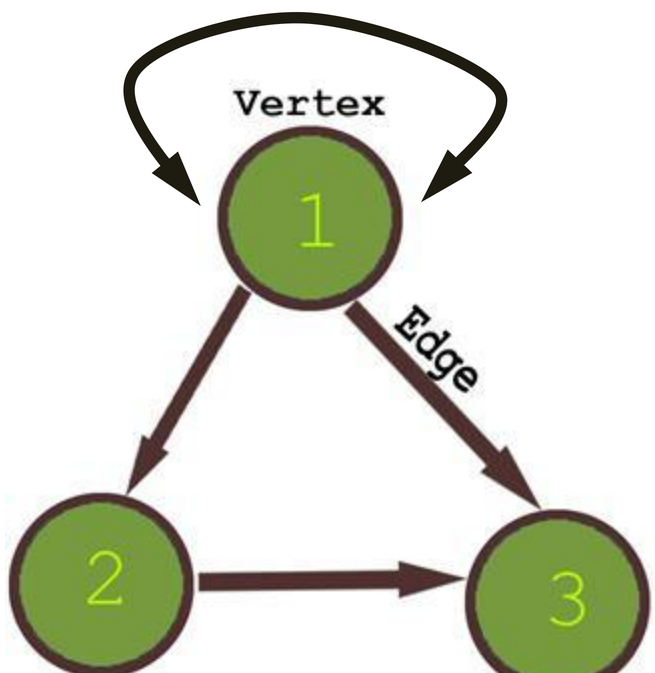
Adjacency Matrix:

D= (from ↓, to →)

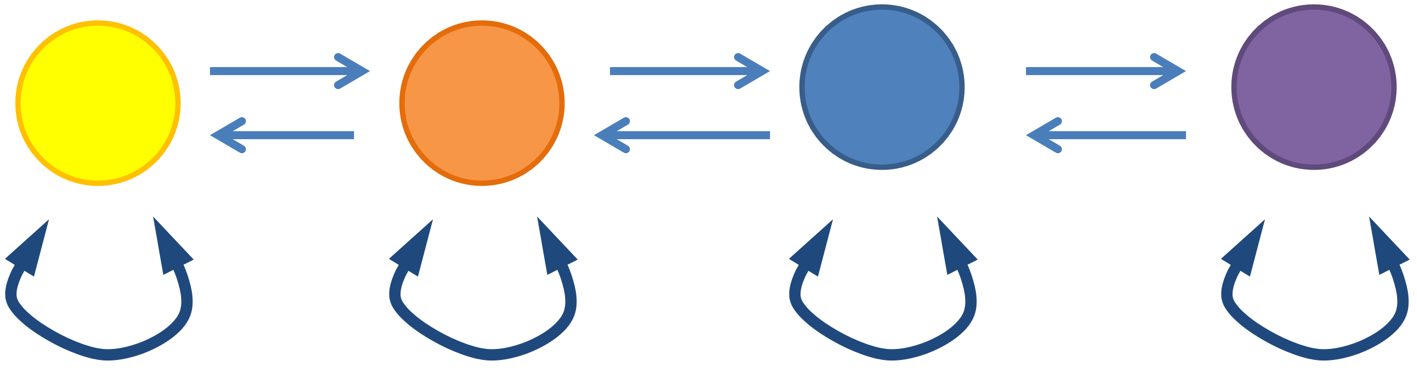
	A	B	C	D
A	0	1	0	0
B	0	0	1	0
C	0	0	0	1
D	1	0	0	0

http://www.mathwarehouse.com/algebra/matrix/matrix-directed-graphs.php

Definition: A *multidigraph* is a directed graph which is permitted to have a vertex targeted to itself.

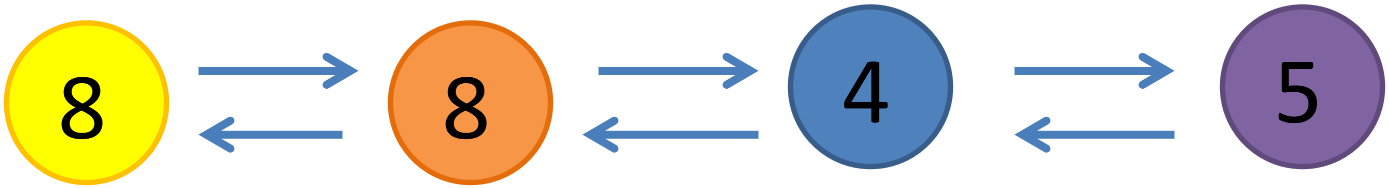


Back to Call of Duty - The dials as a graph, with edges and four vertices:



$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

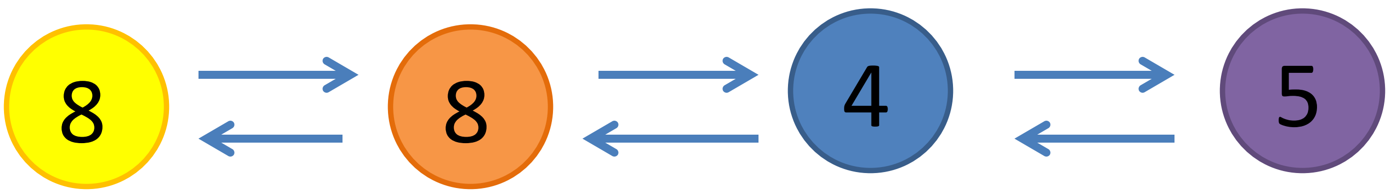
Back to <u>Call of Duty</u> - The dials as a graph, with edges and four vertices:



Rotate every dial once.

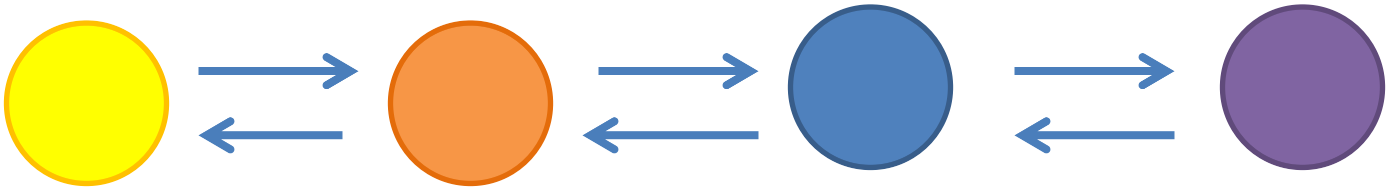
$$\begin{bmatrix}8\\8\\4\\5\end{bmatrix}+\begin{bmatrix}1&1&0&0\\1&1&1&0\\0&1&1&1\\0&0&1&1\end{bmatrix}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}=\begin{bmatrix}\\?\\ \\ \end{bmatrix}$$

Back to <u>Call of Duty</u> - The dials as a graph, with edges and four vertices:



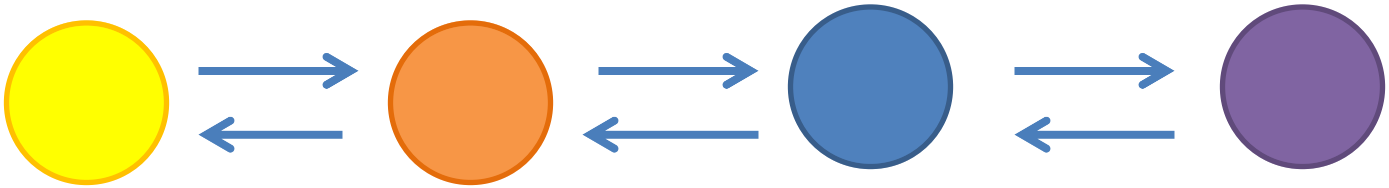
$$\begin{bmatrix} 8 \\ 8 \\ 4 \\ 5 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 11 \\ 7 \\ 7 \end{bmatrix} \equiv \begin{bmatrix} 0 \\ 1 \\ 7 \\ 7 \end{bmatrix} (\text{mod} 10)$$

Back to Call of Duty - The dials as a graph, with edges and four vertices:



$$\begin{bmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \\ \boldsymbol{a}_3 \\ \boldsymbol{a}_4 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \boldsymbol{x}_3 \\ \boldsymbol{x}_4 \end{bmatrix} = \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \\ \boldsymbol{b}_3 \\ \boldsymbol{b}_4 \end{bmatrix}$$

Back to Call of Duty - The dials as a graph, with edges and four vertices:



$$\begin{bmatrix} 8 \\ 8 \\ 4 \\ 5 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \\ 4 \\ 6 \end{bmatrix}$$

In general, a residue matrix, A,
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has a multiplicative inverse
if gcd(det(A), n)=1.

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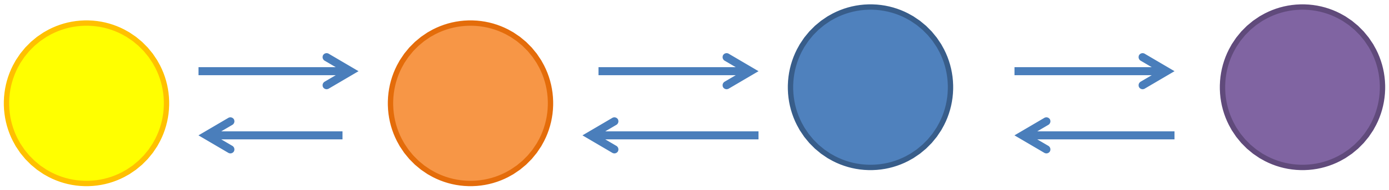
For us, gcd(det(A), 10)=1.

$$\begin{bmatrix}8\\8\\4\\5\end{bmatrix}+\begin{bmatrix}1&1&0&0\\1&1&1&0\\0&1&1&1\\0&0&1&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}2\\7\\4\\6\end{bmatrix}$$



$$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}-5\\-1\\5\\-4\end{bmatrix}\equiv\begin{bmatrix}5\\9\\5\\6\end{bmatrix}(\mathrm{mod}\,10)$$


Questions?