Learning engineering to teach mathematics

January 2010
San Francisco

Josep FERRER*, Carmen ORTIZ** , Marta PEÑA*

*Universitat Politècnica de Catalunya (Spain)
**Universidad de Extremadura (Spain)

Seminar of mathematics for engineering

Introduce examples of engineering use

to increase students interest at learning mathematics.

- Electrical Engineering
- Optimization and Simulation
- Mechanical Engineering
- Elasticity and Materials' Resistance
- Automatics and Control

In the following slides a brief introduction to **electrical engineering** will be carried out as an example of the information collected during the seminar:

- Guideline for teachers
- Exercises for students

Circuit analysis

Study of the system of equations derived from the Ohm's and Kirchoff's laws

Circuit analysis

Circuit

- *M* meshes with *N* nodes and *B* branches.

Objective

- Determine **currents** and **tensions** distribution (i_k, u_k), $1 \leq k \leq B$.

Tools

- Ohm's law: $u_k = R_k i_k + e_k$, each branch where $\mathbf{e}_k$ is the generated tension
- Kirchoffs' law: $\sum u_k = 0$, in each node and $\sum i_k = 0$, in each mesh

Circuit analysis

STEP 1

- Ohm's law establishes an **isomorphism between currents and tensions** so that we just need to refer to one of both variables

  (KCL) $\sum i_k = 0$ in each node.

  (KVL) $\sum R_k i_k = -\sum e_k$ in each mesh.

STEP 2

- **One node equations is redundant** so that we have *N+M-1=B* equations with *B* unknowns.
- It is a **consistent determined system** because so it is its homogeneous associate (if $e_k=0$, there is not energy contribution, meaning that the currents will turn to be null).

STEP 3

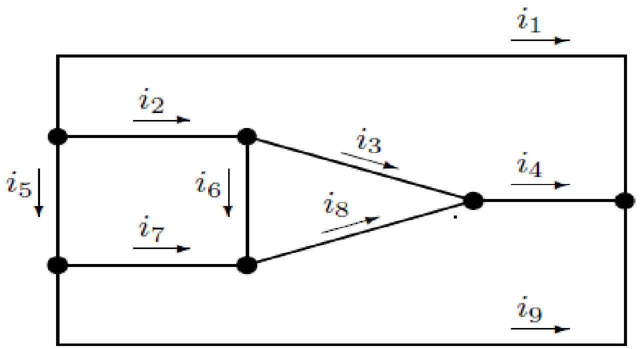
- Therefore:

  dim *{solutions KCL}=B-(N-1)=M*
- **The mesh currents are linearly independent.** Then, they are a basis of this subspace

Circuit analysis

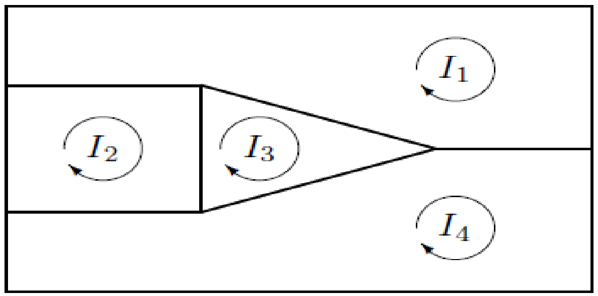
EXERCISE 1 DIMENSION AND BASIS OF THE SUBSPACE OF SOLUTIONS OF KLC

Consider the following mesh



Being *E* the set of possible current's distributions, find out the subset verifying the Kirchoff's law $\sum i_k = 0$

MESH CURRENTS



Prove that it is a subspace parametrized by the mesh currents.

Alternating currents

Use of complex numbers as representation of electrical magnitudes for alternating currents

Alternating currents

Direct current representation
- Set of Real numbers

Generalization →

Alternating current representation
- Set of Complex numbers

Reality

Cosenoidal

$$m(t) = \sqrt{2}M\cos(\omega t + \alpha)$$

M: **root mean square** value (rms)

α: **phase angle**

ω: **frequency** (constant)

Representation

Fasorial

$$\underline{M} = Me^{i\alpha}$$

Alternating currents

Kirchoff's laws (linear correspondence)

- (KVL) $\sum \underline{I}_k = 0$ in each node.
- (KCL) $\sum \underline{U}_k = 0$ in each mesh.

Ohm's law (field structure):

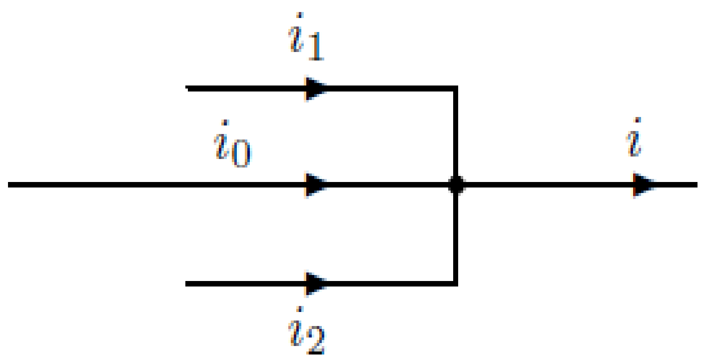
- $\underline{U}_k / \underline{I}_k = Z_k$ Z_k is the impedance.
- Thanks to $m'(t) = j\omega m(t)$ it includes
  - Resistance case $u(t) = Ri(t)$ $\underline{U} = R\underline{I}$
  - Condenser case $i(t) = Cu'(t)$ $\underline{I} = j\omega C\underline{U}$
  - Coil case $u(t) = Li'(t)$ $\underline{U} = j\omega L\underline{I}$

Alternating currents

EXERCISE 2 KLC FOR ALTERNATING CURRENTS

Calculate the current resultant of incrementing it with two others of 75% and 50% of rms, shifted 120º and 90º respectively

$$i_1(t) = 0'75\sqrt{2}I_0\cos\left(\omega t + \frac{\pi}{3}\right) \qquad i_2(t) = 0'50\sqrt{2}I_0\cos\left(\omega t + \frac{\pi}{4}\right)$$

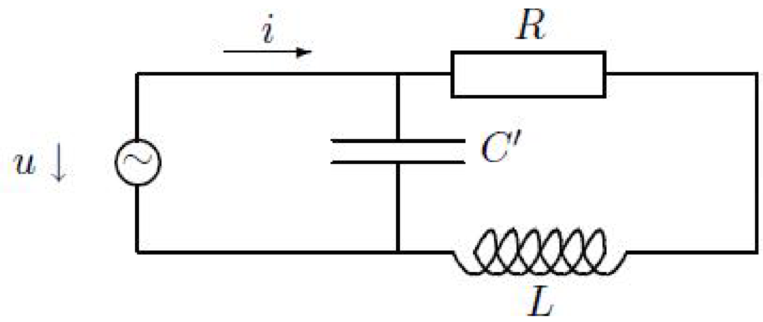


Alternating currents

EXERCISE 3 PARALLEL RESONANCE

Prove for the circuit that the impedance is real if only if

$$\omega L = \sqrt{\frac{L}{C} - R^2} \quad \text{and then} \quad Z = \frac{L}{RC}.$$



Magnetic couplers

Circulant matrices appearing as magnetic couplers of some type of motors and inductance machinery

Magnetic couplers

- Inductance operator (particular case from the matrix called circulant):

$$Z = \begin{pmatrix} c_1 & c_2 & c_3 \\ c_3 & c_1 & c_2 \\ c_2 & c_3 & c_1 \end{pmatrix} \in M_3(\mathbb{C})$$

- The following statements are equivalent:
  - *Z* is a circulant matrix
  - *Z* diagonalized by orthogonal transformation

$$F = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \qquad a = e^{j\frac{2\pi}{3}}$$

- If *A* and *B* are circulant matrices, the matrices *A*+*B* and *AB* are also circulant, and their eigenvalues are the sum and product, respectively, of those of *A* and *B*.
- From a technical point of view, the eigenvalues give the decomposition on monophasics.

Magnetic couplers

EXERCISE 4 EIGENVECTORS OF A CIRCULANT MATRIX

Prove that any circulant matrix

$$A = \begin{pmatrix} a & b & c \\ c & a & b \\ b & c & a \end{pmatrix} \qquad a,b,c \notin \mathbb{C}$$

diagonalizes with the transformation *S*

$$S = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{pmatrix} \qquad \alpha^3 = 1$$

Calculate the diagonal form.