Linear Transformation — A Visual Perspective

(Institution C)

Your Geometer Sketchpad drafts must be complete with all the polygons and at least 3 cases of the different values of the parameters in the matrix of the transformation. Your reflections must be written up **clearly, legibly, in complete sentences**, primarily focusing on **explaining your reasoning, your thoughts while completing the module.** For this particular assignment, most credit will be awarded for satisfying these conditions.

Consider the following polygons with vertices at

a) (2,1) (4,3) and (3,4)
b) (2,1) (4,3) (3,4) and (5,4)
c) (2,1) (4,3) (3,4) (6,4) and (5,5)
d) your choice of a hexagon

Define a transformation $T: R^2 \to R^2$ by $T(y) = Xy$ where X are the following elementary matrices:

A. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

C. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

D. $\begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix}$ $a > 1$ or $0 < a < 1$

E. $\begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix}$ $a > 1$ or $0 < a < 1$

F. $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ $a \neq 0$

G. $\begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix}$ $a \neq 0$

H. $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ $a \neq 0$ and $b \neq 0$

I. $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Part I

1. Apply transformations A–C to polygons a–d.

2. Discuss what happens when you apply different elementary matrices, as well as when you change the parameters *a*, *b*, and $\boldsymbol{\theta}$ in matrices D–I.
3. Discuss the domain and range of the transformations. Is this what you expected the domain and range to be? Is the domain and range consistent with the domain and range of a function in other courses that you have taken? Explain.
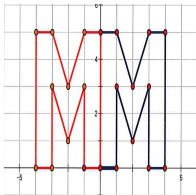4. Can you reverse the process? If yes, explain how.

Part II

1. Discuss what you observe as you change the parameters *a*, *b*, and $\boldsymbol{\theta}$ in matrices D–I. Change the parameters *a*, *b*, and $\boldsymbol{\theta}$ and predict what the image will be before you do the computation. Check your prediction by doing the computation.
2. Discuss the domain and range of the transformations as you keep changing the parameters. . Is this what you expected the domain and range to be? Is the domain and range consistent with the domain and range of a function in other courses that you have taken? Explain.
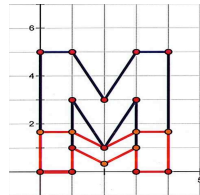3. Can you reverse the process? If yes, explain how.

Part III

1. Consider the following images from Geometer's Sketchpad. Examine these images and describe the properties of the linear transformation.
2. Find the elementary matrix of each of these transformations.
3. Discuss the domain and range of these transformations. Is this what you expected the domain and range to be? Is the domain and range consistent with the domain and range of a function in other courses that you have taken? Explain.
4. Can you reverse the process? If yes, explain how.
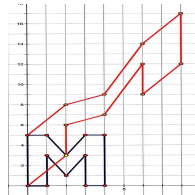5. Are any of these transformations an isomorphism? Explain! What about isometries? Explain!

a.

b.

c.

d.