Weekly MATLAB labs in Linear Algebra

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Labs in Mathematics - Premise

- Most STEM disciplines presume a lab component is necessary for complete comprehension of material.
- Mathematics is one of the only STEM disciplines where a Laboratory component is rarely presumed or implemented.
- The "computer competency" requirement has all but vanished from the US higher-ed curriculum.
- Meanwhile: USM (Univ. Sains Malaysia) has incorporated lab components in mathematics courses since 2000.

Labs in Mathematics

- Technologies are in place to enable enhanced learning of concepts from basic to upper level undergraduate math.
  - Graphic calculators, e.g.:TI-84, Nspire CX (algebra, trig, calculus, statistics)
  - Matlab (linear algebra, numerical analysis, calculus)
  - Mathematica (calculus, real analysis)
  - Minitab (statistics)
- Instructors have integrated explorations that implement these tools into math classrooms around the world.

Concerns

- Explorations must be integrated into current classroom format or material
- Incorporation of new course items means a sacrifice; topics or depth of coverage
- Budget issues disallow additions of lab components in mathematics:
  - 3 hr courses become 4 hr with lab
  - requires additional faculty budget lines (i.e., positions)
  - increased student course hours (e.g., Texas has 120 hr program limits, adding additional hours is not encouraged)

Some Technology Experiences

- ATLAST (Augmenting Teaching Linear Algebra with Software Tools) workshops in 1995, 1996, Seattle.
- Technology (MATLAB, TI-92/V200) integrated into linear algebra since 1997, often in a computer classroom.
- Retention and comprehension of theories or concepts has improved significantly.
- Yet, issues abound:
  - Persistent holes in student comprehension, e.g. - span, linear combinations, vector spaces and norms, applicability.
  - The plethora of software and on-line sites has increasingly become a distraction, surfing, emailing, etc…
  - Topical depth, sometimes coverage, reduced to allow time to integrate explorations into the classroom.

A Resolution

- Weekly laboratory, currently Fridays.
- Class meetings: three 50 minute periods, 2 in a traditional classroom, *lab day* in a computer classroom with MATLAB.
- Positives:
  - Lab time focused on enhancement of theory with technology
  - More opportunity to interact with the students
  - Student comprehension of topics has improved, exceeding results of classroom integration of MATLAB explorations
  - Student attention seems better, retention is up
  - Students work together and help teach each other

A Resolution

- Weekly Friday laboratory.
- Class meetings: three 50 minute periods, 2 in a traditional classroom, *lab day* in a computer classroom with MATLAB.
- Shortcomings:
  - Reduced time on lecture days for questions
  - Tons of grading
  - Some students miss labs that never miss classroom days
  - Shock of intro to vector space concepts seems greater, yet are grasped faster. I suspect because they cannot just "put it in the computer" or "rref it", novel applications of "rref" are tried.

Lab Assignments Linear Algebra

Fall 2012

- lab 1 - introduction to matlab – (Seth, Lay)
- lab 2 – Solving systems of equations, rref, vector forms (Seth)
- lab 3 - Span and linear combinations (Seth)
- lab 4 – Balancing Chemical Equations (Seth, Adsmond(chemist))
- lab 5 – Rank and Linear Independence (Seth)
- lab 6 – Solving Systems with Inverses (Seth)
- lab 7 – Determinants (Seth, some ideas of Hill, ATLAST, Lay)
- lab 8 – Owls (Lay) and magic squares (ATLAST)

Lab Assignments Linear Algebra

Fall 2012

- lab 9 - Coordinate vectors and basis (Seth, Hill, ATLAST)
- lab 10 – Eigenvalues and eigenvectors (Lay, Seth)
- lab 11 – Eigenvalues – Owls and systems of DE's (Seth, Lay)
- lab 12 – norms and inner products – (Seth, ATLAST, Hill)
- lab 13 – orthogonal vectors and grahm Schmidt – (Seth, Lay)

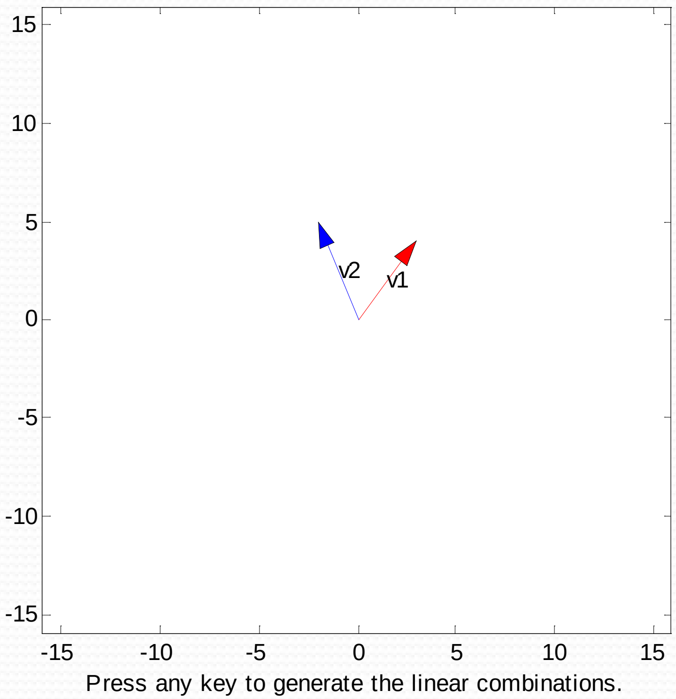
Other labs or explorations, other semesters

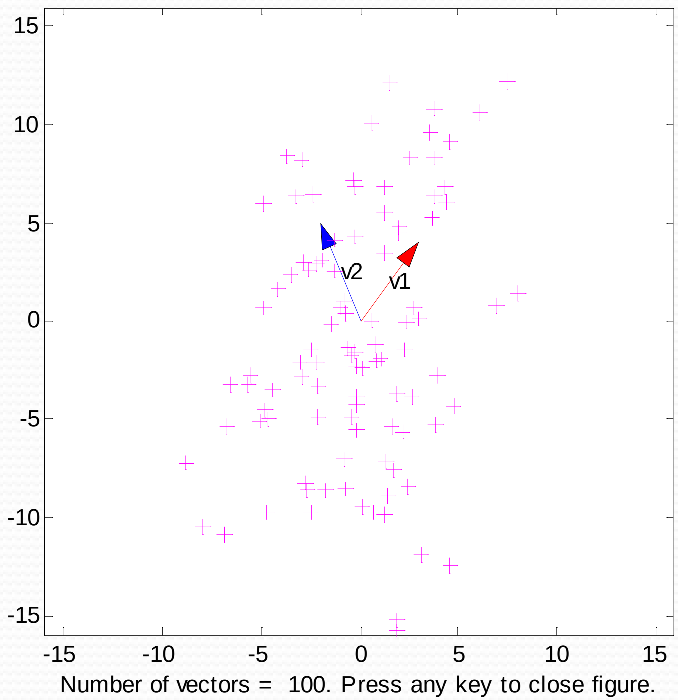
- Linear Algebra and Least Squares
- Linear Transformations and Animation
- Modeling ski slopes – polynomial interpolation
- Matrix multiplications and graph theory (airplane connections)

Samples from Labs:
Linear Combinations and Span

- With 100 linear combinations:

14. Use *drawlc.m* to plot multiple linear combinations of $u = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and $v = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$. Have the routine draw 5, 50, 100, and 500 different linear combinations. Discuss your observations. Based on the plot for multiple linear combinations of u and v, what conclusion can you draw about $span\{u, v\}$?

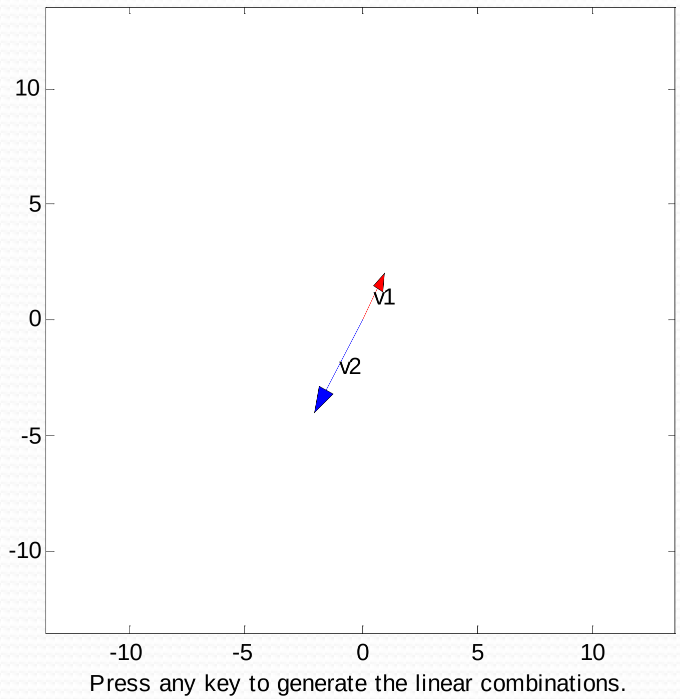


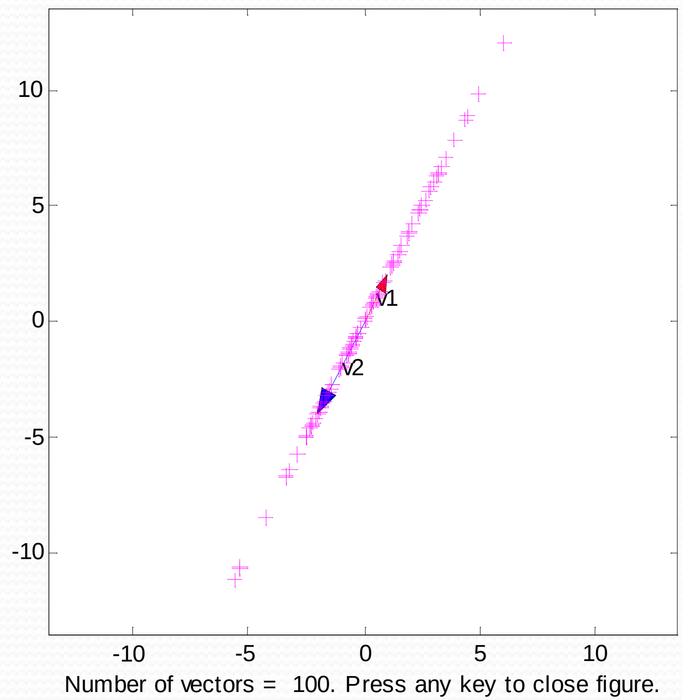


Linear Combinations and Span

- With 100 linear combinations:

16. Use *drawlc.m* to plot multiple linear combinations of $u = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $v = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$. Have the routine draw 5, 50, 100, and 200 different linear combinations. Discuss your observations. Based on the plot for multiple linear combinations of u and v, what conclusion can you draw about $span\{u, v\}$?





Samples from Labs:
Owls lifespan

t = 0.1800

a =

0	0	0.3300
0.1800	0	0
0	0.7100	0.9400

x0 =

100

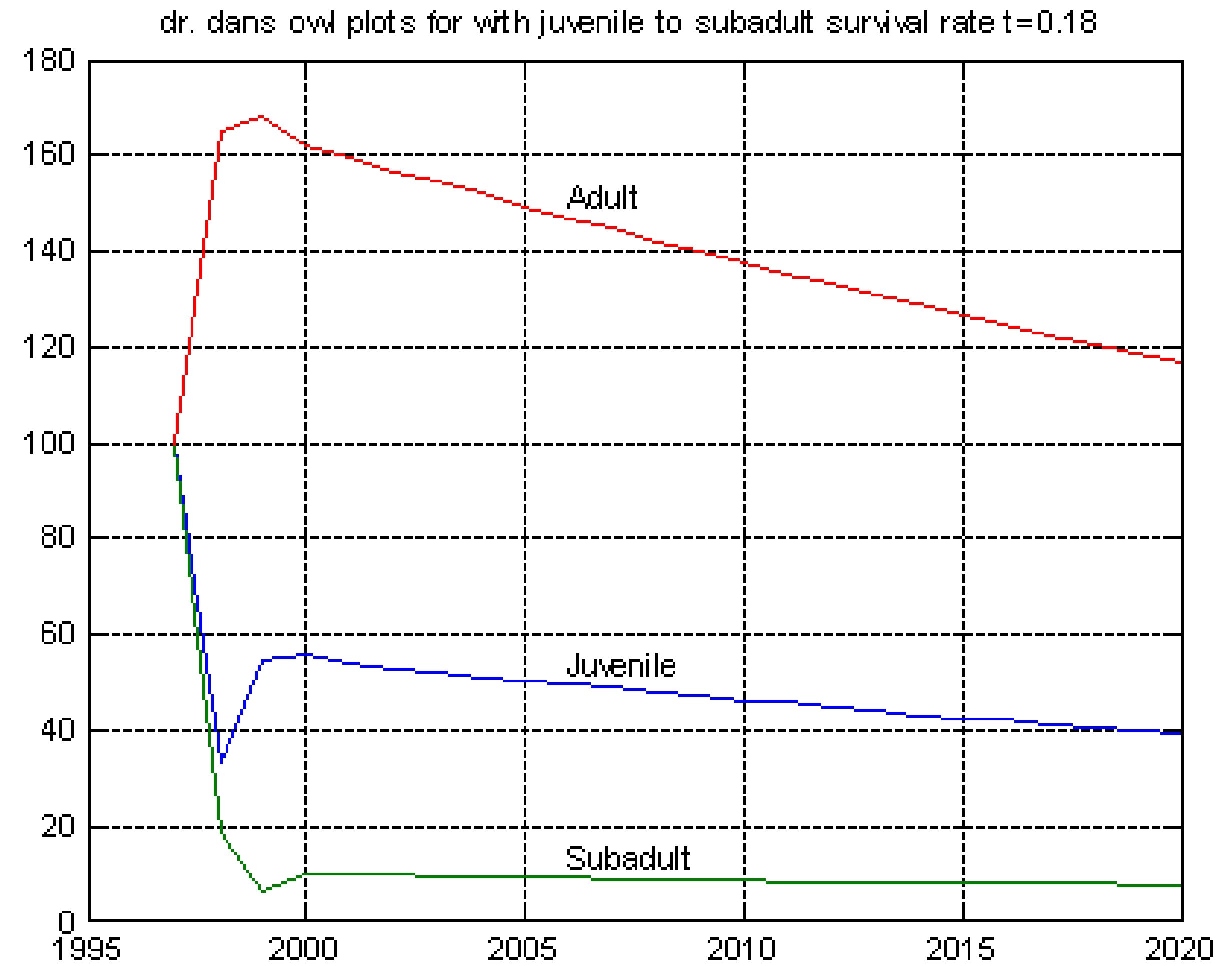
100

100

a*xn =

k=1997 +	0	1	2	3	10	20
juveniles	100.0000	33.0000	54.4500	55.4004	46.0797	39.0538
subadults	100.0000	18.0000	5.9400	9.8010	8.4327	7.1469
adults	100.0000	165.0000	167.8800	162.0246	137.3445	116.4030

Owls lifespan



Owls lifespan

t = 0.3000

a =

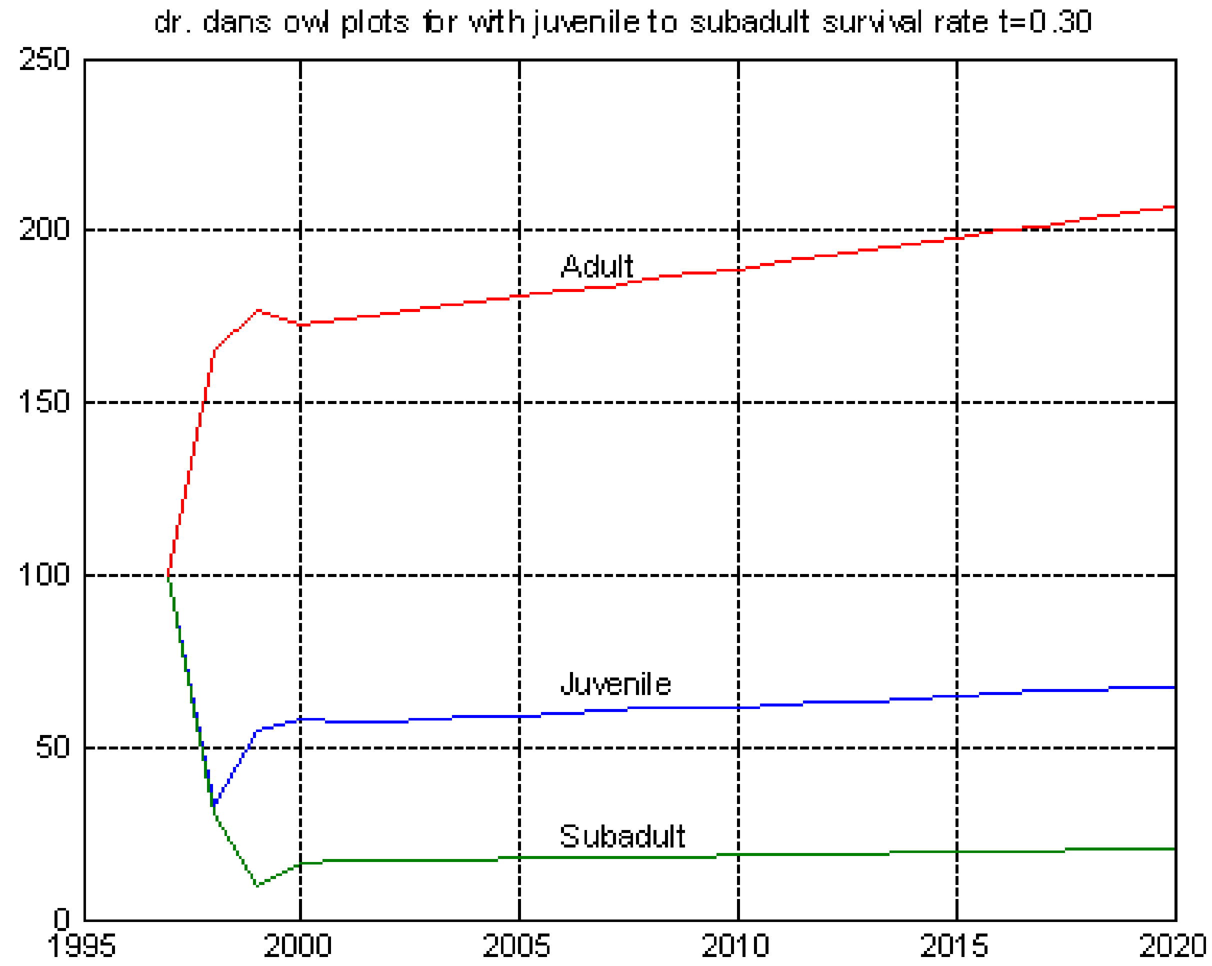
0	0	0.3300
0.3000	0	0
0	0.7100	0.9400

x0 =

100
100
100

k=1997 +	0	1	2	3	10	20
juveniles	100.0000	33.0000	54.4500	58.2120	61.8555	67.6781
subadults	100.0000	30.0000	9.9000	16.3350	18.3905	20.1216
adults	100.0000	165.0000	176.4000	172.8450	189.1348	206.9383

Owls lifespan



Magic Squares

Square 2: $\begin{bmatrix} 1 & x_1 & x_2 \\ x_3 & 2 & x_4 \\ x_5 & x_6 & 3 \end{bmatrix}$

The system of equations:

$$1 + x_1 + x_2 = 6$$
$$x_3 + 2 + x_4 = 6$$
$$x_5 + x_6 + 3 = 6$$
$$1 + x_3 + x_5 = 6$$
$$x_1 + 2 + x_6 = 6$$
$$x_2 + x_4 + 3 = 6$$
$$x_2 + 2 + x_5 = 6$$

Augmented matrix of system of equations:

[1	1	0	0	0	0	5
0	0	1	1	0	0	4
0	0	0	0	1	1	3
0	0	1	0	1	0	5
1	0	0	0	0	1	4
0	1	0	1	0	0	3
0	1	0	0	1	0	4],

Magic Squares

Reduced row echelon form:

[1	0	0	0	0	0	-1
0	1	0	0	0	0	4
0	0	1	0	0	0	9
0	0	0	1	0	0	5
0	0	0	0	1	0	7
0	0	0	0	0	1	0
0	0	0	0	0	0	0]

Solution of system of equations:

$x_1 = -t + 4$

$x_2 = t + 1$

$x_3 = t + 2$

$x_4 = -t + 2$

$x_5 = -t + 3$

$x_6 = t$

free variable, infinite solutions with integers:

Select $t = 1$, then: $\begin{bmatrix} 1 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

Select $t = 10$, then: $\begin{bmatrix} 1 & -6 & 11 \\ 12 & 2 & -8 \\ -7 & 10 & 3 \end{bmatrix}$

Eigenvalues and Owls

Survival rates and eigenvalues.

- table of extinction rates, leading eigenvalues

Survival rate juv - subadult	t =	.18	.20	.22	.24	.25	.26	.28	.30
Dominant eigenvalue	λ	0.9836	0.9880	0.9923	0.9966	0.9987	1.0008	1.0050	1.0090

Critical value of t: $t = .26, \quad \lambda_1 = 1.0008$

Eigenvalue of "steady state": lambda1 = 1.0008

v1 =

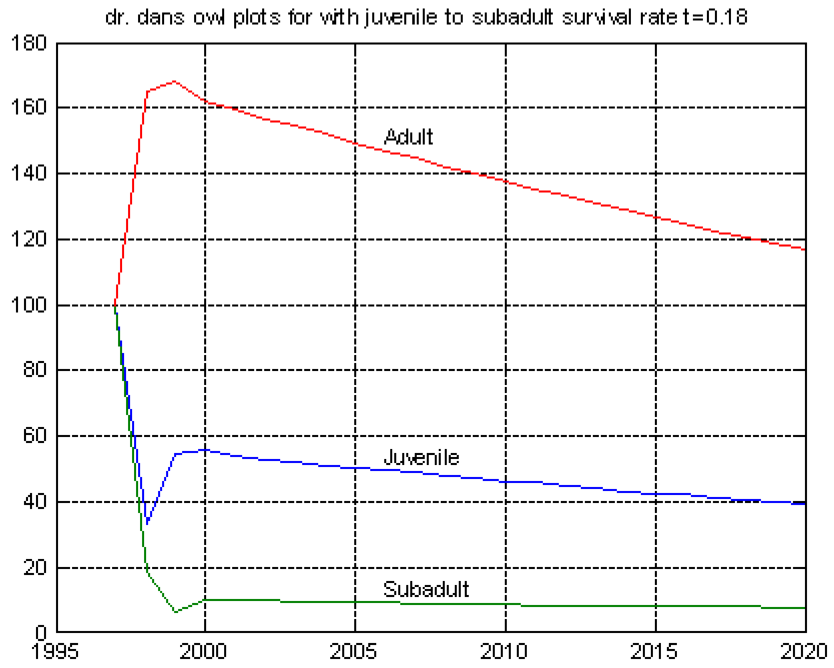
0.3121

0.0811

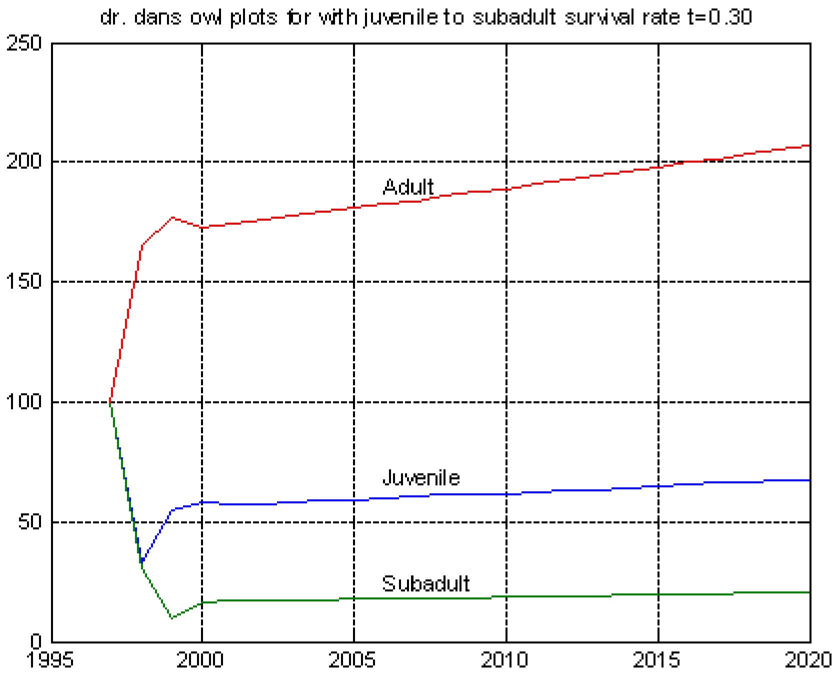
0.9466

Eigenvalues and Owls

(a) t below critical:



(b) t above critical:



Eigenvalues and DE

Solution and plot of system of DE's.

A = X0 =

1	-2		3
3	-4		2

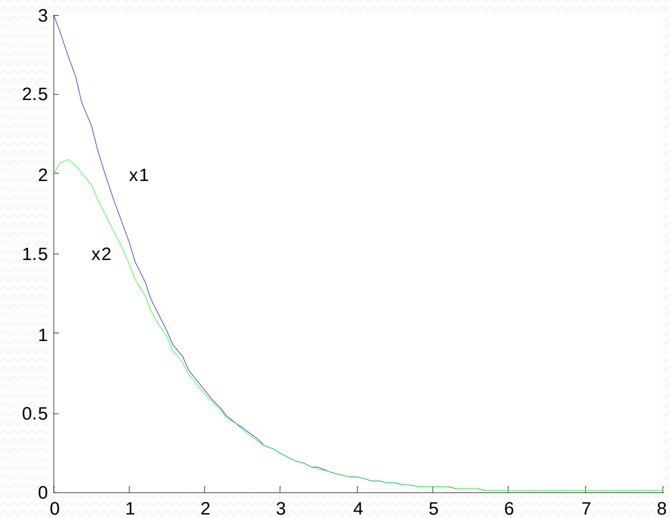
u = d = c = u\X0 =

0.7071	0.5547	-1.0000	0	7.0711
0.7071	0.8321	0	-2.0000	-3.6056

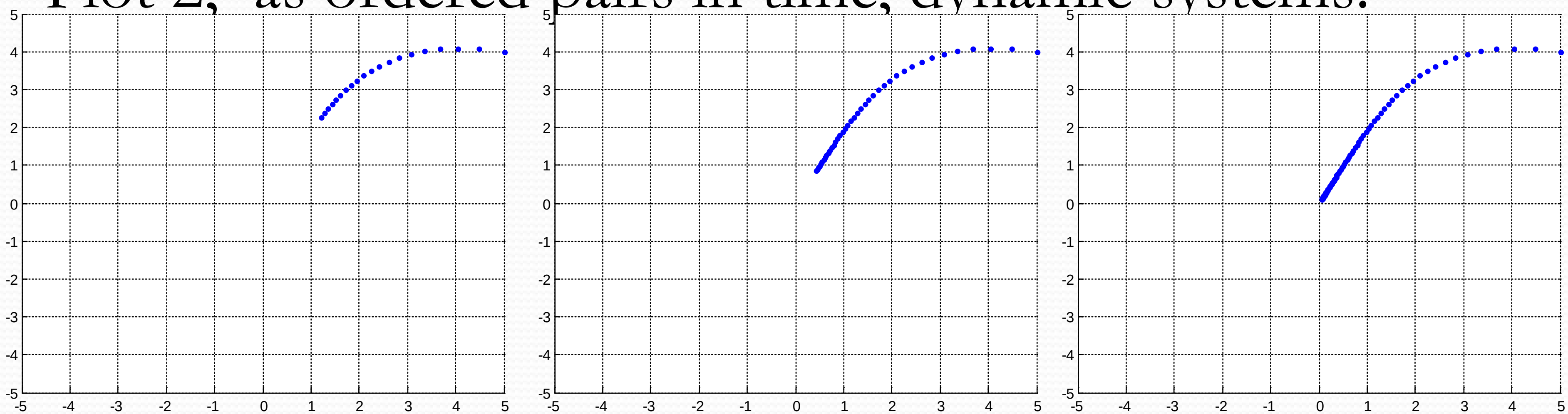
- Plot 1, as separate functions of time:
- Plot 2, as ordered pairs in time, dynamic systems:

Eigenvalues and DE

- plot 1, as separate functions of time:



- Plot 2, as ordered pairs in time, dynamic systems:



Assessment and Retention

Retention

- Fall 2012 (2^{nd} with lab): 46 started, 3 D, 1 failed
- Fall 2008 (no lab): 27 started, 4 D, 2 failed

Assessment

- *Common* final exam, 3-4 question coverage variations

	2008	2009	2010	2011	2012
Final Exam Passed (%)	81	83	95	87	100
Final average of D	37	11	8	10	11
Failed Final Exam	19	19	17	5	0

Future and References

- Some tasks:
  - Clean up the labs, e.g., reduce lengths and drudgery, add minor subtopics
  - Incorporate linear transformations and animation (CS friendly)
  - Develop and implement a more complete assessment process
- Good Lab Book References

  - *Instructors MATLAB Manual*, J. Case and J. Day, supplement to *Linear Algebra*, D. Lay, Pearson Education (2006)
  - *ATLAST Computer Exercises for Linear Algebra*, S. Leon, E. Herman, and R. Faulkenberry, Prentice Hall (2002)
  - *Linear Algebra Labs with MATLAB*, 3rd Ed., D. Hill and D. Zitarelli, Prentice Hall (2003)