LINE: Linear Algebra in New Environments

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Guiding Question:

How can the work of mathematics education researchers influence the teaching and learning of undergraduate mathematics?

NDSU Module 2

1. (a) Give an example of a linear map $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that $T(2,3) = (7,8)$ and $T(3,4) = (10,11)$. (If such a linear map does not exist, explain why.) For your example, does there exist $(a,b) \in \mathbb{R}^2$ such that $T(a,b) = (1,1)$?

   (b) How many linear maps $T : \mathbb{R}^2 \to \mathbb{R}^2$ with $T(2,3) = (7,8)$ and $T(3,4) = (10,11)$ exist?

2. (a) Give an example of a linear map $T : \mathbb{R}^3 \to \mathbb{R}^2$ such that $T(2,3,1) = (7,8)$ and $T(3,4,2) = (10,11)$. (If such a linear map does not exist, explain why.)

   (b) How many linear maps $T : \mathbb{R}^3 \to \mathbb{R}^2$ with $T(2,3,1) = (7,8)$ and $T(3,4,2) = (10,11)$ exist?

NDSU Module 2

3. Let U, V be finite dimensional vector spaces over $\mathbb{F}$ and let $(u_1, \ldots, u_n)$ be a list of linearly independent vectors in U and $v_1, \ldots, v_n \in V$. Prove that there exists a linear map $T : U \to V$ such that $T(u_i) = v_i$ for all $i = 1, \ldots, n$.

4. Let U, V be finite dimensional vector spaces over $\mathbb{F}$ and let $(u_1, \ldots, u_n)$ be a list of vectors in U. Assume that for every $v_1, \ldots, v_n \in V$ there exists a linear map $T : U \to V$ such that $T(u_i) = v_i$ for all $i = 1, \ldots, n$. Prove that the vectors $u_1, \ldots, u_n$ are linearly independent.

5. Let U, V be finite dimensional vector spaces over $\mathbb{F}$, W a subspace of U, and let $T : W \to V$ be a linear map. Prove that there exists a linear map $S : U \to V$ that extends T. (S extends T means $S(w) = T(w)$ for all $w \in W$.)

APOS

- An **action** is a transformation of a mathematical object according to an explicit algorithm seen as externally driven. It may be a manipulation of objects or acting upon a memorized fact.
- When one reflects upon an action, constructing an internal operation for a transformation, the action begins to be interiorized. A **process** is this internal transformation of an object. Each step may be described or reflected upon without actually performing it.
- A person may reflect on actions applied to a particular process and become aware of the process as a totality. At this point, the individual has reconstructed a process as a cognitive **object**. In this case we say that the process has been encapsulated into an **object**.
- A **schema** is a collection of actions, processes, objects, and other previously constructed schemata which are coordinated and synthesized to form mathematical structures utilized in problem situations.

Other Modules

- Systems of Equations
- Vector Spaces and Subspaces
- Linear Independence, Span
- Basis, Change of Basis, Dimension
- Inner Products
- Operators on Complex Vector Spaces

References

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