A Nickel and Dime Example

David Strong
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A Nickel and Dime Example

(Actually nickels and pennies)

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Pepperdine University

Where used?

- First or second day of class.
- In discrete math (including linear algebra) for business students
- Also used in “real” linear algebra
- This talk is basically what students see in class

Example: coins

- Given some coins, say 15 pennies and 6 nickels, determine how many pennies and nickels are needed to satisfy one, two or all three of the given conditions:
  - The total number of coins you have is 6.
  - You have five times as many pennies as nickels.
  - Your coins add up to a total of 30 cents.
- Use your handout.

Name: ____

Suppose you had 15 pennies and 6 nickels. Find a solution to each of the following seven problems by finding how many *pennies* and *nickels* would be needed to satisfy the condition(s) **in bold**.

Pennies	Nickels	
		The total number of coins you have is 6. You have five times as many pennies as nickels. Your coins add up to a total of 30 cents.
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Suppose you had 15 pennies and 6 nickels. Find a solution to each of the following seven problems by finding how many *pennies* and *nickels* would be needed to satisfy the condition(s) **in bold**.

Pennies **Nickels**

The total number of coins you have is 6.
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Pennies/Nickels

0/6, 1/5, 2/4, 3/3, 4/2, 5/1, …

Name: ____

Suppose you had 15 pennies and 6 nickels. Find a solution to each of the following seven problems by finding how many *pennies* and *nickels* would be needed to satisfy the condition(s) **in bold**.

Pennies/Nickels

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0/6, 1/5, 2/4, 3/3, 4/2, 5/1, …

0/0, 5/1, 10/2, 15/3, …

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0/0, 5/1, 10/2, 15/3, …

0/6, 5/5, 10/4, 15/3, …

5/1

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Pennies	Nickels	Conditions	Pennies/Nickels
		The total number of coins you have is 6. You have five times as many pennies as nickels. Your coins add up to a total of 30 cents.	(0/6) 1/5, 2/4, 3/3, 4/2, 5/1, …
		The total number of coins you have is 6. **You have five times as many pennies as nickels.** Your coins add up to a total of 30 cents.	0/0, 5/1, 10/2, 15/3, …
		The total number of coins you have is 6. You have five times as many pennies as nickels. **Your coins add up to a total of 30 cents.**	(0/6) 5/5, 10/4, 15/3, …
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Restrictions/relationships = Equations

- Each **restriction** (relationship between unknowns) corresponds to an **equation**.
- Where: p is number of pennies

  n is number of nickels

Restriction/relationship	Equation
Total number of coins is 6.	$p + n = 6$
Five times as many pennies as nickels.	$p - 5n = 0$
Coins add up to 30 cents.	$p + 5n = 30$

Functions vs. relationships

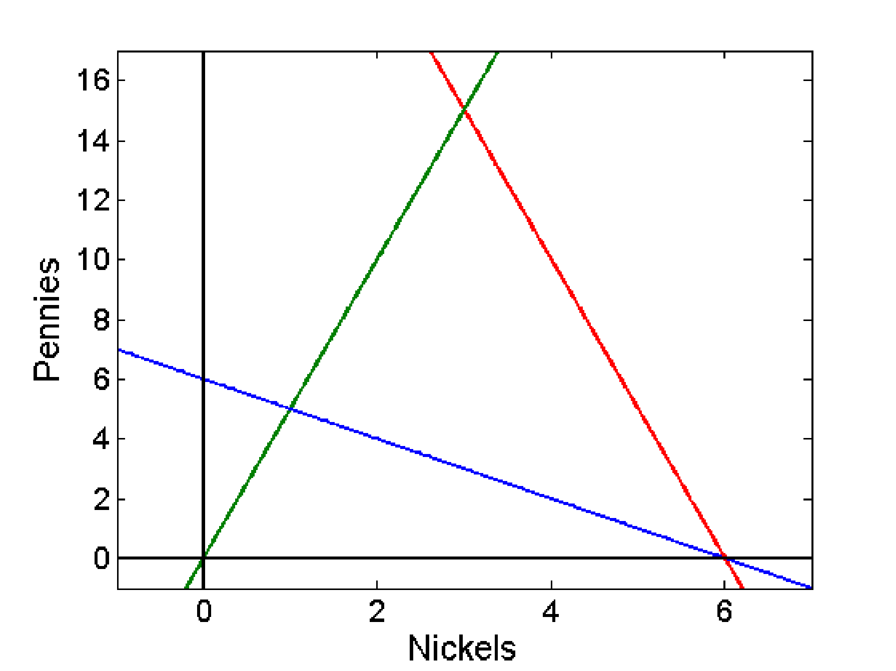
- Sometimes there is simply a relationship between the two values (variables).
  That is, one value is not really a function of the other. This is the case for the pennies and nickels equations.
- Sometimes one value (variable) really is a function of the other, in which case we can solve for one variable in terms of the other. We can do that here as well.

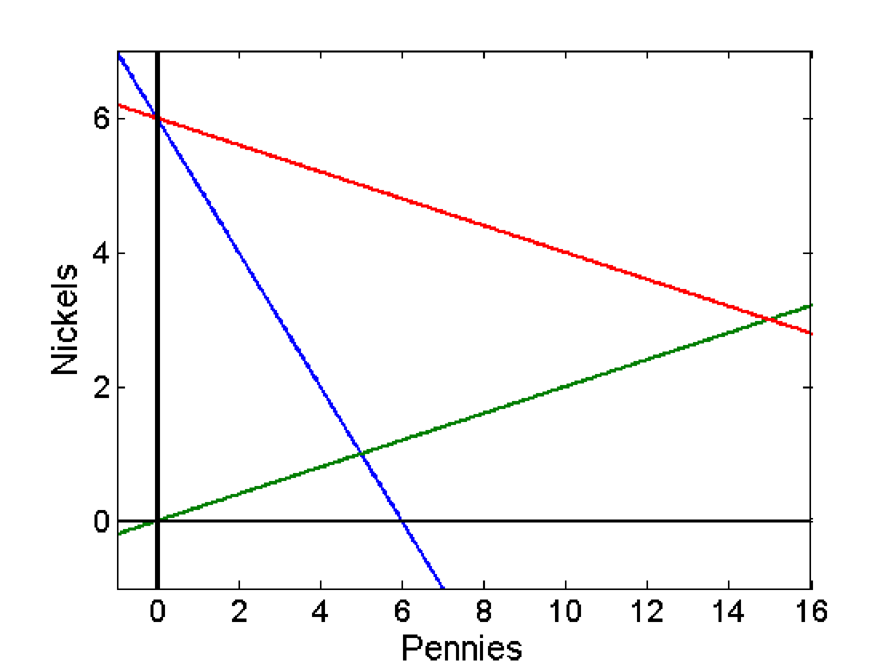
Equations in **standard** form

$$p + n = 6$$

$$p - 5n = 0$$

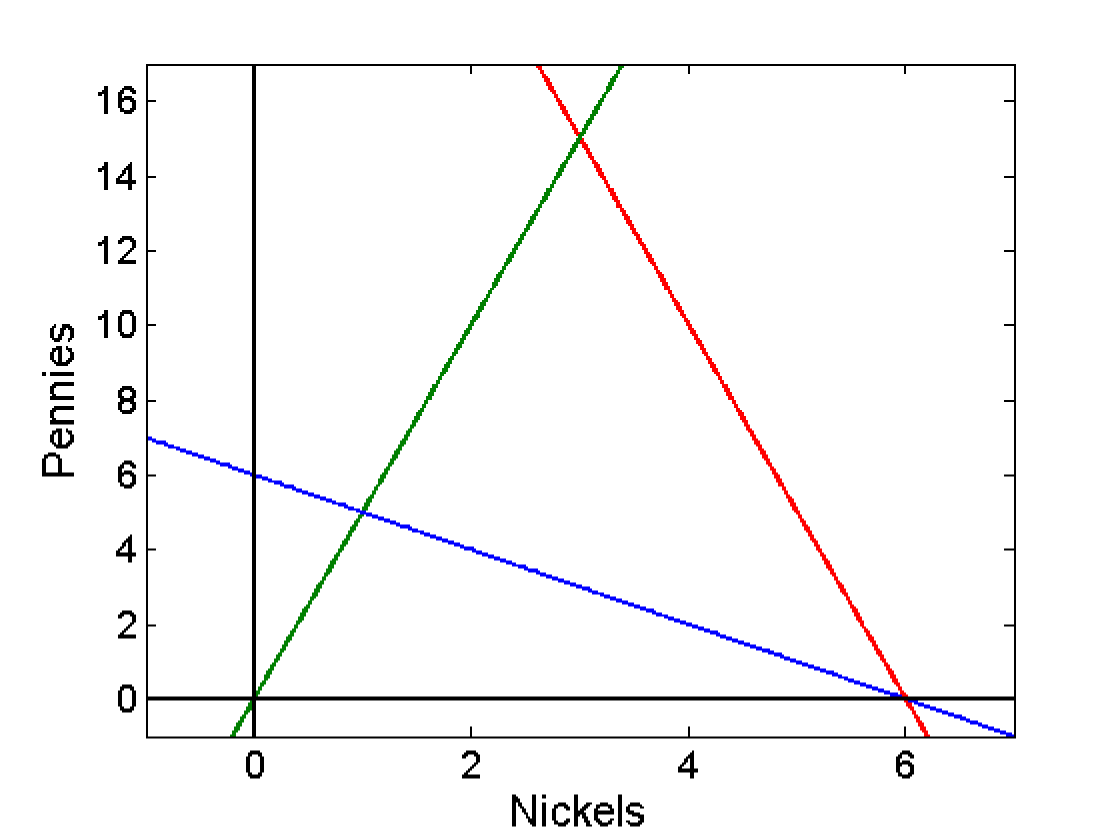
$$p + 5n = 30$$





Equations with ***p*** in terms of ***n***

$$\begin{aligned} p + n &= 6 \\ p - 5n &= 0 \\ p + 5n &= 30 \end{aligned} \quad \Longrightarrow \quad \begin{aligned} p &= -n + 6 \\ p &= 5n \\ p &= -5n + 30 \end{aligned}$$

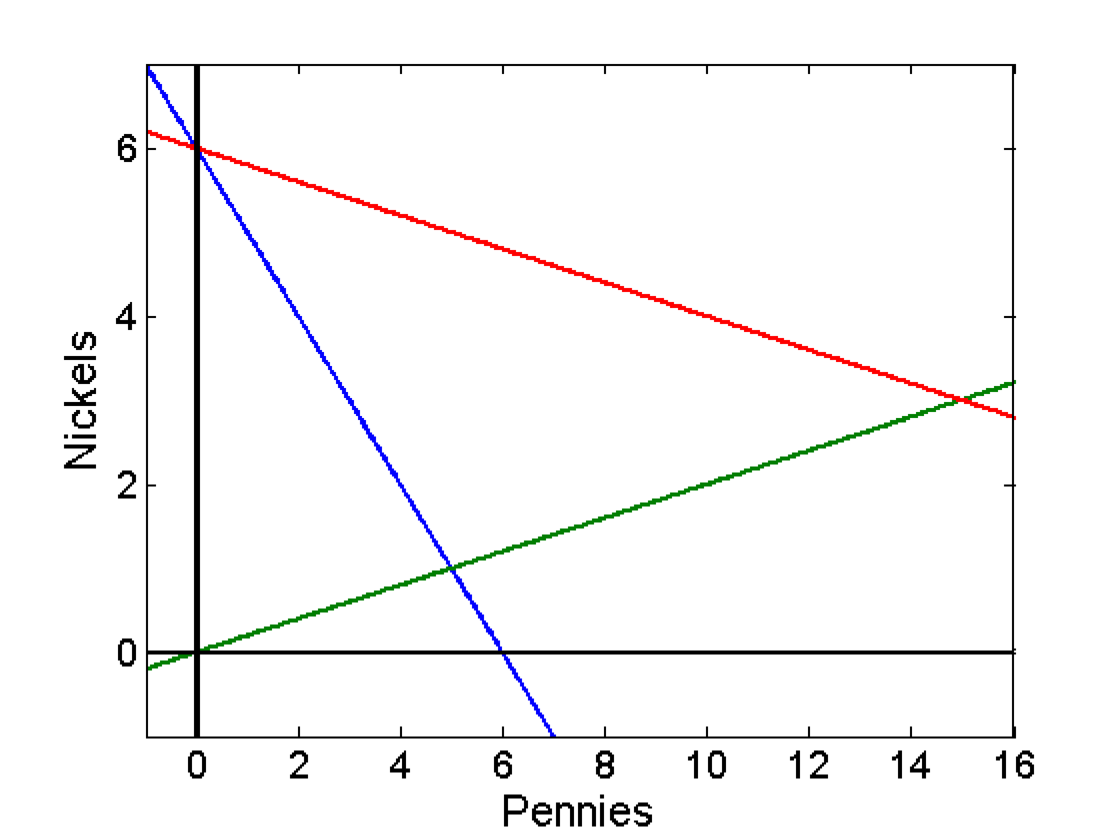


Equations with ***n*** in terms of ***p***

$$p + n = 6 \quad\Rightarrow\quad n = -p + 6$$

$$p - 5n = 0 \quad\Rightarrow\quad n = \tfrac{1}{5}p$$

$$p + 5n = 30 \quad\Rightarrow\quad n = -\tfrac{1}{5}p + 6$$



Equations in **standard** form

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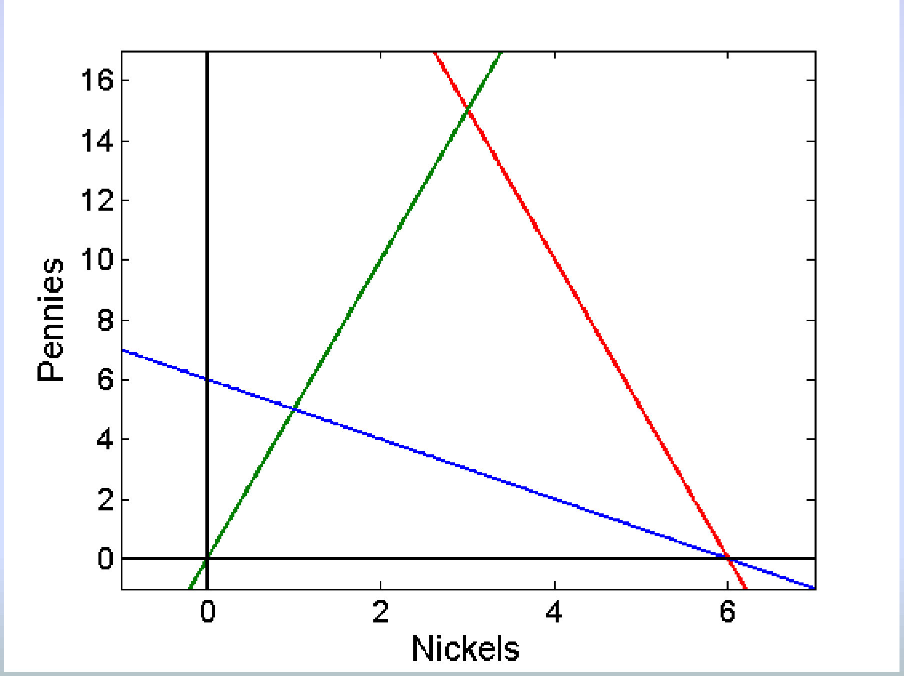

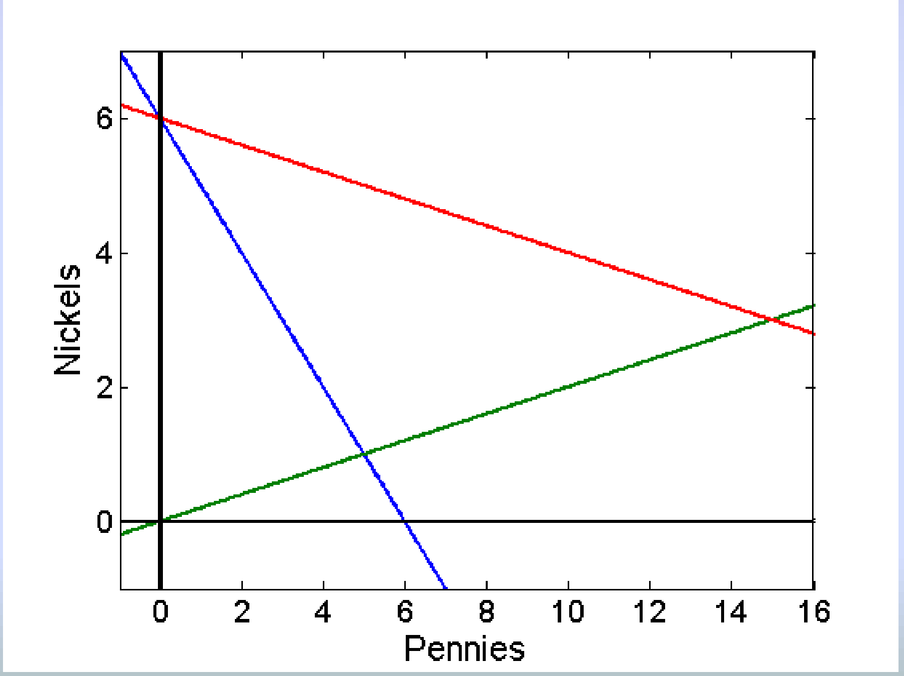

Number of equations vs. number of unknowns

- Each **unknown** is a **degree of freedom**, each **equation** is a **restriction**.
- In general, if there are the **same** number of equations (restrictions, conditions) as unknowns (variables), then there is **one** solution.

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- Each **unknown** is a **degree of freedom**, each **equation** is a **restriction**.
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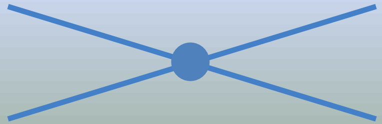
- Example: since an equation with two variables is a line, then two equations with two variables are two lines, and usually two lines intersect at a single point: the single solution. So if there are two equations each with two variables, there will generally be a single solution.



Number of equations vs. number of unknowns

- In general, where

  m = number of equations

  n = number of unknowns

  then:

m vs. n	# solutions	# equations	For n = 2

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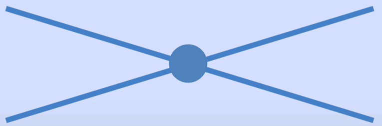
$m = n$	1	Just right	

Number of equations vs. number of unknowns

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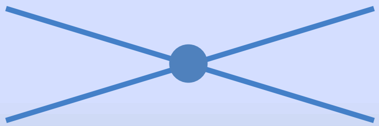
m vs. *n*	# solutions	# equations	For n = 2
$m > n$	0	Too many	
$m = n$	1	Just right	

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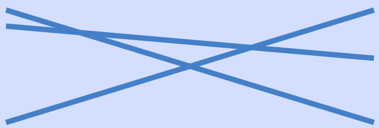
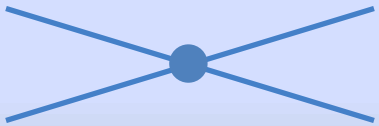
$m > n$	0	Too many	
$m = n$	1	Just right	

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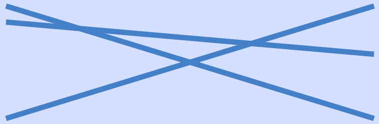
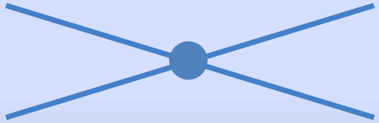
$m > n$	0	Too many	
$m = n$	1	Just right	
$m < n$	∞	Not enough	

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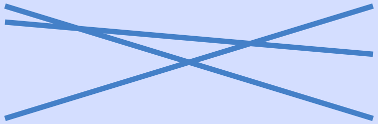
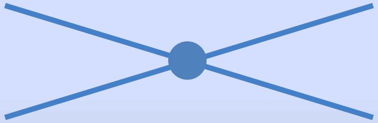
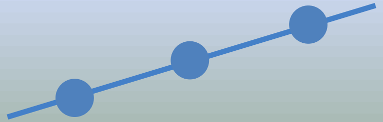
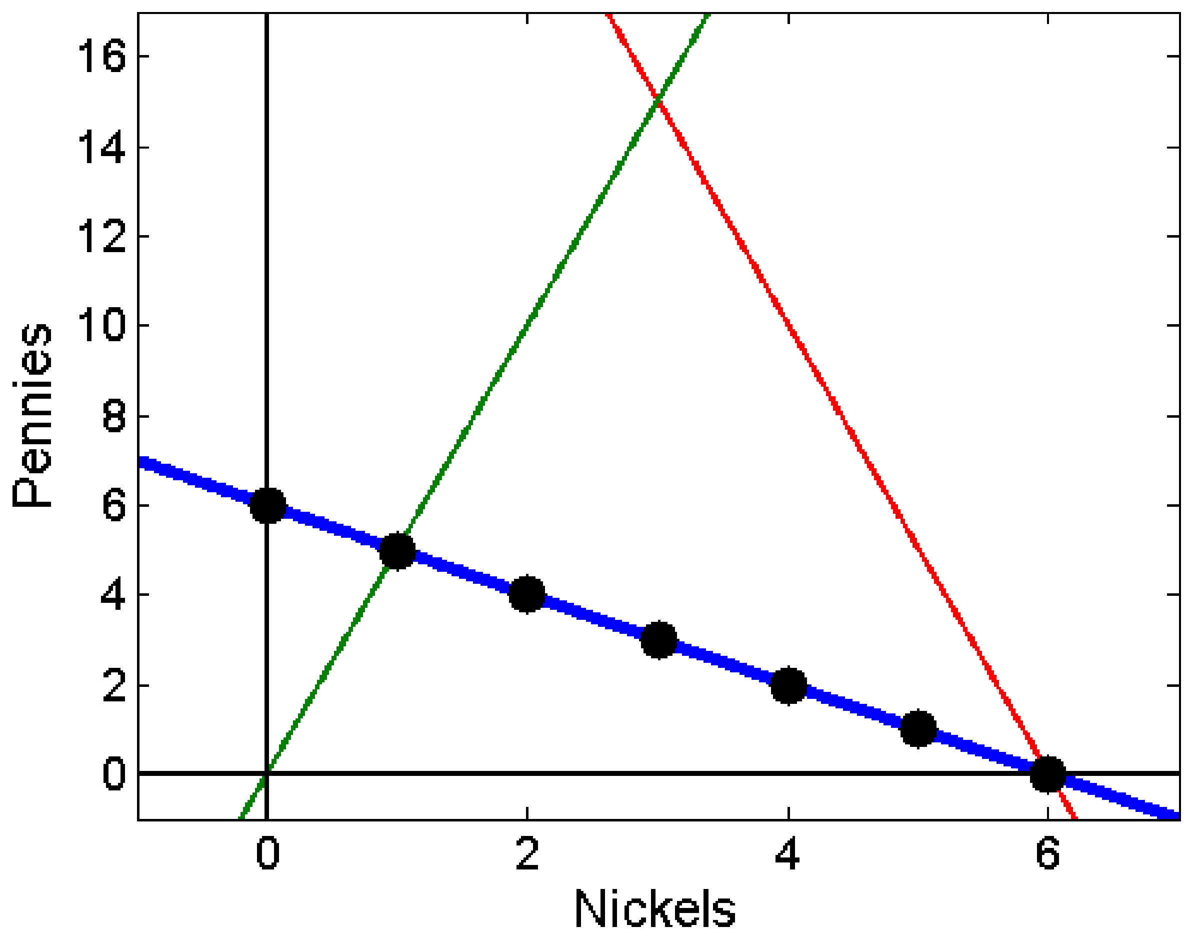
$m > n$	0	Too many	
$m = n$	1	Just right	
$m < n$	∞	Not enough	

$$p + n = 6$$

$$p - 5n = 0$$

$$p + 5n = 30$$



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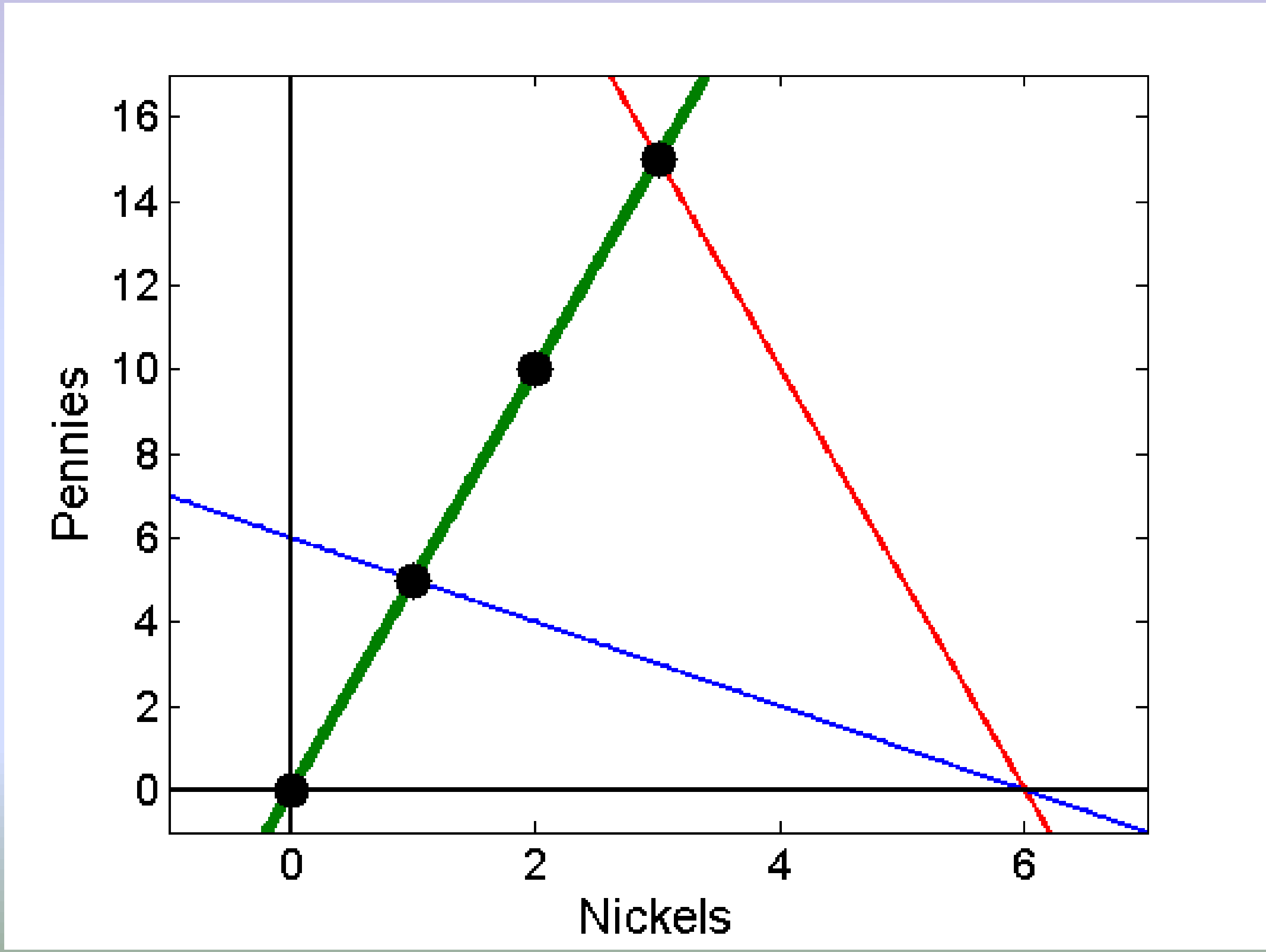

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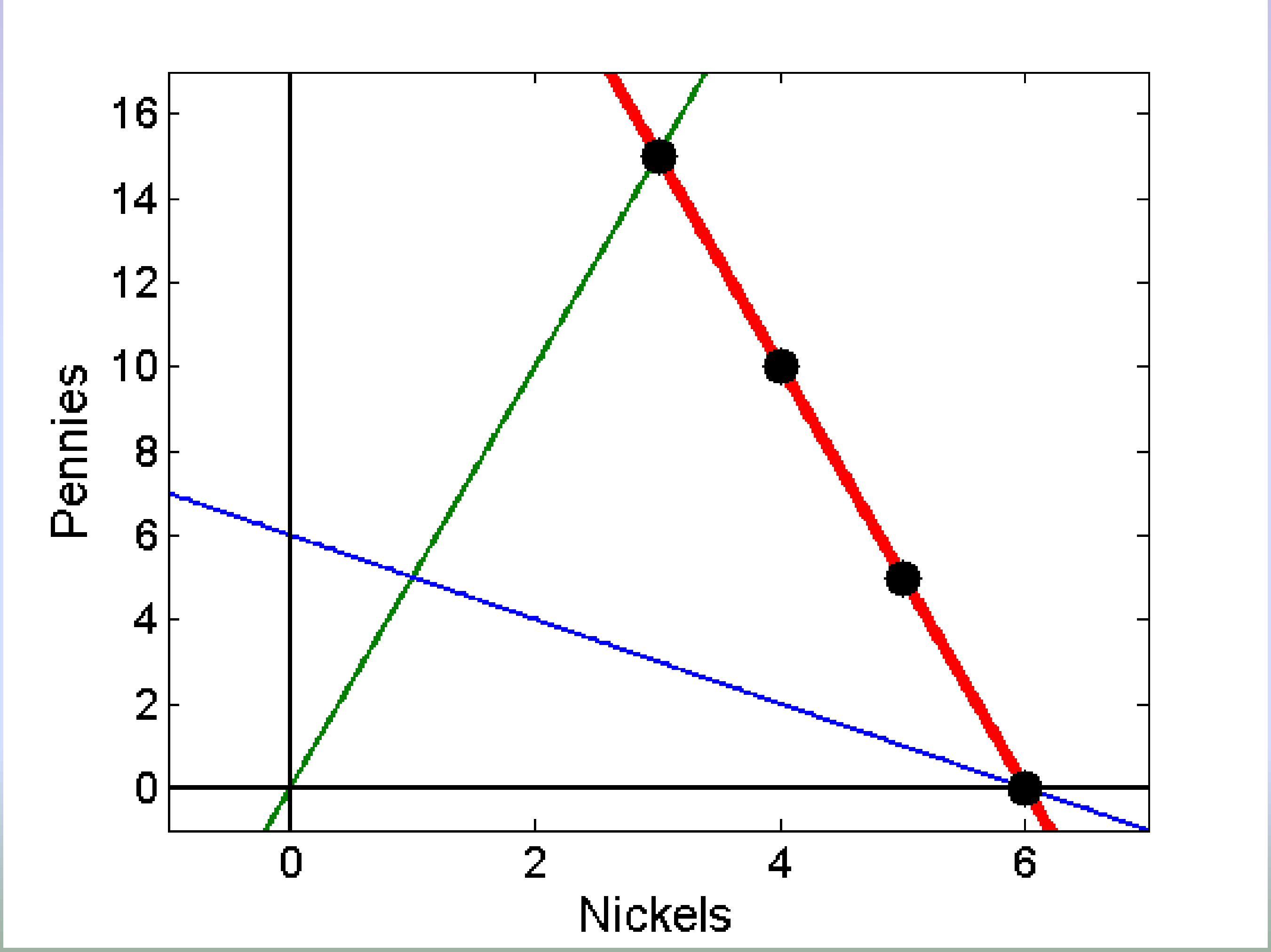

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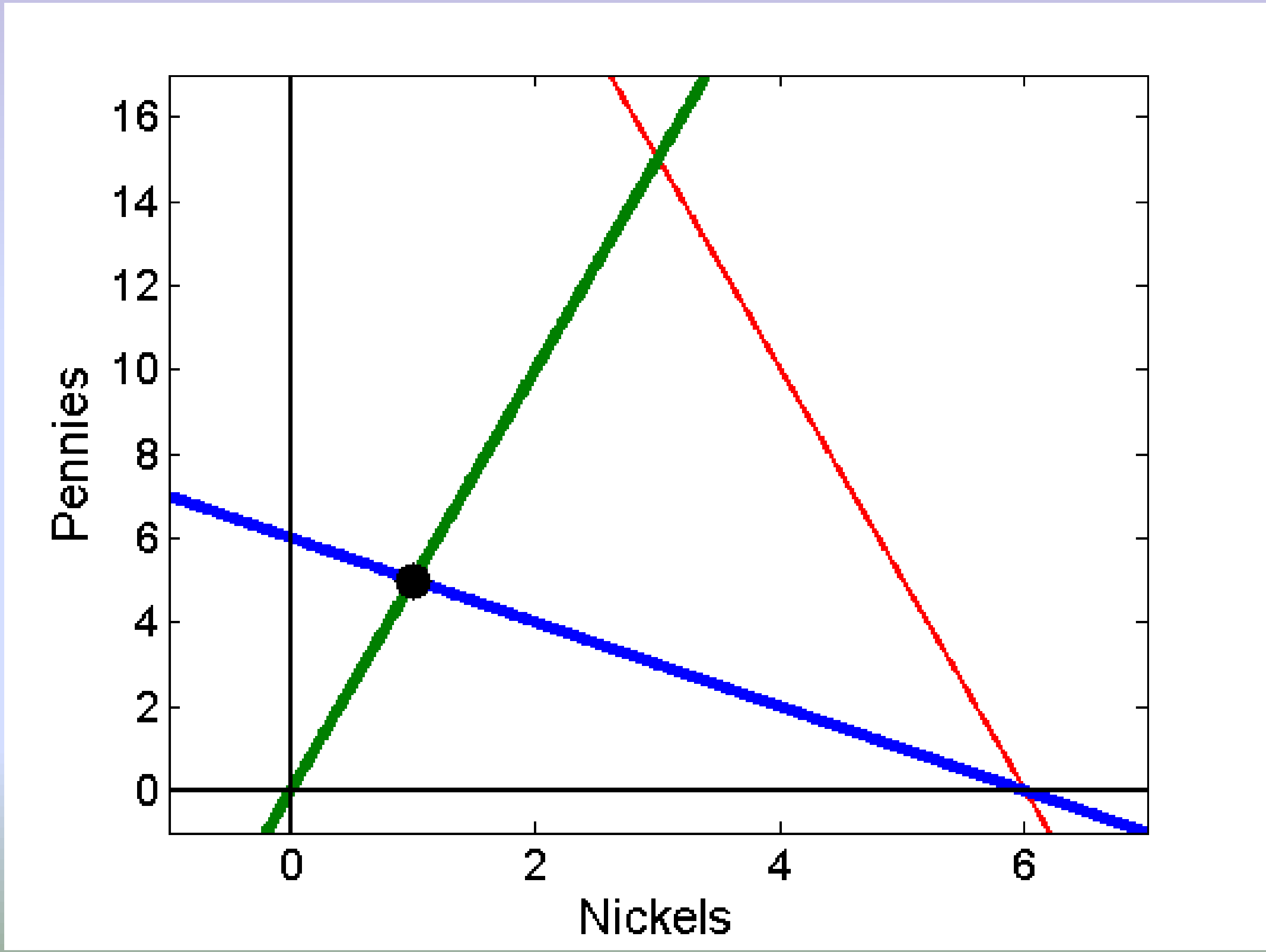

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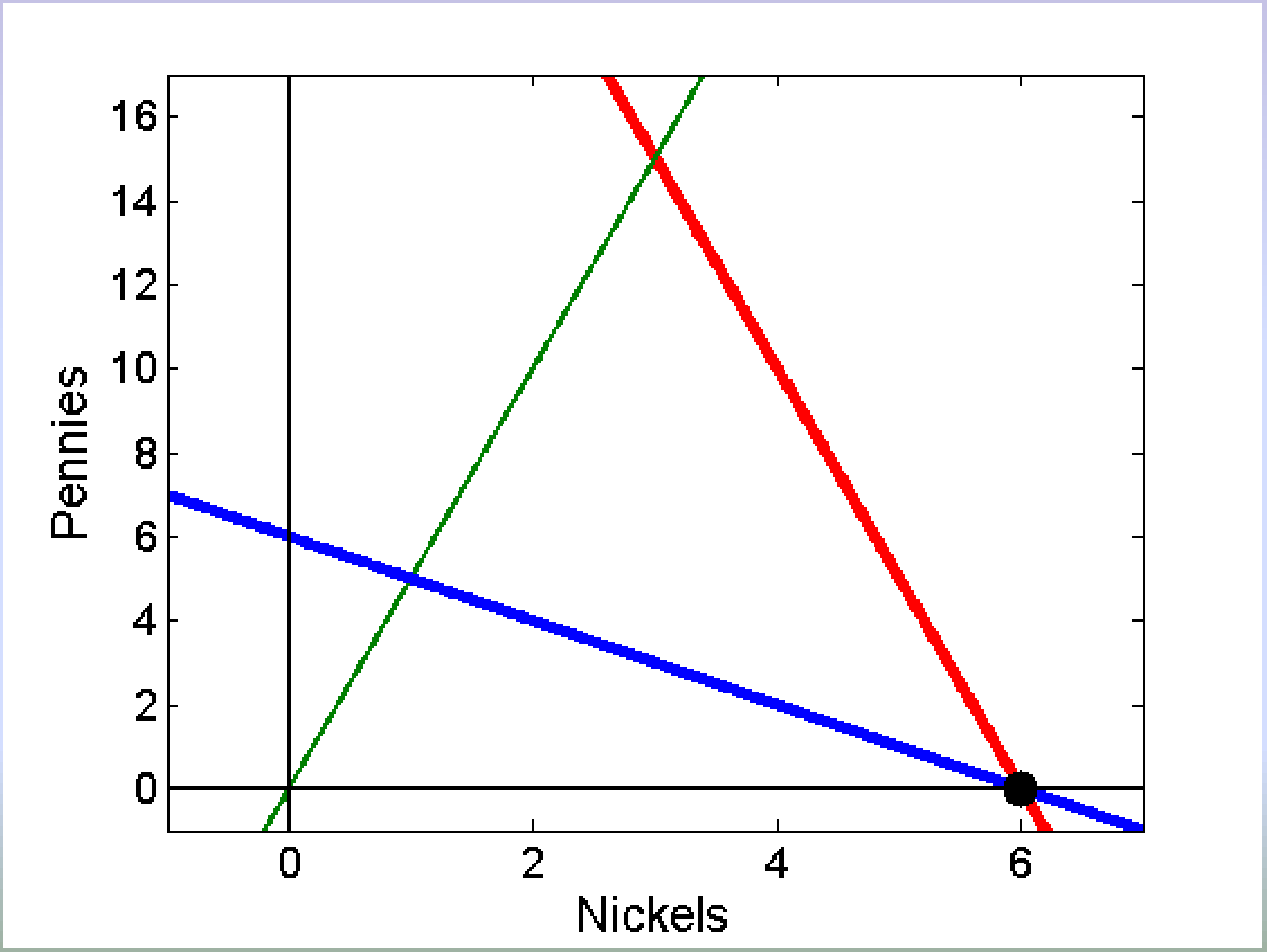

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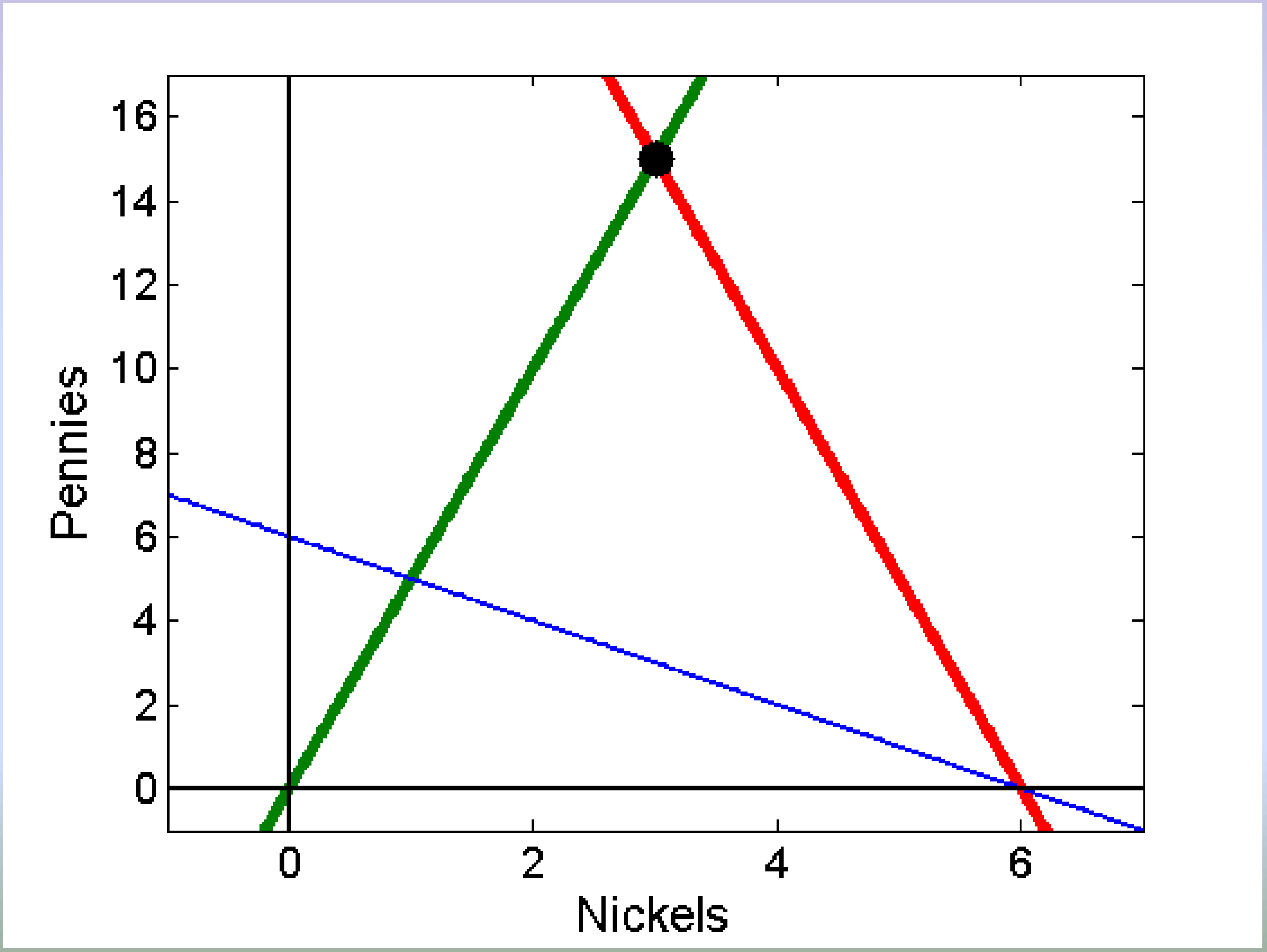

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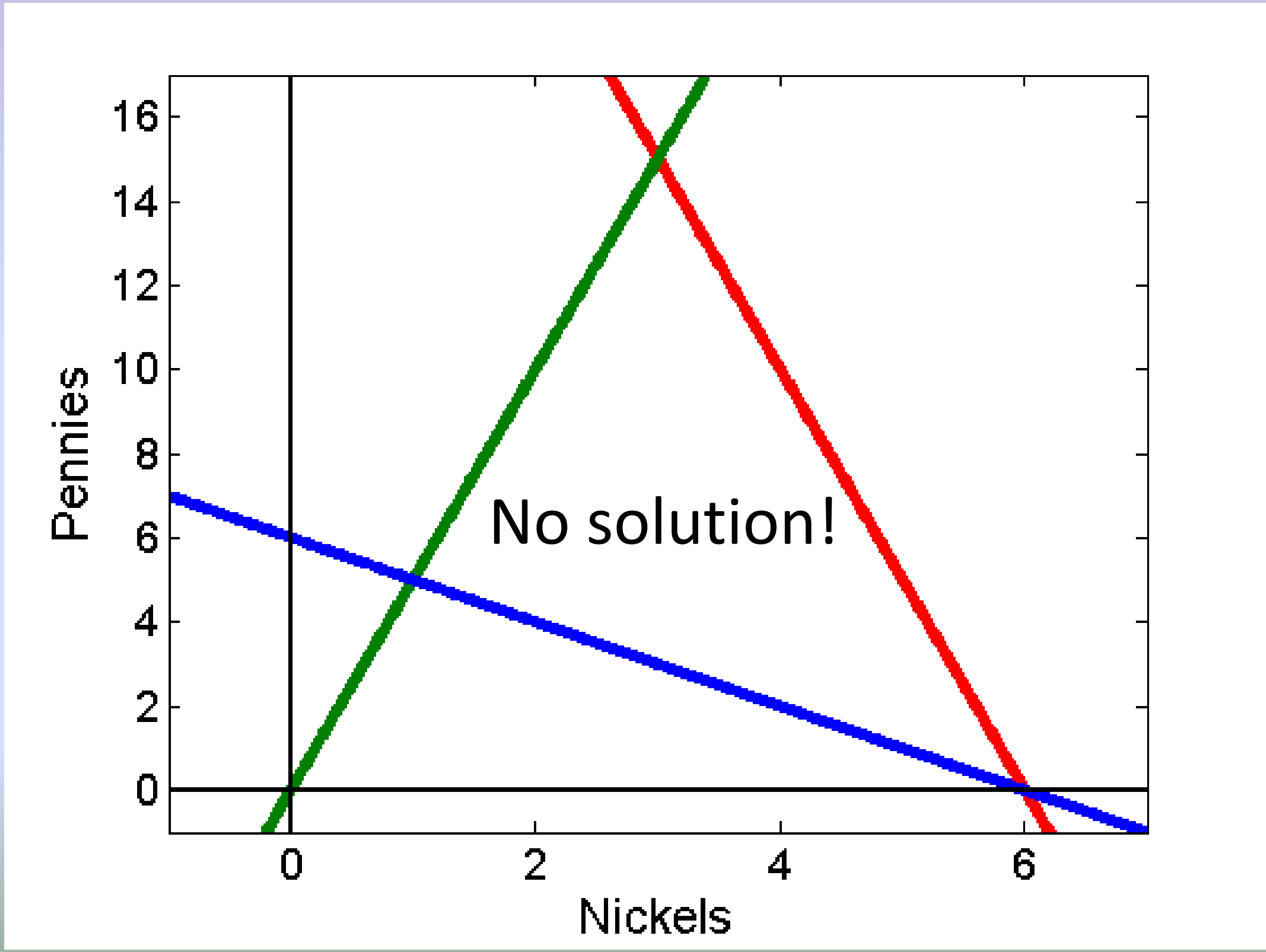

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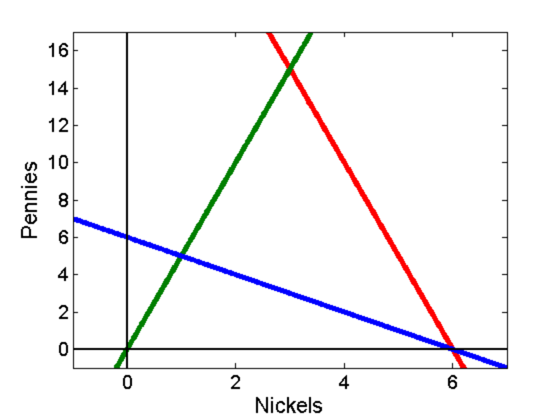

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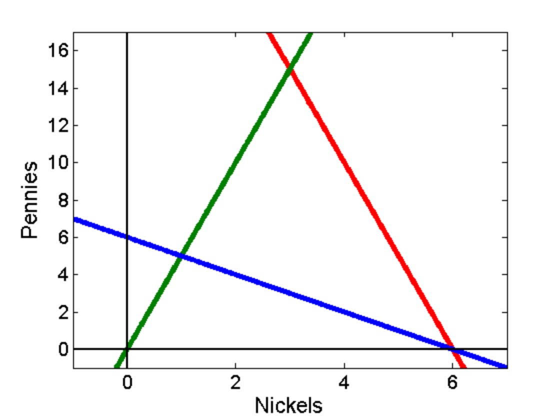
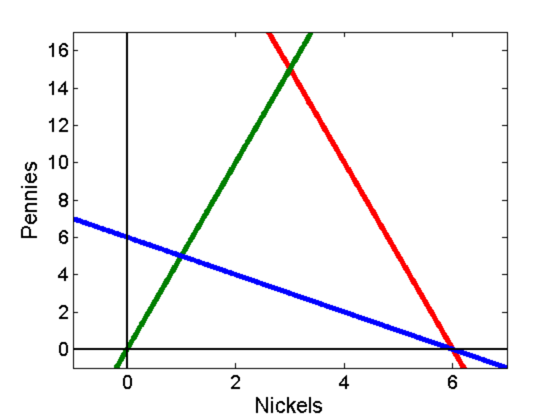


$$\left[\begin{array}{cc|c} 1 & 1 & 6 \\ 1 & -5 & 0 \\ 1 & 5 & 30 \end{array}\right] \xrightarrow[R3-R1]{R2-R1} \left[\begin{array}{cc|c} 1 & 1 & 6 \\ 0 & -6 & -6 \\ 0 & 4 & 24 \end{array}\right]$$

$$\xrightarrow[\frac{1}{4}R3]{-\frac{1}{6}R2} \left[\begin{array}{cc|c} 1 & 1 & 6 \\ 0 & 1 & 1 \\ 0 & 1 & 6 \end{array}\right] \xrightarrow[R3-R2]{R1-R2} \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & 5 \end{array}\right]$$

$$p + n = 6$$
$$p - 5n = 0$$
$$p + 5n = 30$$

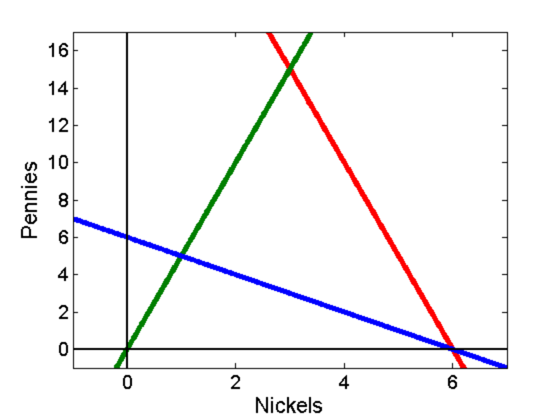


$$\left[\begin{array}{cc|c} 1 & 1 & 6 \\ 1 & -5 & 0 \\ 1 & 5 & 30 \end{array}\right] \xrightarrow[R3-R1]{R2-R1} \left[\begin{array}{cc|c} 1 & 1 & 6 \\ 0 & -6 & -6 \\ 0 & 4 & 24 \end{array}\right]$$

$$\xrightarrow[\frac{1}{4}R3]{-\frac{1}{6}R2} \left[\begin{array}{cc|c} 1 & 1 & 6 \\ 0 & 1 & 1 \\ 0 & 1 & 6 \end{array}\right] \xrightarrow[R3-R2]{R1-R2} \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & 5 \end{array}\right]$$

$0p + 0n = 5$. No solution.

$$p + n = 6$$
$$p - 5n = 0$$
$$p + 5n = 30$$



What if we change 30 to 10?

$$\left[\begin{array}{cc|c} 1 & 1 & 6 \\ 1 & -5 & 0 \\ 1 & 5 & 30 \end{array}\right] \xrightarrow[R3-R1]{R2-R1} \left[\begin{array}{cc|c} 1 & 1 & 6 \\ 0 & -6 & -6 \\ 0 & 4 & 24 \end{array}\right]$$

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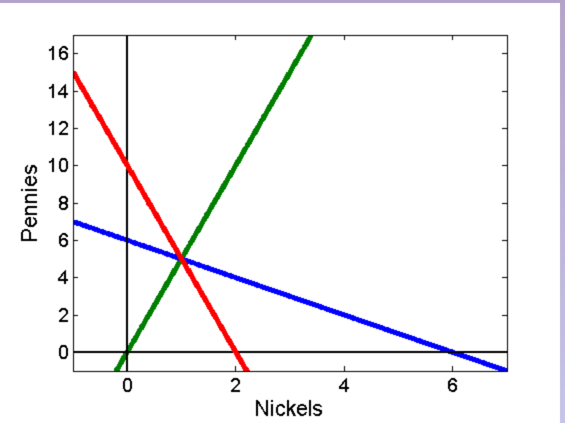
$0p + 0n = 5$. No solution.

$$p + n = 6$$

$$p - 5n = 0$$

$$p + 5n = 10$$



What if we change 30 to 10?

$$\left[\begin{array}{cc|c} 1 & 1 & 6 \\ 1 & -5 & 0 \\ 1 & 5 & 30 \end{array}\right] \xrightarrow[R3-R1]{R2-R1} \left[\begin{array}{cc|c} 1 & 1 & 6 \\ 0 & -6 & -6 \\ 0 & 4 & 24 \end{array}\right]$$

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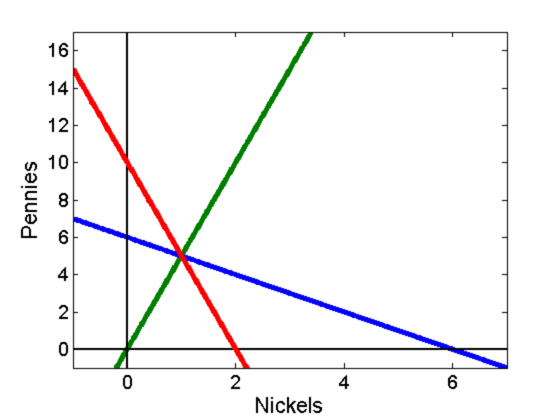
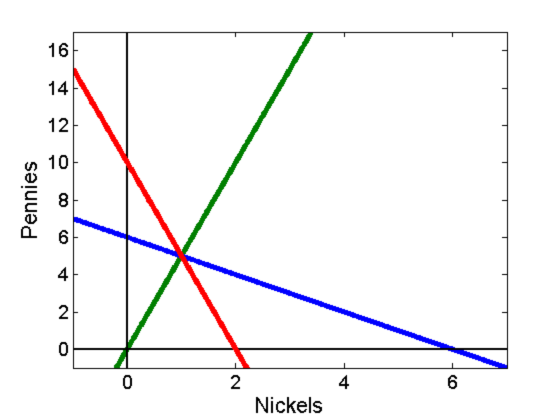


$$\left[\begin{array}{cc|c} 1 & 1 & 6 \\ 1 & -5 & 0 \\ 1 & 5 & 30 \end{array}\right] \xrightarrow{\substack{R2-R1 \\ R3-R1}} \left[\begin{array}{cc|c} 1 & 1 & 6 \\ 0 & -6 & -6 \\ 0 & 4 & 24 \end{array}\right]$$

$$\xrightarrow{\substack{-\frac{1}{6}R2 \\ \frac{1}{4}R3}} \left[\begin{array}{cc|c} 1 & 1 & 6 \\ 0 & 1 & 1 \\ 0 & 1 & 6 \end{array}\right] \xrightarrow{\substack{R1-R2 \\ R3-R2}} \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & 5 \end{array}\right]$$

$$p + n = 6$$
$$p - 5n = 0$$
$$p + 5n = 10$$



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$$p - 5n = 0$$

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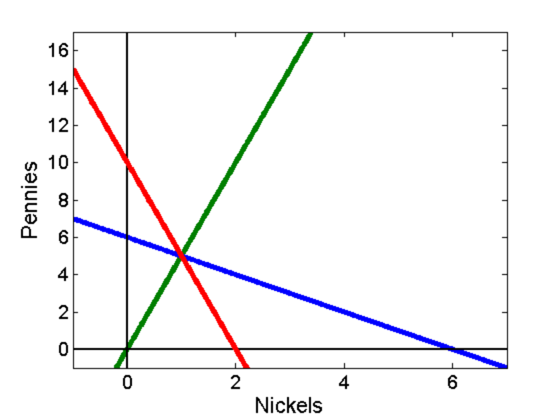

$$\left[\begin{array}{cc|c} 1 & 1 & 6 \\ 1 & -5 & 0 \\ 1 & 5 & 10 \end{array}\right] \xrightarrow[R3-R1]{R2-R1} \left[\begin{array}{cc|c} 1 & 1 & 6 \\ 0 & -6 & -6 \\ 0 & 4 & 4 \end{array}\right]$$

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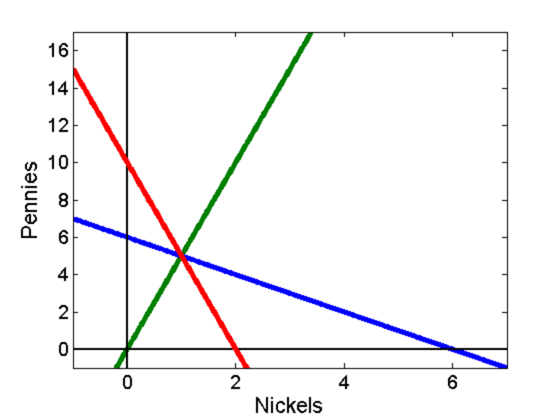

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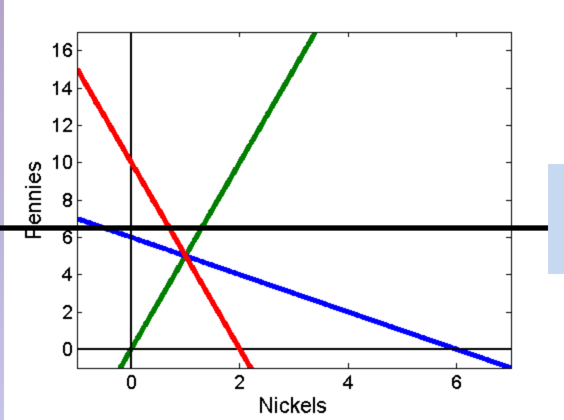
$p = 5$
$n = 1$

$0p + 0n = 0$. This is OK.

$$\left.\begin{array}{l} p + n = 6 \\ p - 5n = 0 \\ p + 5n = 10 \end{array}\right]$$



Check!

$$\left[\begin{array}{cc|c} 1 & 1 & 6 \\ 1 & -5 & 0 \\ 1 & 5 & 10 \end{array}\right] \xrightarrow[R3-R1]{R2-R1} \left[\begin{array}{cc|c} 1 & 1 & 6 \\ 0 & -6 & -6 \\ 0 & 4 & 4 \end{array}\right]$$

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$p = 5$

$n = 1$

$0p + 0n = 0$. This is OK.

Students come up with examples (with three unknowns)

	No solution	One solution	Infinite solutions
# equations < # unknowns			
# equations = # unknowns			
# equations > # unknowns			

Concepts discovered and discussed

- Restrictions/relationships $\leftrightarrow$ equations
- Functions vs. equations (variables vs. unknowns)
- Standard form vs. slope-intercept form
- Number of equations (restrictions) vs. number of unknowns (freedom):
  - Typical number of solutions
  - Exceptions
- In standard form, coefficients determine slope and right hand side determines y- (or x-) intercept
- Gaussian elimination is driven by coefficients, not by right hand side
- Systems of linear equations have 0, 1 or ∞ solutions

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- All with a simple 15-minute example

Thanks for your interest.