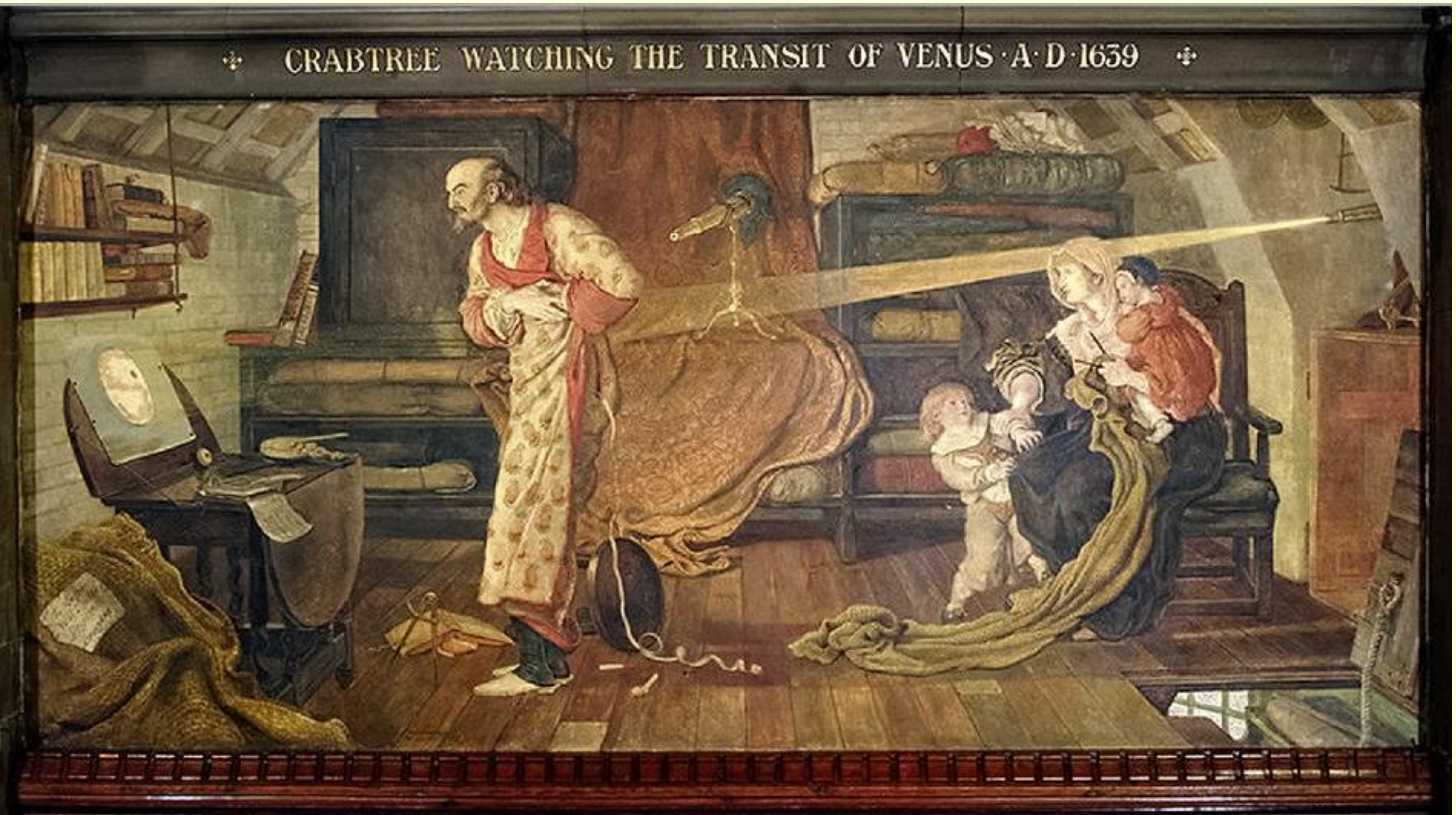
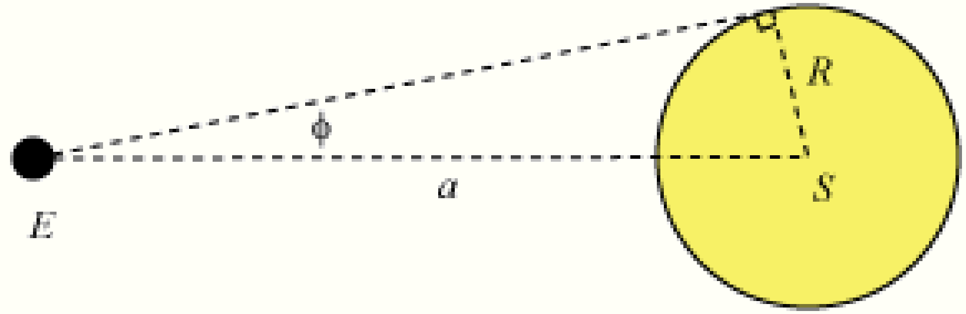
Tracking the Transit of Venus

Andrew Simoson
King College
ajsimoso@king.edu

- Predicted transit for1631
- Estimated period at 120 years

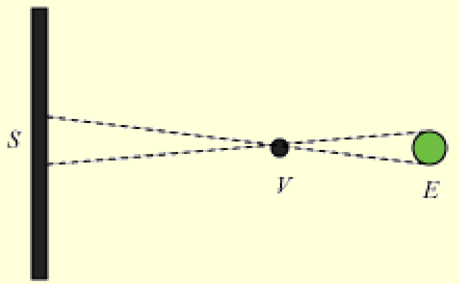
First recorded observation--1639

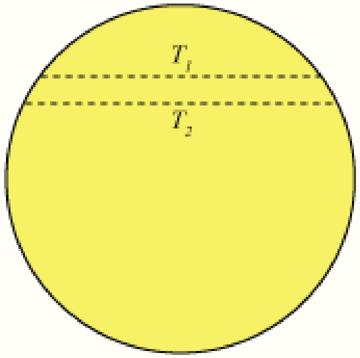


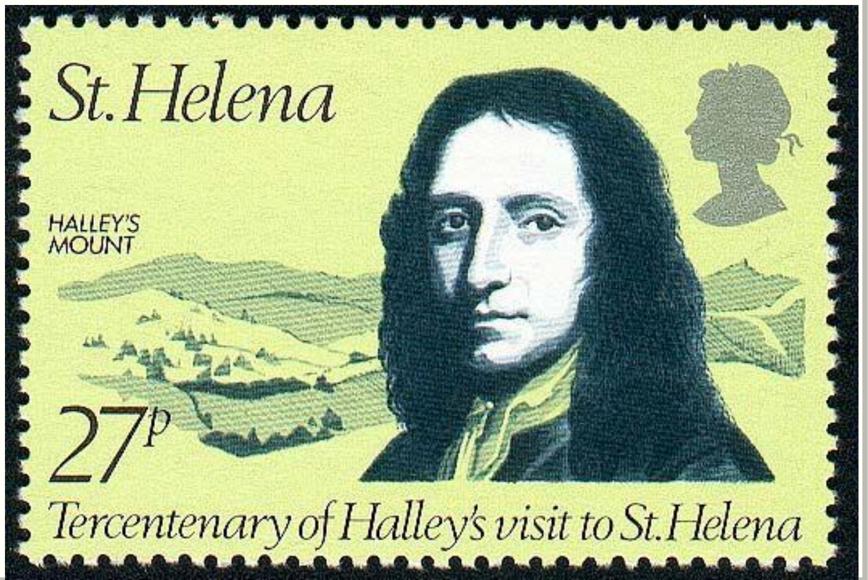


(1638--1675)

(1656—1742)





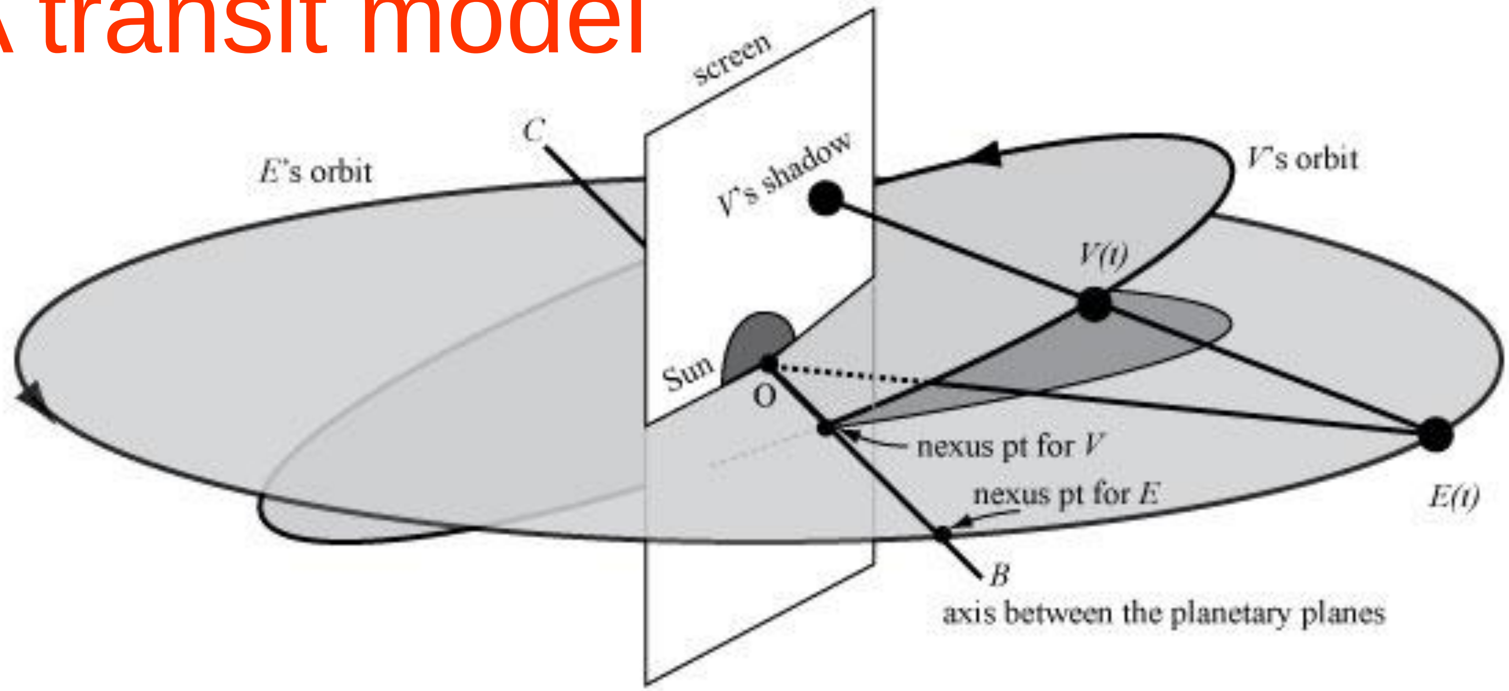


$$T_e = 365.26 \text{ days} \qquad T_v = 224.70 \text{ days}$$

$$\sigma = T_e/T_v \approx 1.62555 \qquad a^3 = T^2$$

$$\lambda = 0.723 \text{ AU} \qquad \mu = 3.39^\circ$$

A transit model



$$E(t) = \begin{bmatrix} \cos(2\pi t) \\ \sin(2\pi t) \\ 0 \end{bmatrix} \qquad P(u,t) = (V(t) - E(t))u + E(t)$$

$$X \cdot E(t) = 0$$

$$V(t) = \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\mu & -\sin\mu \\ 0 & \sin\mu & \cos\mu \end{bmatrix} \begin{bmatrix} \cos(2\pi\sigma t) \\ \sin(2\pi\sigma t) \\ 0 \end{bmatrix}$$

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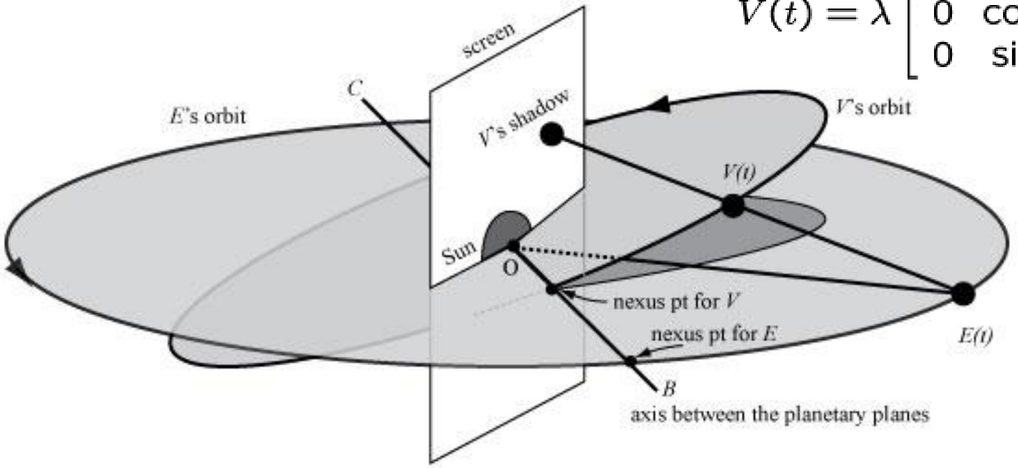
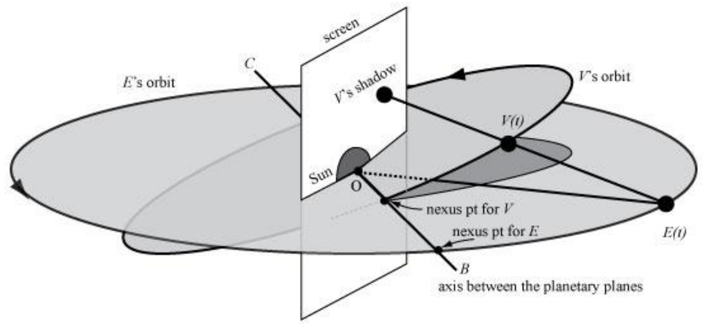


$$\begin{cases} x = (\lambda\cos(2\pi\sigma t) - \cos(2\pi t))u + \cos(2\pi t) \\ y = (\lambda\cos\mu\sin(2\pi\sigma t) - \sin(2\pi t))u + \sin(2\pi t) \\ z = \lambda\sin\mu\sin(2\pi\sigma t)u \\ 0 = x\cos(2\pi t) + y\sin(2\pi t) \end{cases}$$



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$$\widehat{X}(t) = (x,\ y,\ z,\ u)$$

$$A\widehat{X}(t) = \hat{E}(t)$$

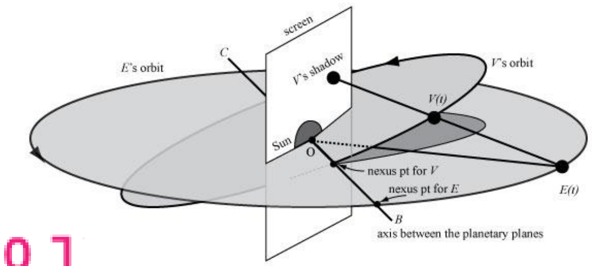
$$\hat{E}(t) = (\cos(2\pi t), \sin(2\pi t),\ 0,\ 0)$$

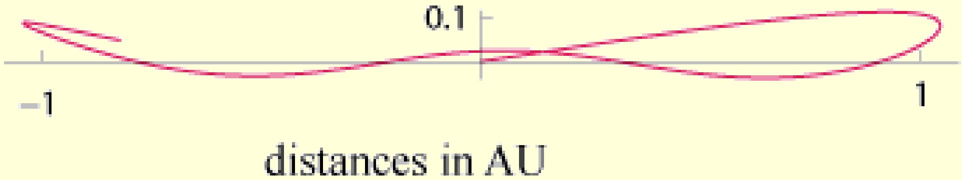
$$A = \begin{bmatrix} 1 & 0 & 0 & \cos(2\pi t) - \lambda\cos(2\pi\sigma t) \\ 0 & 1 & 0 & \sin(2\pi t) - \lambda\cos\mu\sin(2\pi\sigma t) \\ 0 & 0 & 1 & -\lambda\sin\mu\sin(2\pi\sigma t) \\ \cos(2\pi t) & \sin(2\pi t) & 0 & 0 \end{bmatrix}$$

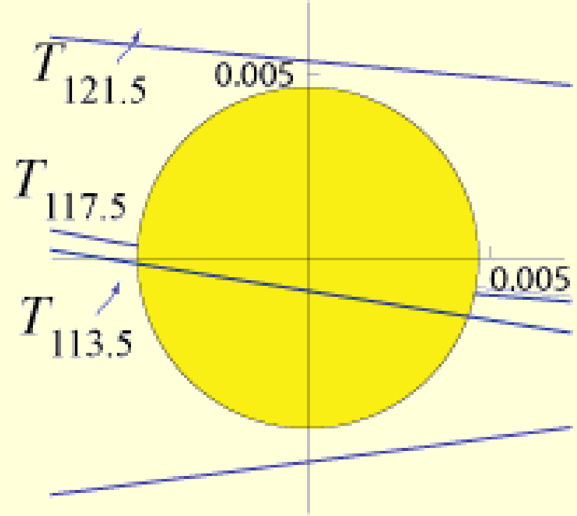
$$\widehat{X}(t) = A^{-1}\hat{E}(t)$$

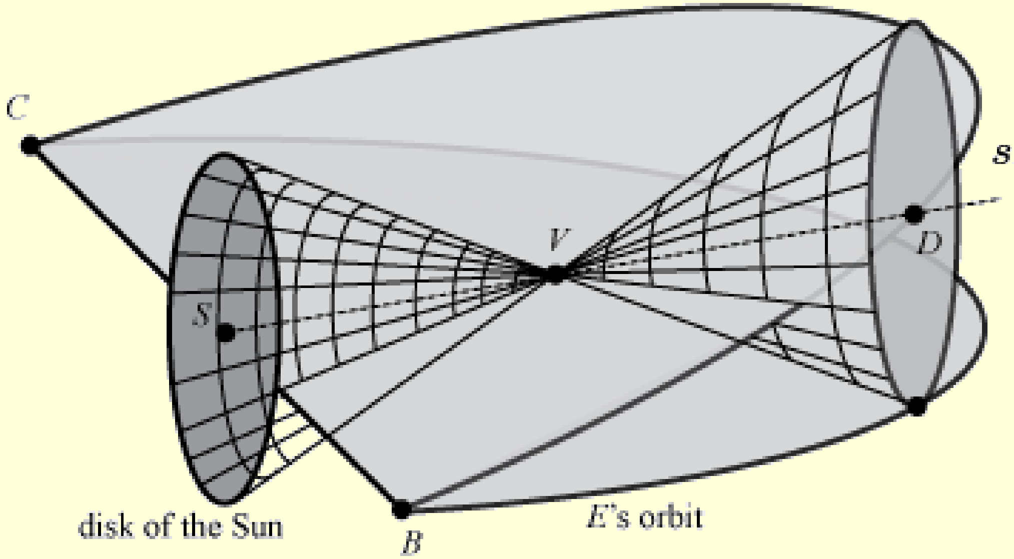
$$W(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \cos(2\pi t) & \sin(2\pi t) & 0 & 0 \\ -\sin(2\pi t) & \cos(2\pi t) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A^{-1}\hat{E}(t)$$

$$W(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \cos(2\pi t) & \sin(2\pi t) & 0 & 0 \\ -\sin(2\pi t) & \cos(2\pi t) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A^{-1}\widehat{E}(t)$$









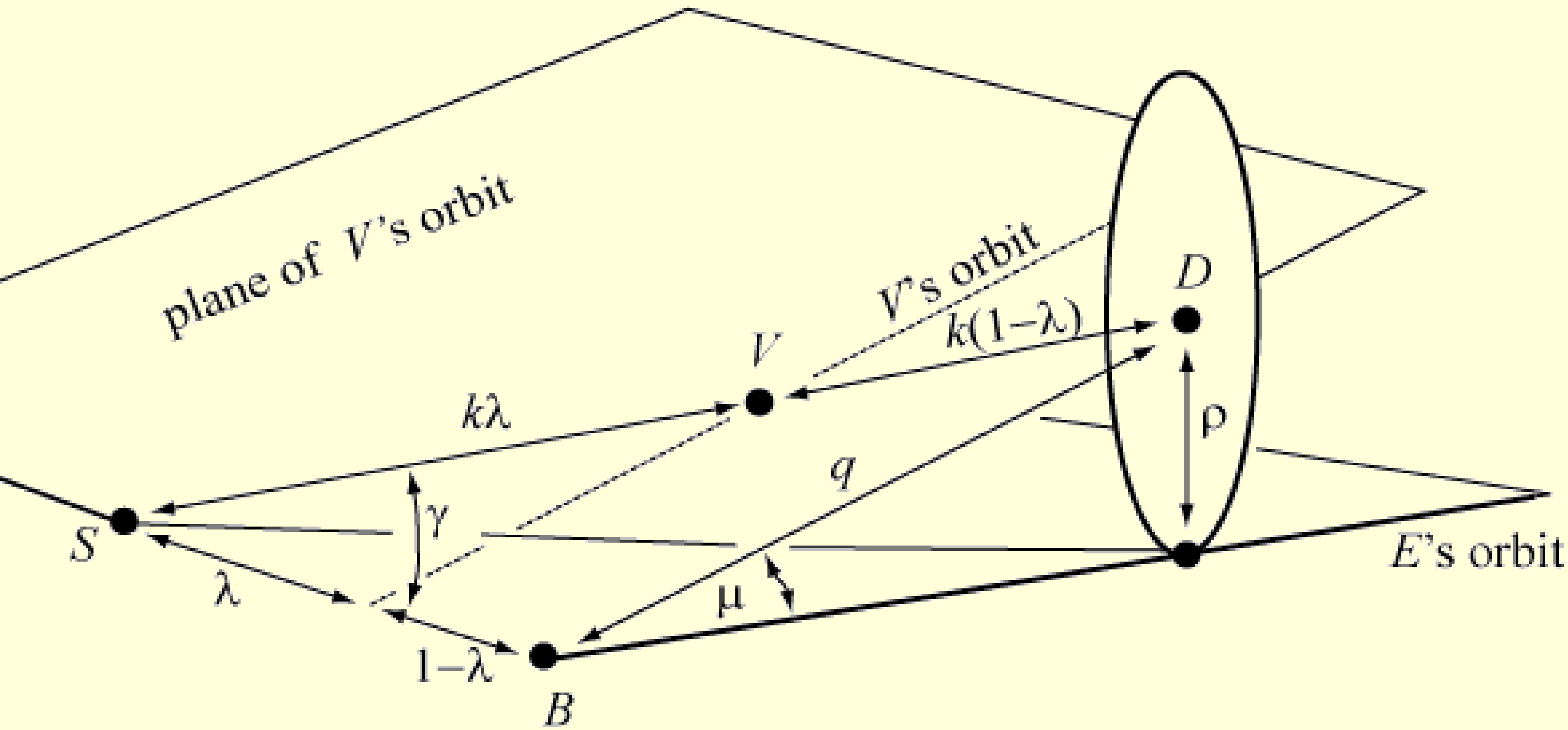
$s \approx 0.00465$ AU

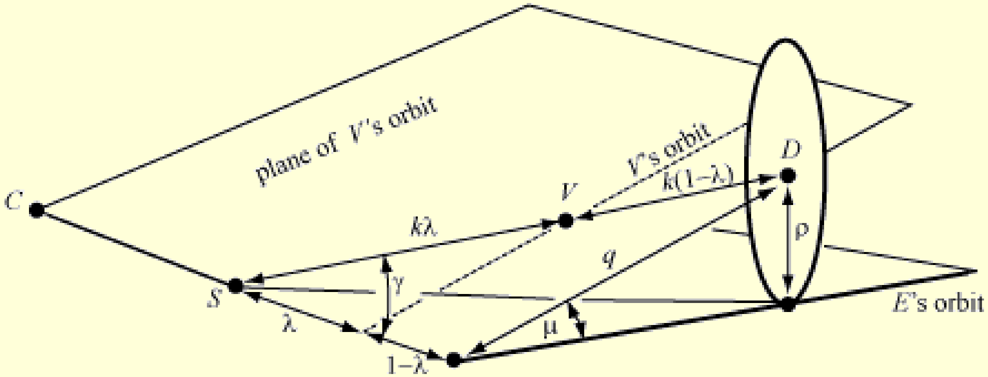
$$\frac{s}{k\lambda} = \frac{\rho}{k(1-\lambda)}$$

$\rho \approx 0.0178$ AU

$$\sin\mu = \frac{\rho}{q} \qquad \tan\gamma = q$$

$$\gamma = \tan^{-1}\left(\frac{s(1-\lambda)}{\lambda \sin\mu}\right) \approx \frac{s(1-\lambda)}{\lambda\mu} \approx 0.0301$$

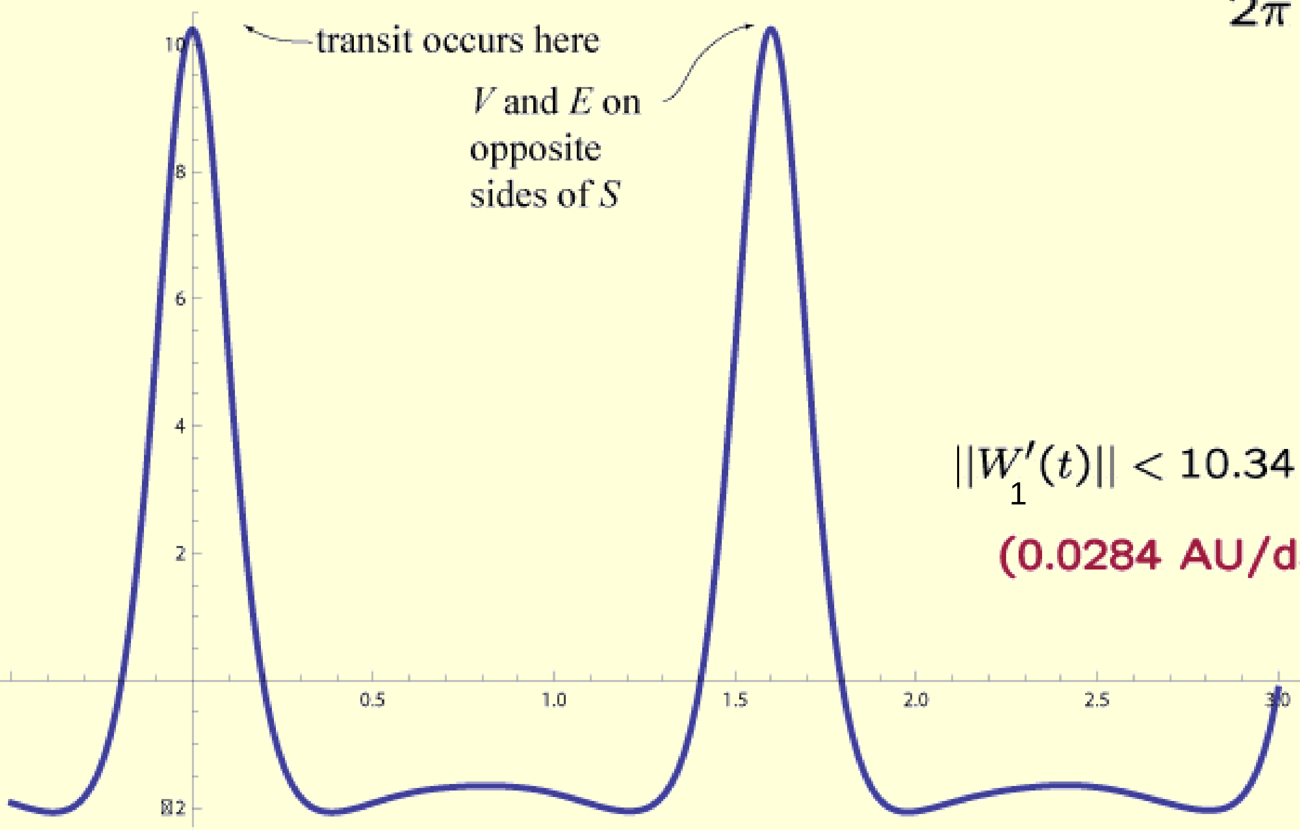


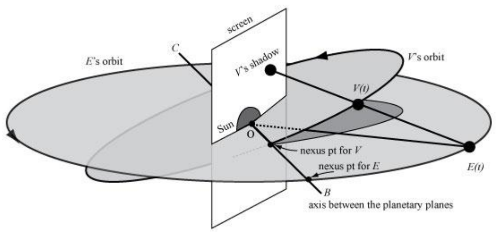


$$\gamma \approx \frac{s(1-\lambda)}{\lambda\mu}$$

$$L_v \approx \frac{s(1-\lambda)}{2\pi\lambda\sigma\mu} \approx 1.08 \text{ days}$$

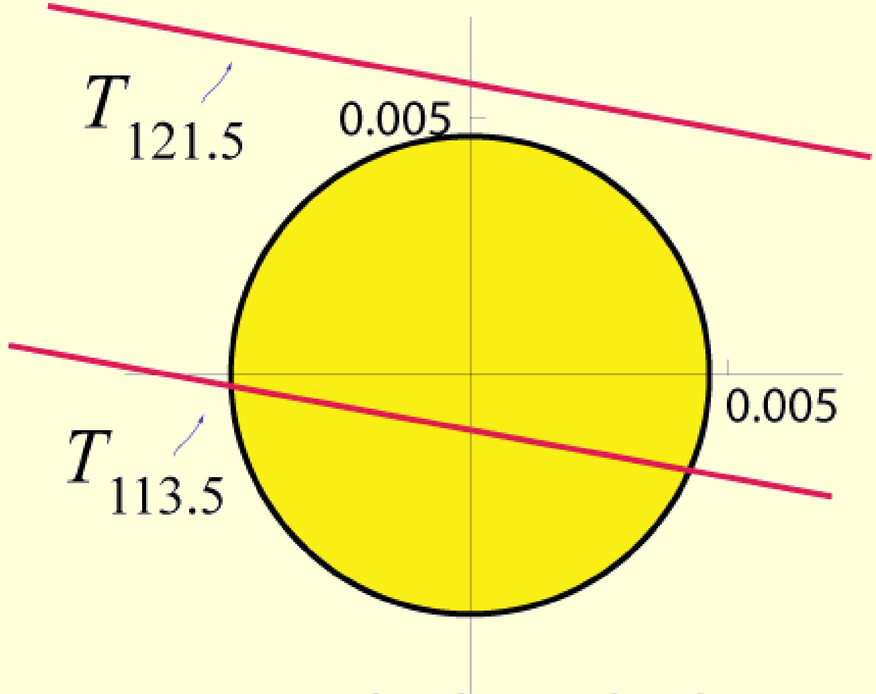
$$L_e = \frac{\gamma}{2\pi} \approx 42 \text{ hours} < 2 \text{ days}$$





$$||W_1'(t)|| < 10.34 \text{ AU/year} < 0.0284 \text{ AU/day}$$

$$(0.0284 \text{ AU/day})(42 \text{ hours}) < 0.05\text{AU}$$



$$W(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \cos(2\pi t) & \sin(2\pi t) & 0 & 0 \\ -\sin(2\pi t) & \cos(2\pi t) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A^{-1}\widehat{E}(t)$$

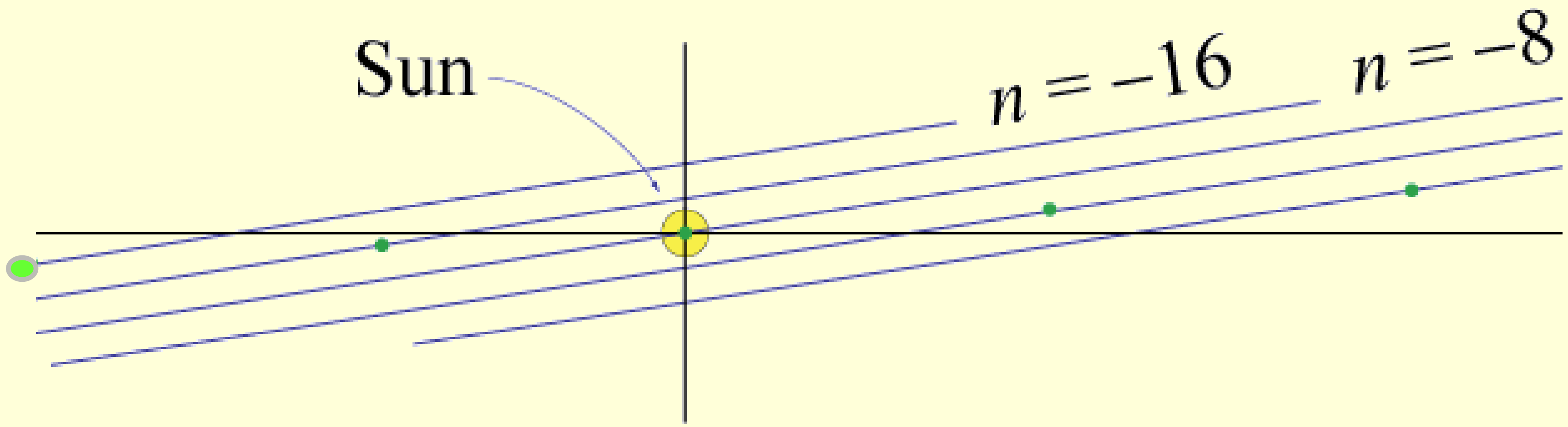
$$||W(n)|| < .05 \text{ or } ||W(n+\frac{1}{2})|| < 0.05$$

0	113.5	227	(340.5, 348.5)	(454, 462)	575.5	689
802.5	916	1029.5	(1143, 1151)	(1256.5, 1264.5)	1378	1491.5
1605	1718.5	1832	(1945.5, 1953.5)			

But, we seek a *more natural period* b/c

- How is 1605 related to σ ?
- How does 1605 give the time lapse between transits?
- How does 8 related to 1605?

branch j	0	1	2	3	4	5	6	7	0
transit year n	0	227	(454, 462)	689	916	(1143, 1151)	1378	1605	1832
n mod 8	0	3	6	1	4	7	2	5	0
$3j$ mod 8	0	3	6	1	4	7	2	5	0



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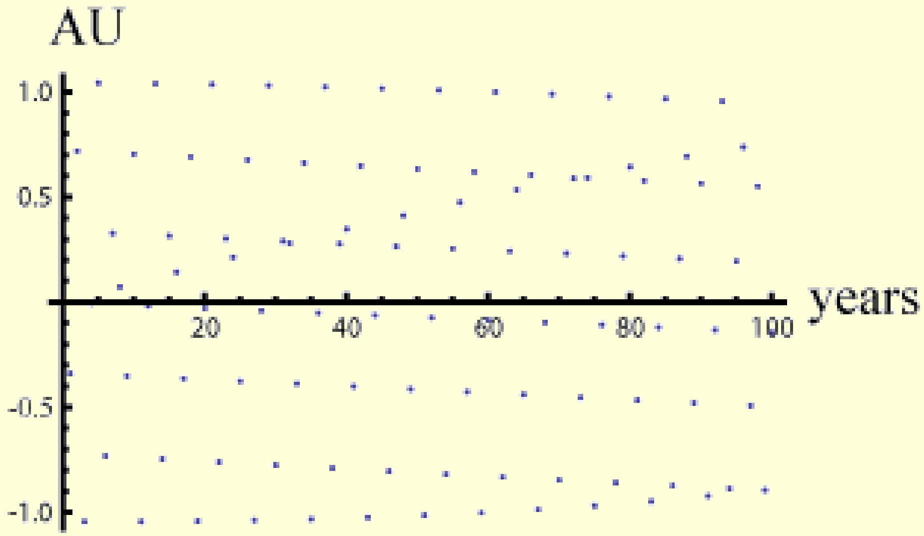
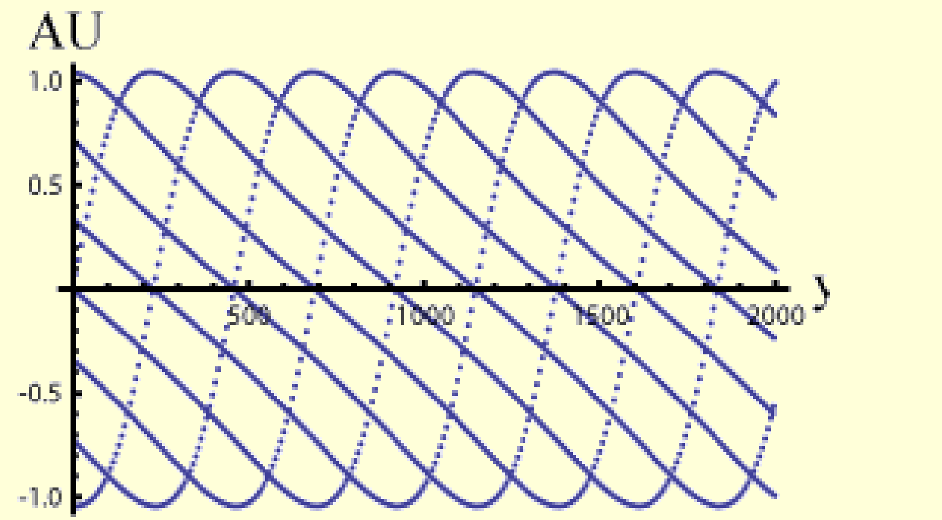
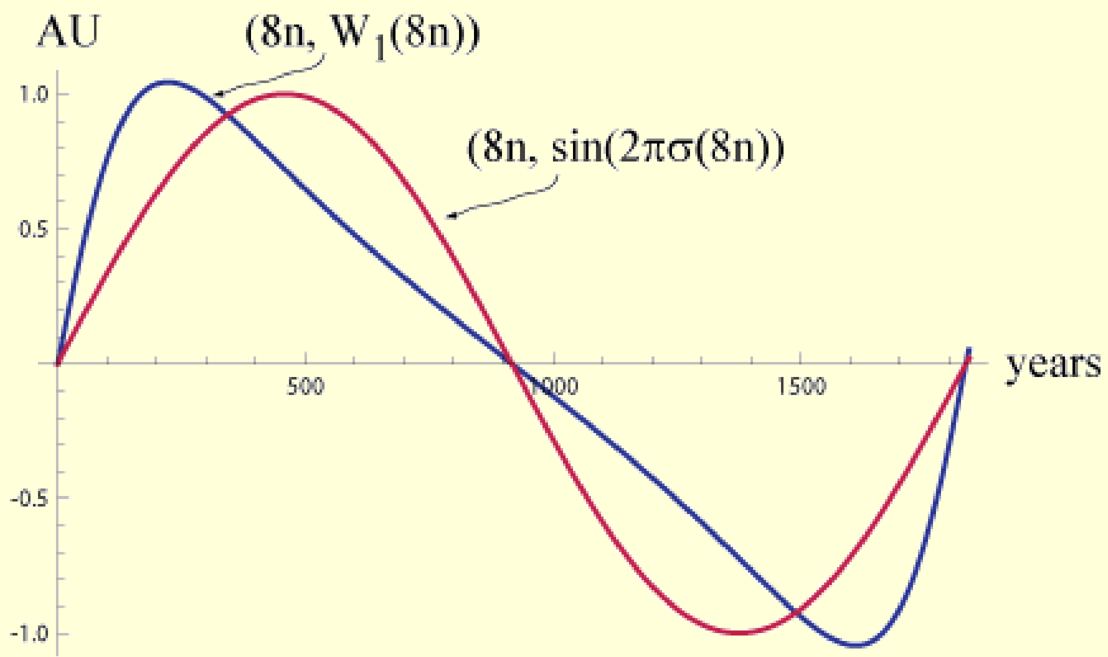
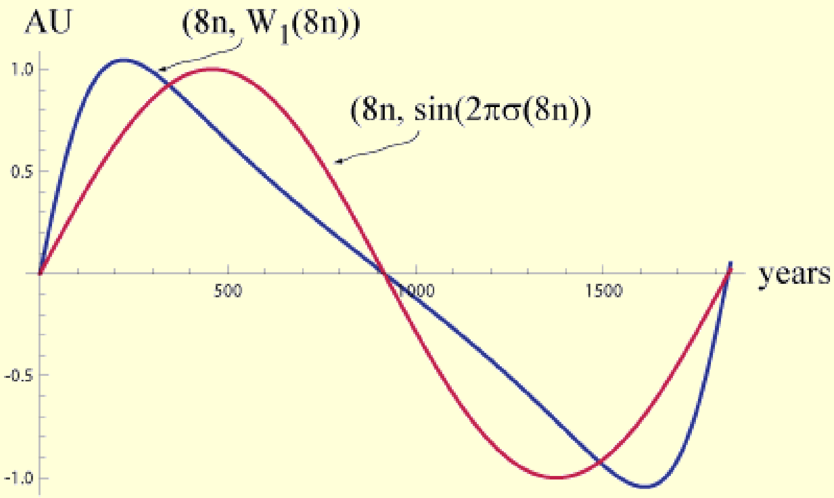








Find $y_j = \sin(\alpha(t - \beta j))$

$$(2\pi\sigma)(8) = (2\pi)(13) + \psi \qquad \psi \approx 0.0274 < \frac{\pi}{2}$$

$$\sin(\alpha(8n)) = \sin\left(\left(\frac{\sin^{-1}(\sin((2\pi\sigma)(8)))}{8}\right)(8n)\right) = \sin((2\pi\sigma)(8n))$$

$$\alpha = \frac{\sin^{-1}(\sin(16\pi\sigma))}{8} = \frac{\psi}{8} \qquad \sigma - \frac{13}{8} = \frac{\alpha}{2\pi}$$

$$T = \frac{2\pi}{\alpha} \approx 1834.29 \text{ years}$$

$$\frac{1605}{7} \approx 229.286 \approx \beta$$

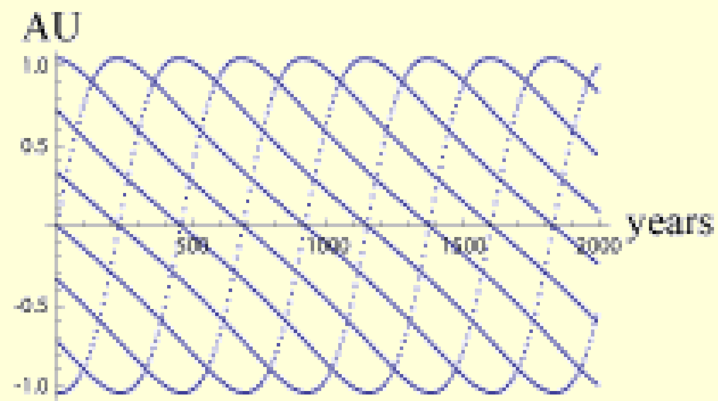
$$\beta = \frac{T}{8} \approx 229.286 \text{ years}$$

1605 ≡ a *lucky seven* multiple of β

Residue corresponding to branch-1:

$$\sin(2\pi\sigma q) = \sin(\alpha\, q - \frac{2\pi}{8}) \longrightarrow q = 3$$

y_1 passes through data for years $8n + 3$



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