Connecting *STE* to *M*

Robert Rogers

SUNY Fredonia

FREDONIA
STATE UNIVERSITY OF NEW YORK

JMM
January 11, 2015

MAED 310 – Reading/Writing Mathematics

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- Created by the department to partially satisfy the 6 hour literacy requirement for all pre-service teachers in NYS

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- I use this course to promote STEM education and help answer the question

  "Does anyone actually use this stuff?"

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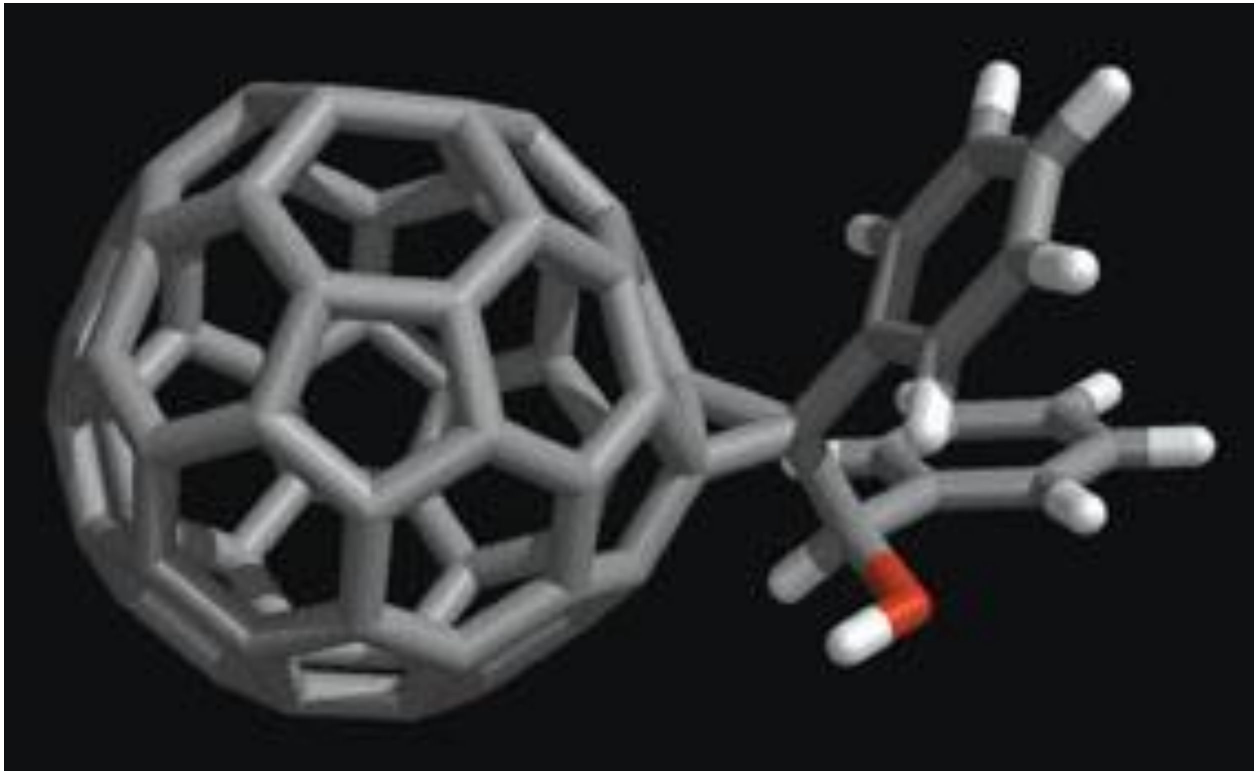

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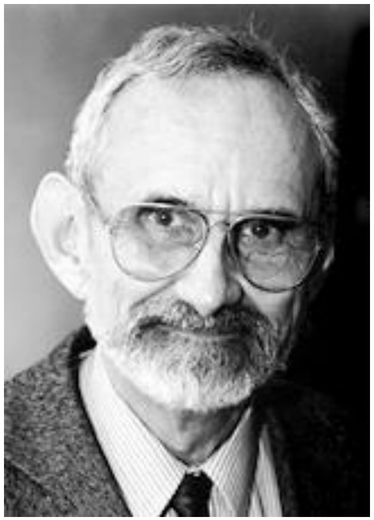
The Nobel Prize in Chemistry 1996

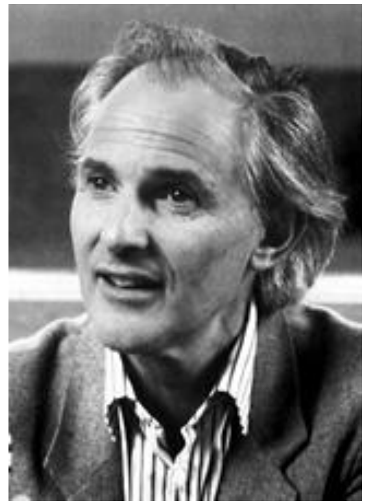


Robert F. Curl Jr.

1/3 of the prize
USA

Rice University
Houston, TX, USA

b. 1933

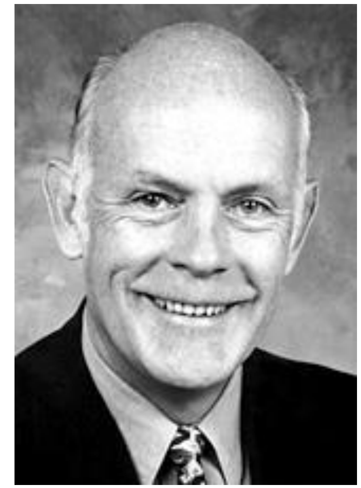


Sir Harold W. Kroto

1/3 of the prize
United Kingdom

University of Sussex
Brighton, United Kingdom

b. 1939



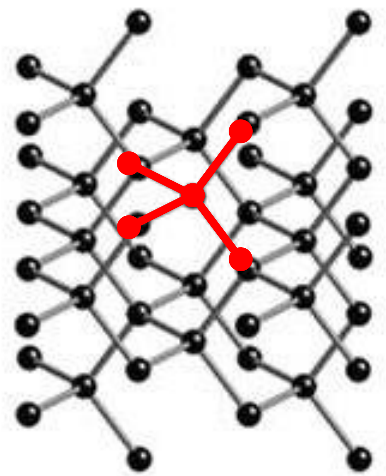
Richard E. Smalley

1/3 of the prize
USA

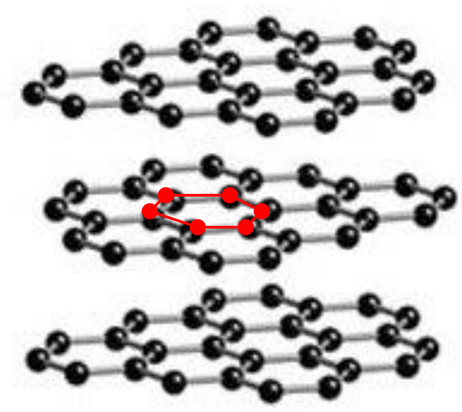
Rice University
Houston, TX, USA

b. 1943
d. 2005

Before 1985, scientists knew of only two forms of pure carbon.

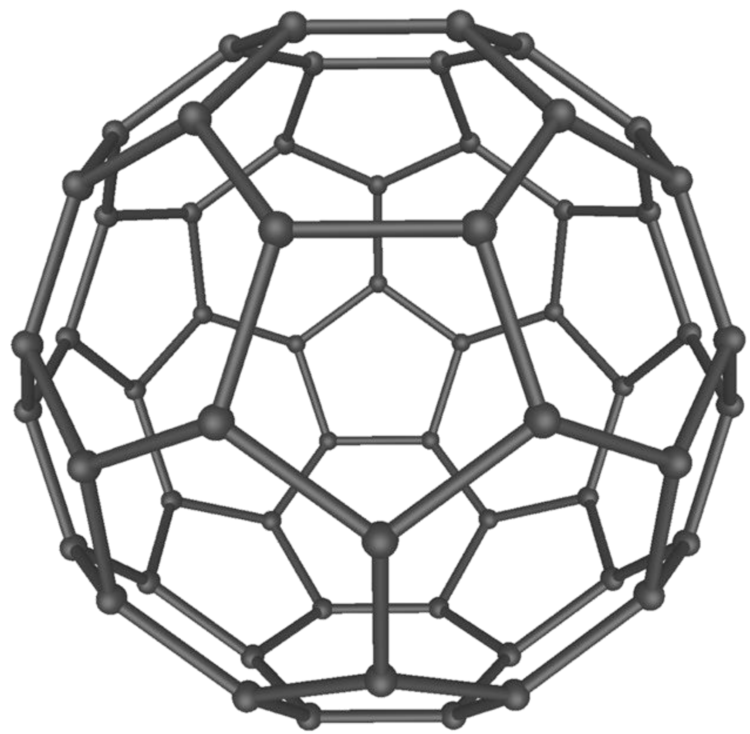


Diamond



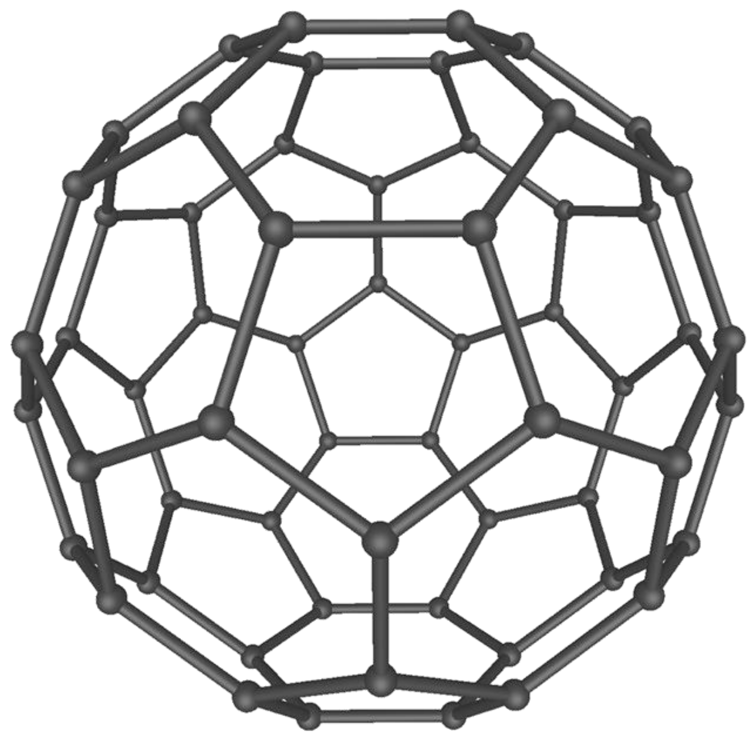
Graphite

In 1985, Curl, Kroto, and Smalley discovered a new pure carbon molecule.



C_{60}

They called it a Buckminsterfullerene, in honor of the architect Richard Buckminster Fuller.



C_{60}

Fuller pioneered the design of geodesic domes.

U. S. Pavilion, Expo 67

Montreal

1. Who are Robert F. Curl, Jr., Richard E. Smalley, and Sir Harold W. Kroto, what is a Buckminsterfullerene, and what does this have to do with polyhedra? Briefly describe how a Buckyball is produced from graphite and it makes sense (molecularly) that graphite would be used. Name three uses/potential uses for a Buckyball.

2. Briefly describe how a Fullerene nanotube is made from a Buckyball. What is the length of the longest nanotube to date? Suppose a ¼ inch diameter straw had the same length to diameter ratio as the longest nanotube to date. How many miles long would it be? Name three uses/potential uses for a nanotube.

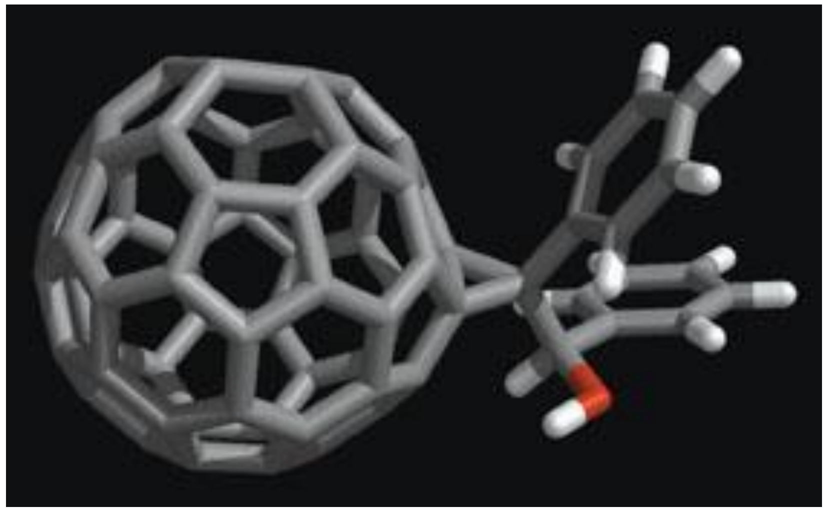
3. Use Euler's formula for a planar graph to prove the Euler-Descartes Formula for a hollow polyhedron.

Euler-Descartes Formula: Given a hollow polyhedron with v vertices, f faces, and e edges

$$v + f - e = 2$$

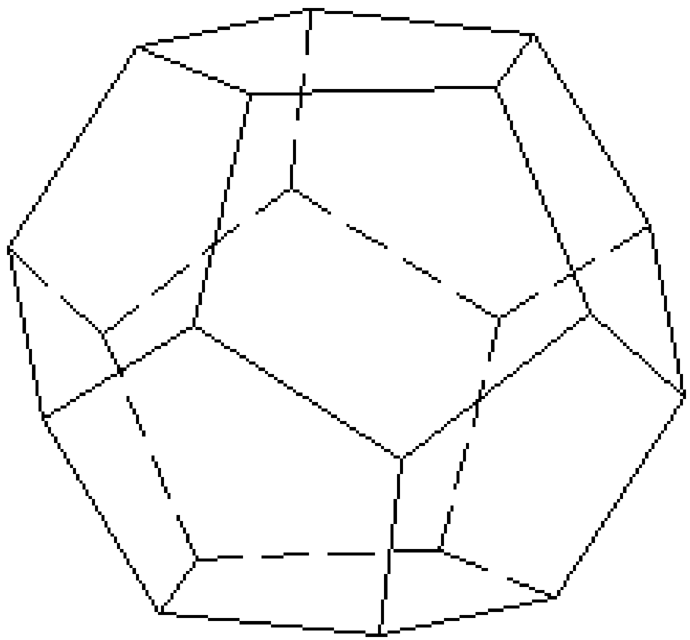
Some Uses of a “Buckyball”:

Medicine:



BUCKY DRUG. *Model of a fullerene-based HIV protease inhibitor recently designed by Simon Friedman.*

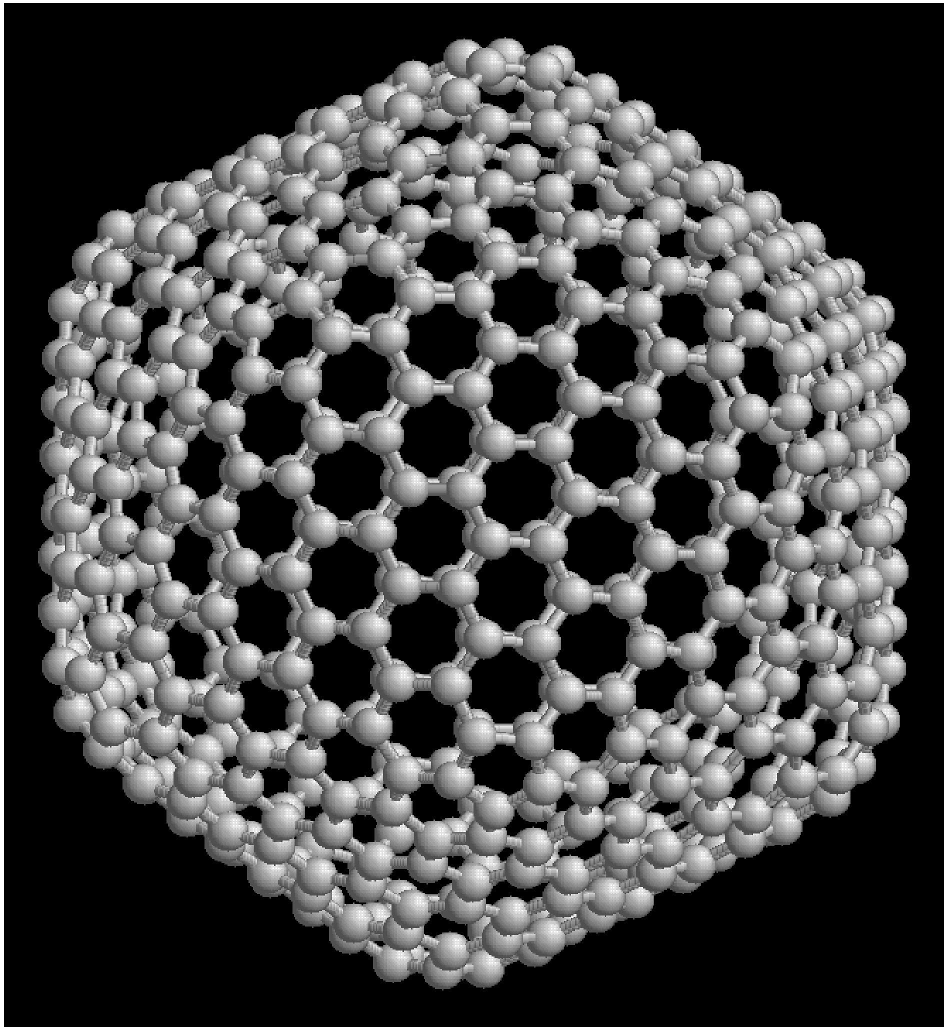
Scientists also want to use buckyballs as a “passkey” to inject drugs into cancer cells.



How many pentagons are in this Dodecahedron

How many pentagons are in a soccer ball?

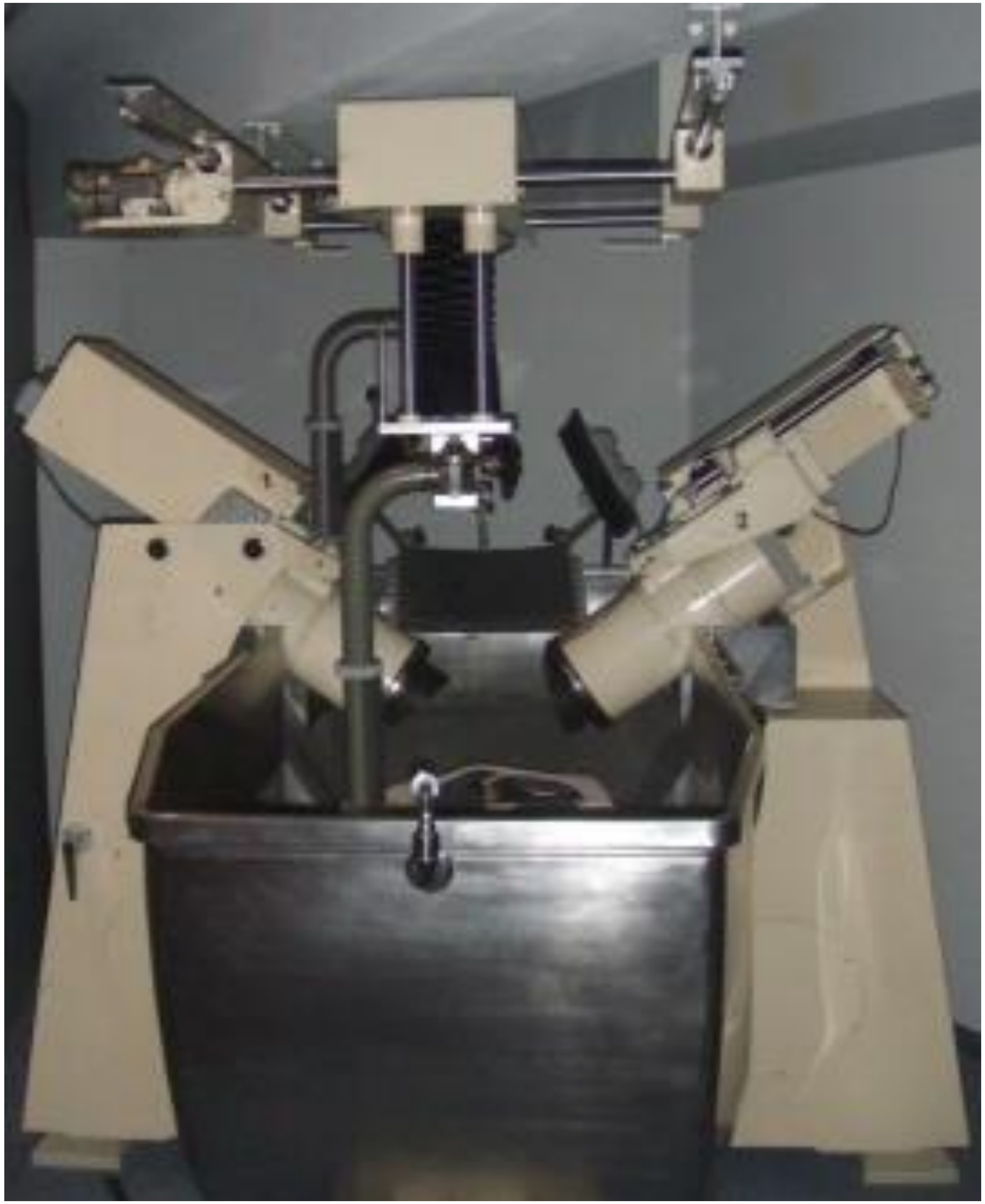
How many pentagons are in this Callaway HX Hot Plus golf ball?

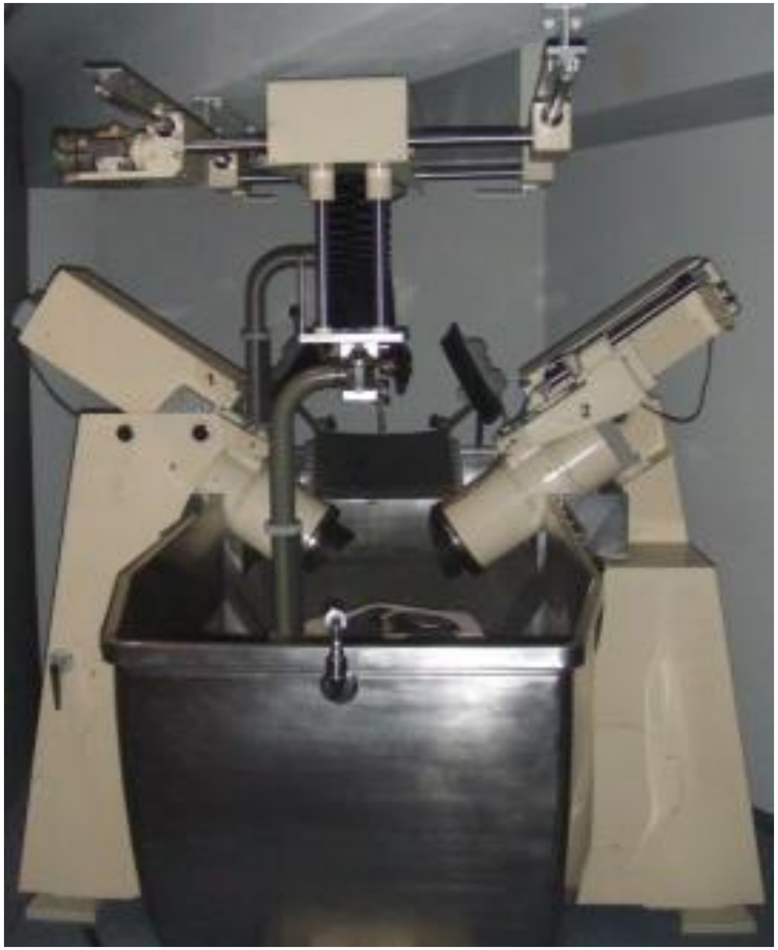


How many pentagons are in this C_{540} carbon molecule?

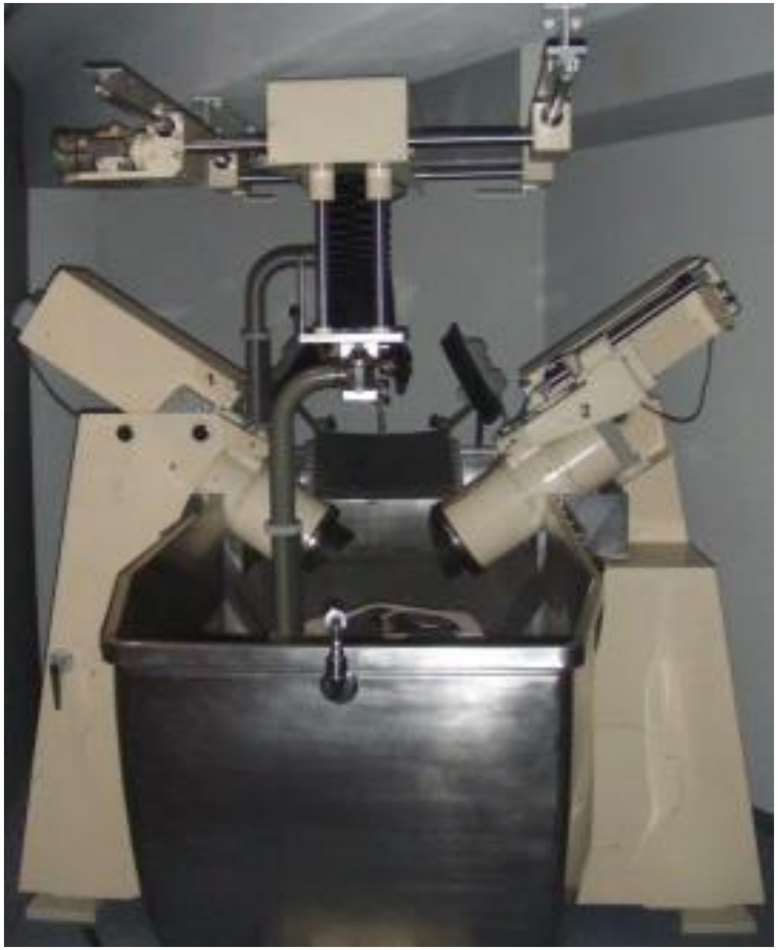
4. Show that a hollow polyhedron whose faces consist of either pentagons or hexagons must have exactly 12 pentagons. (One can show that such a polyhedron must have three edges emanating from each vertex, but you can just assume this.)

5. In light of Problem #4, both a soccer ball and a Callaway HX Hot golf ball contain 12 pentagons. How many hexagons does each contain?



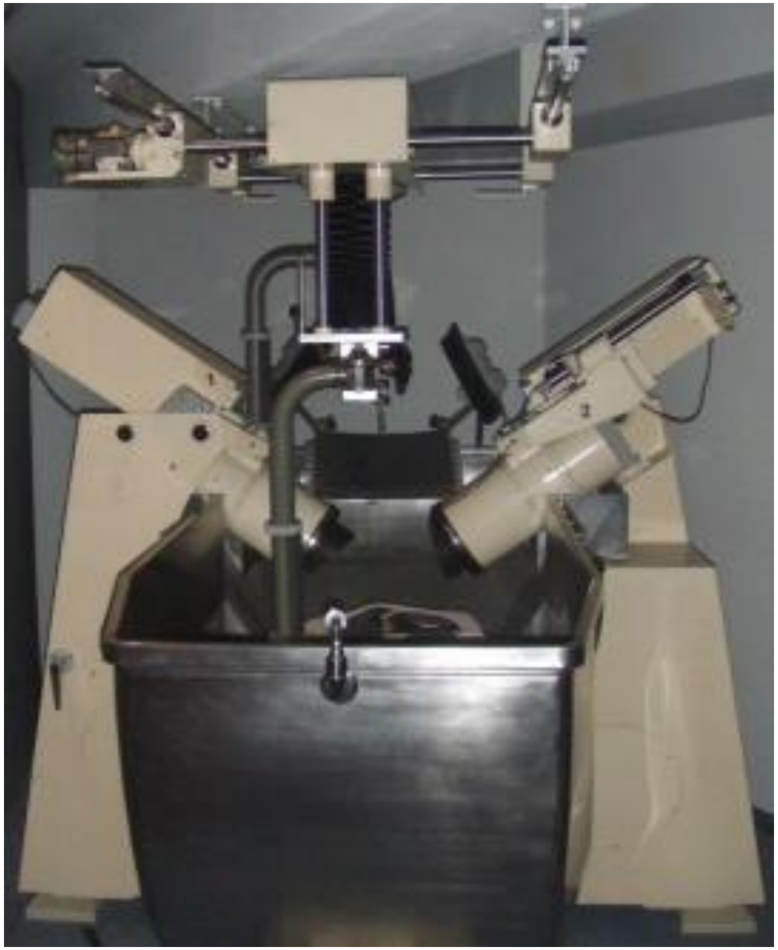


This machine is:



This machine is:

a. an industrial bakery dough mixer.



This machine is:

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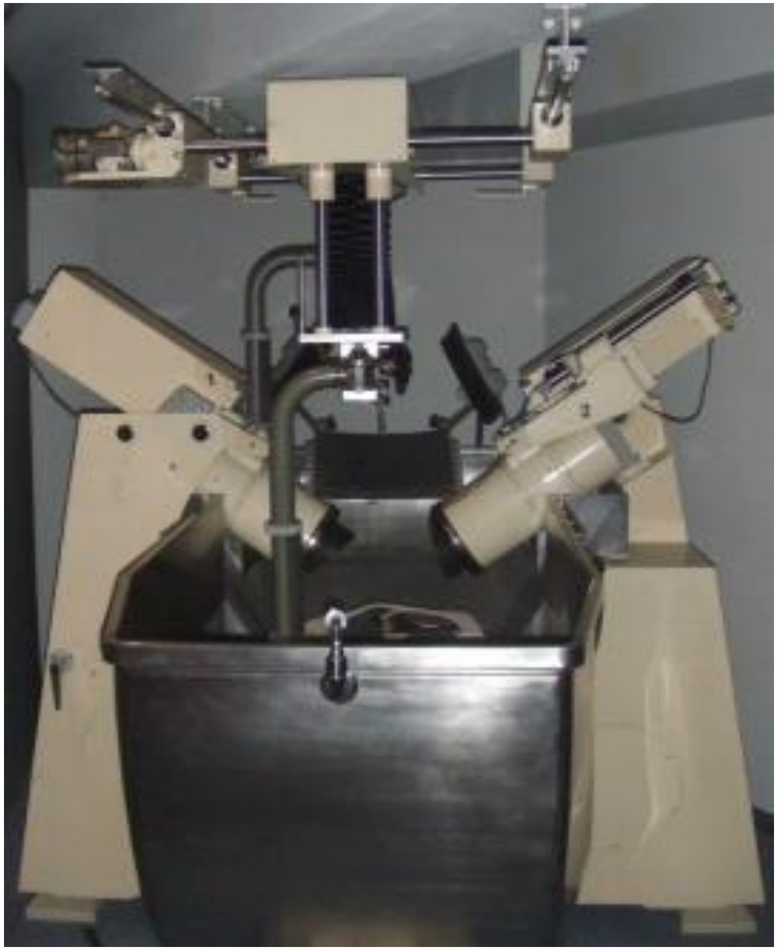
b. a device for treating kidney stones.



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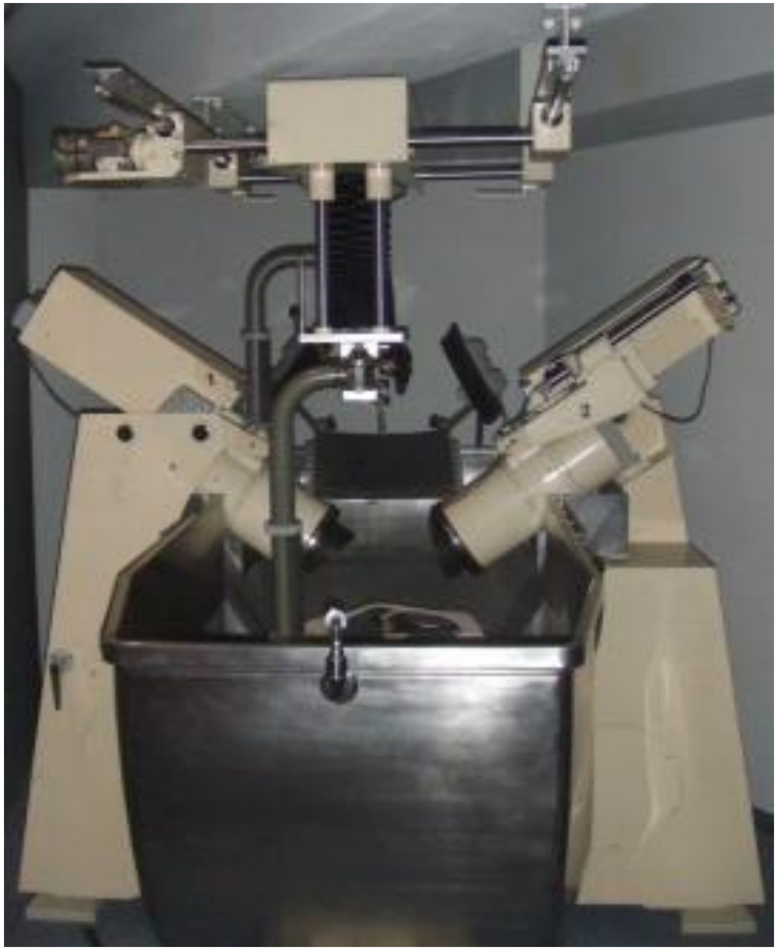

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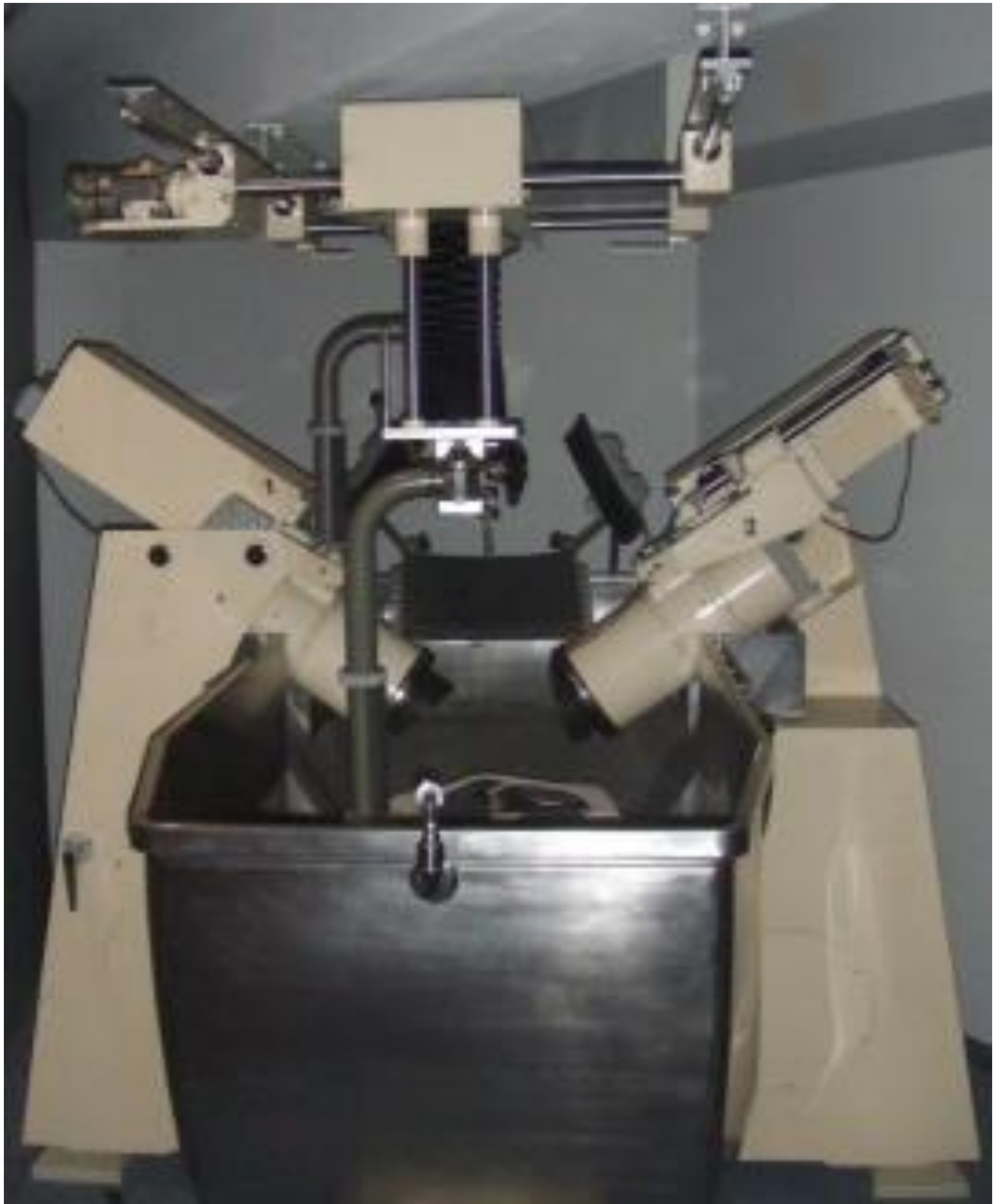
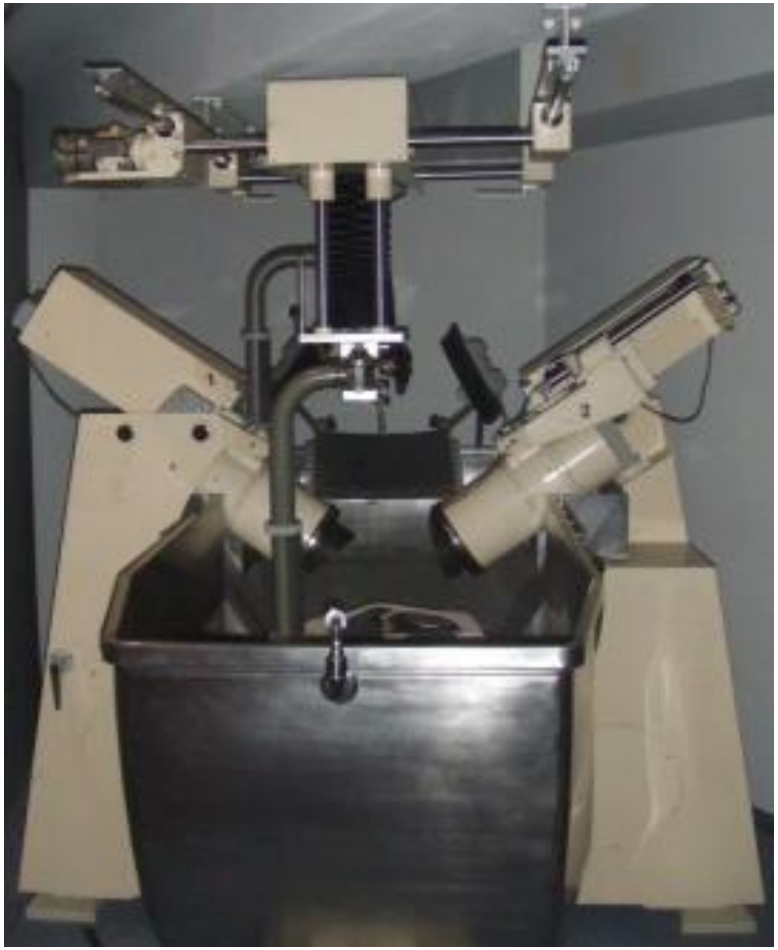
d. a device for testing the effects of vibrations on small airplane components.



Dornier HM-1
lithotripter
(1980)



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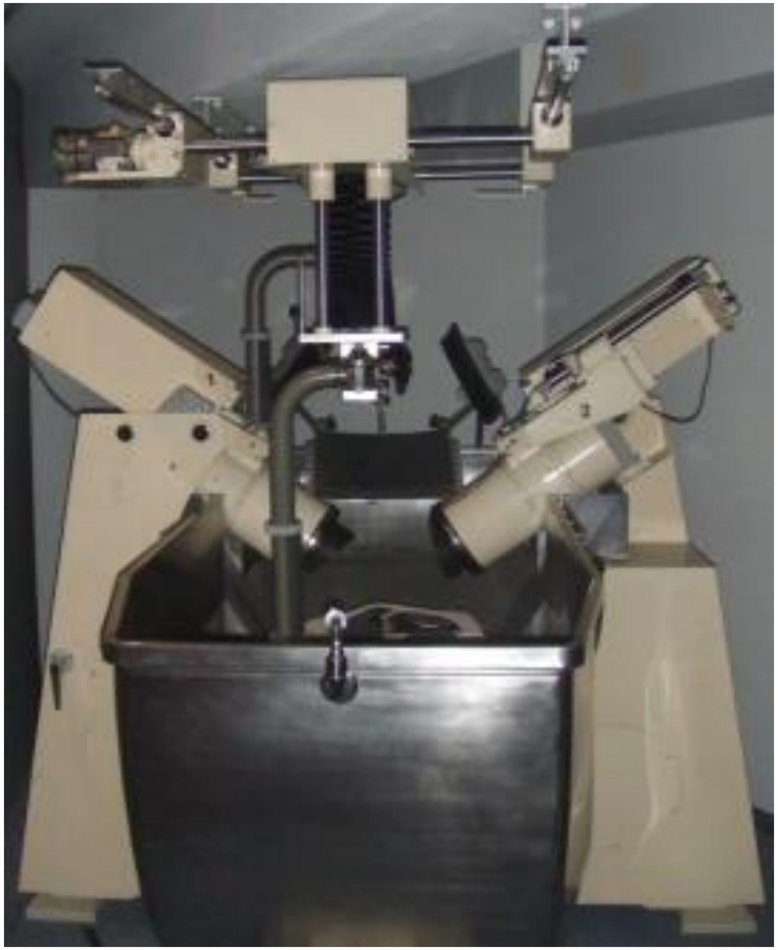

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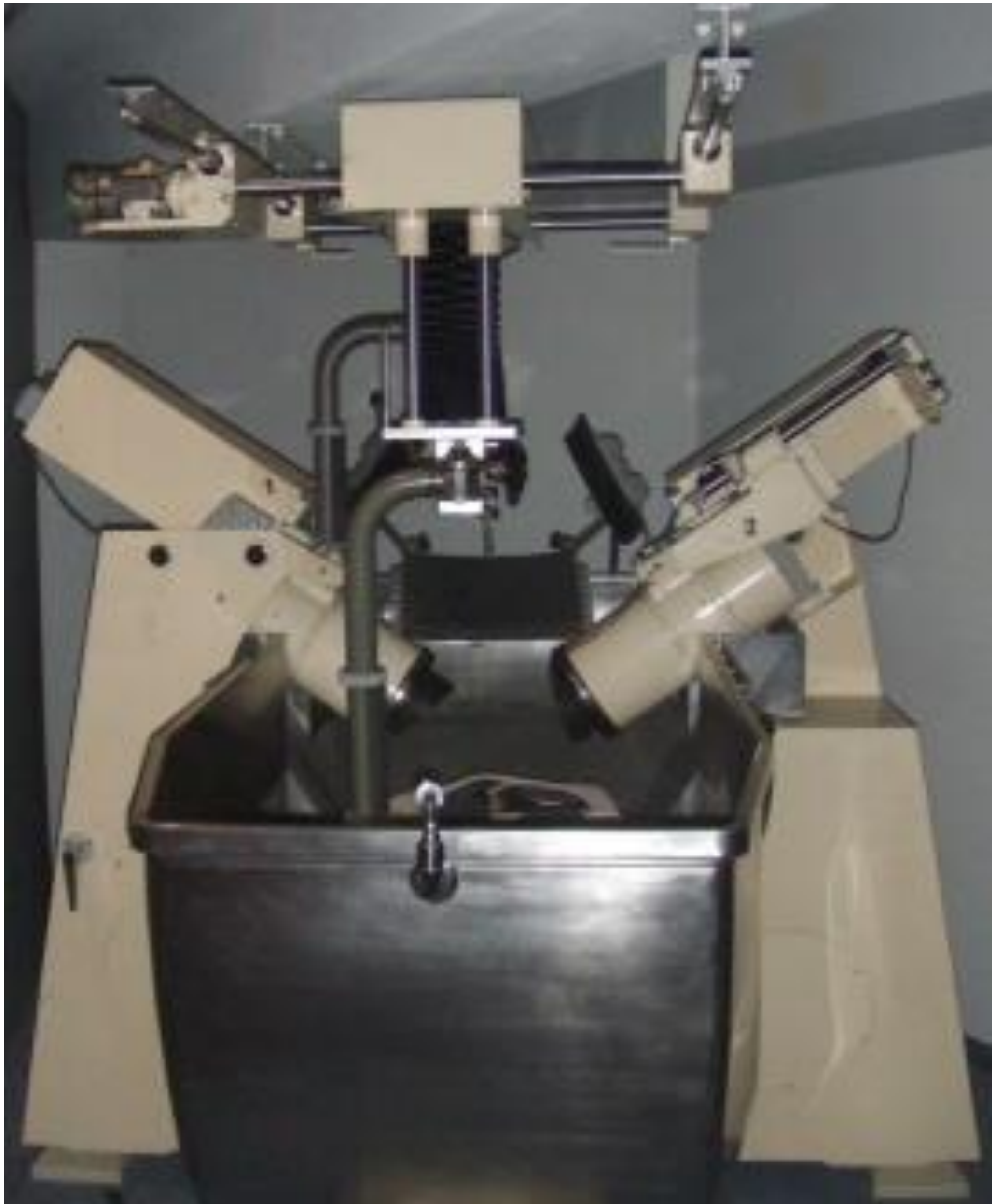

Dornier HM-1 lithotripter (1980)

[Extra-corporeal shock wave lithotripsy (ESWL)]

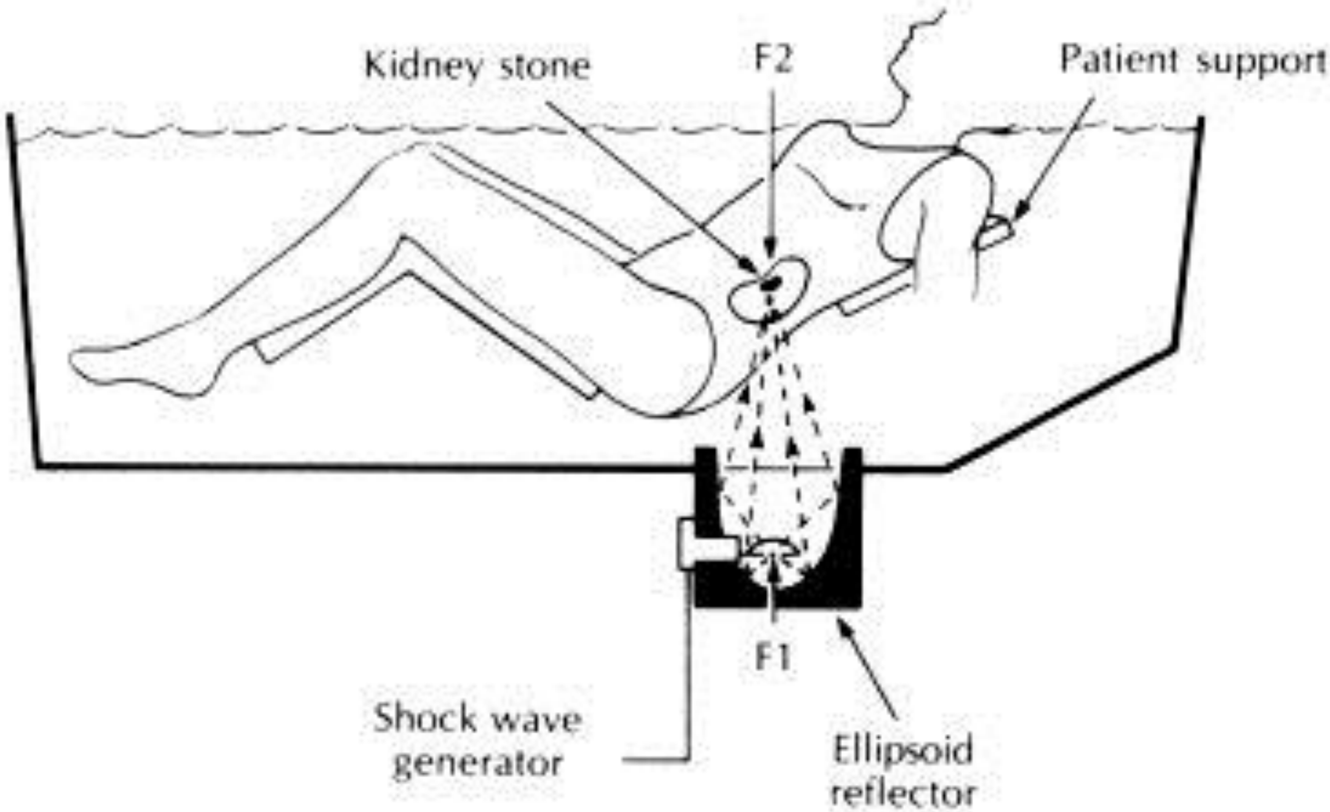

Diagram of patient positioning for ESWL

Question from NYS June 2007 Math B Exam

30 A landscape architect is working on the plans for a new horse farm. He is laying out the exercise ring and racetrack on the accompanying graph. The location of the circular exercise ring, with point *R* as its center, has already been plotted.

Write an equation that represents the outside edge of the exercise ring.

The equation of the outside edge of the racetrack is

$$\frac{x^2}{144} + \frac{y^2}{36} = 1$$

Sketch the outside edge of the racetrack on the graph.

There is a history of asking silly “contextual” math questions.

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Liber abbaci (1228) – Leonardo of Pisa (Fibonacci)

There is a lion at the bottom of a pit 50 feet deep. The lion climbs up 1/7 of a foot each day and then falls back 1/9 of a foot each night. How long will it take the lion to climb out of the pit?

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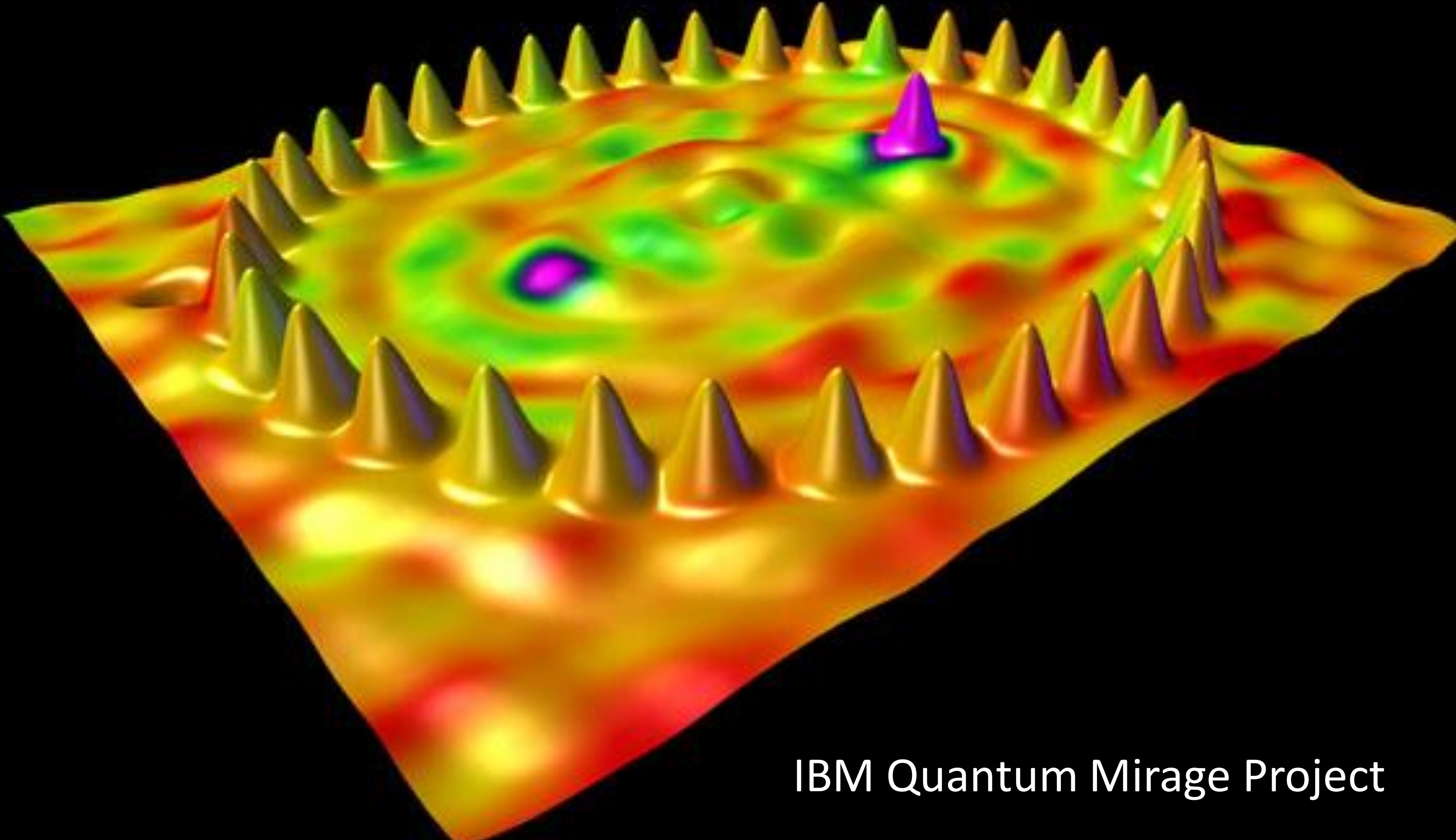
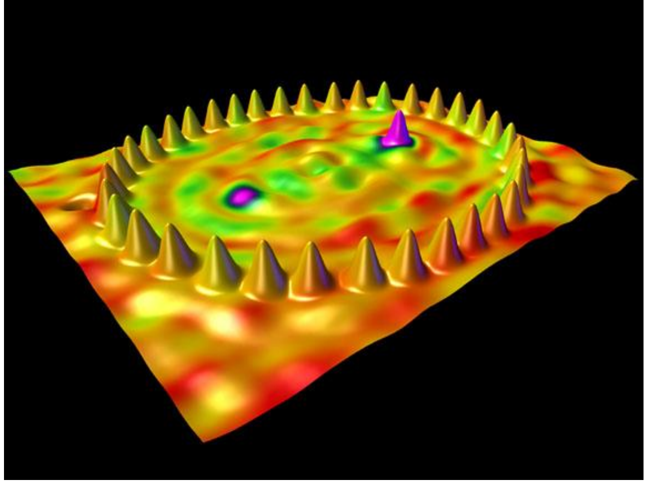
Leonardo gives the incorrect answer of 1575 days. (Actual answer is 1573 days.)



IBM Quantum Mirage Project



IBM researchers have placed 36 cobalt atoms in an elliptical ring on a copper substrate in their efforts to design nanoprocessors for nanobots. An equation for such an ellipse is given by

$$\frac{x^2}{100} + \frac{y^2}{25} = 1$$

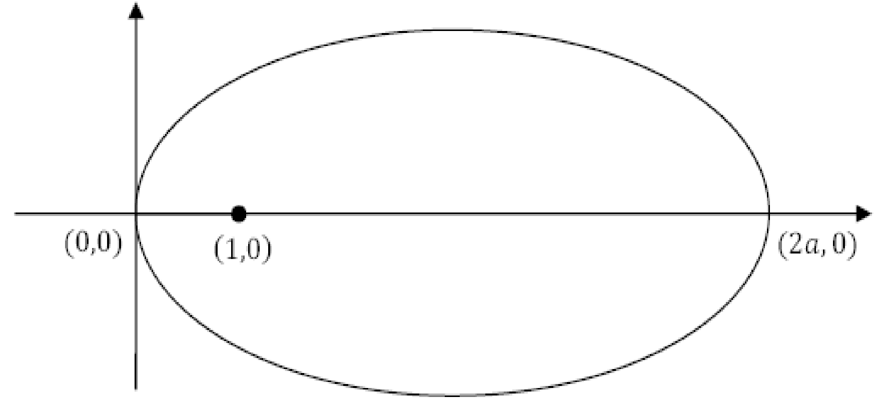
Plot the graph of this ellipse on the accompanying axes (units are in nanometers) and give the lengths of the major and minor axes of this ellipse.

1. Consider the ellipse whose equation is given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$. Find the coordinates of the foci and show how you obtained them.

2. Who coined the term “quantum mirage” and when did he/she do this? What was his/her occupation at the time? What is the length and width of the smallest quantum corral to date? Briefly describe how IBM intends to use quantum corrals?

3. When did Dornier build the first commercial lithotripter and what was the model number. Briefly describe how it uses ellipses to break up kidney stones.

4. Consider the following ellipse.

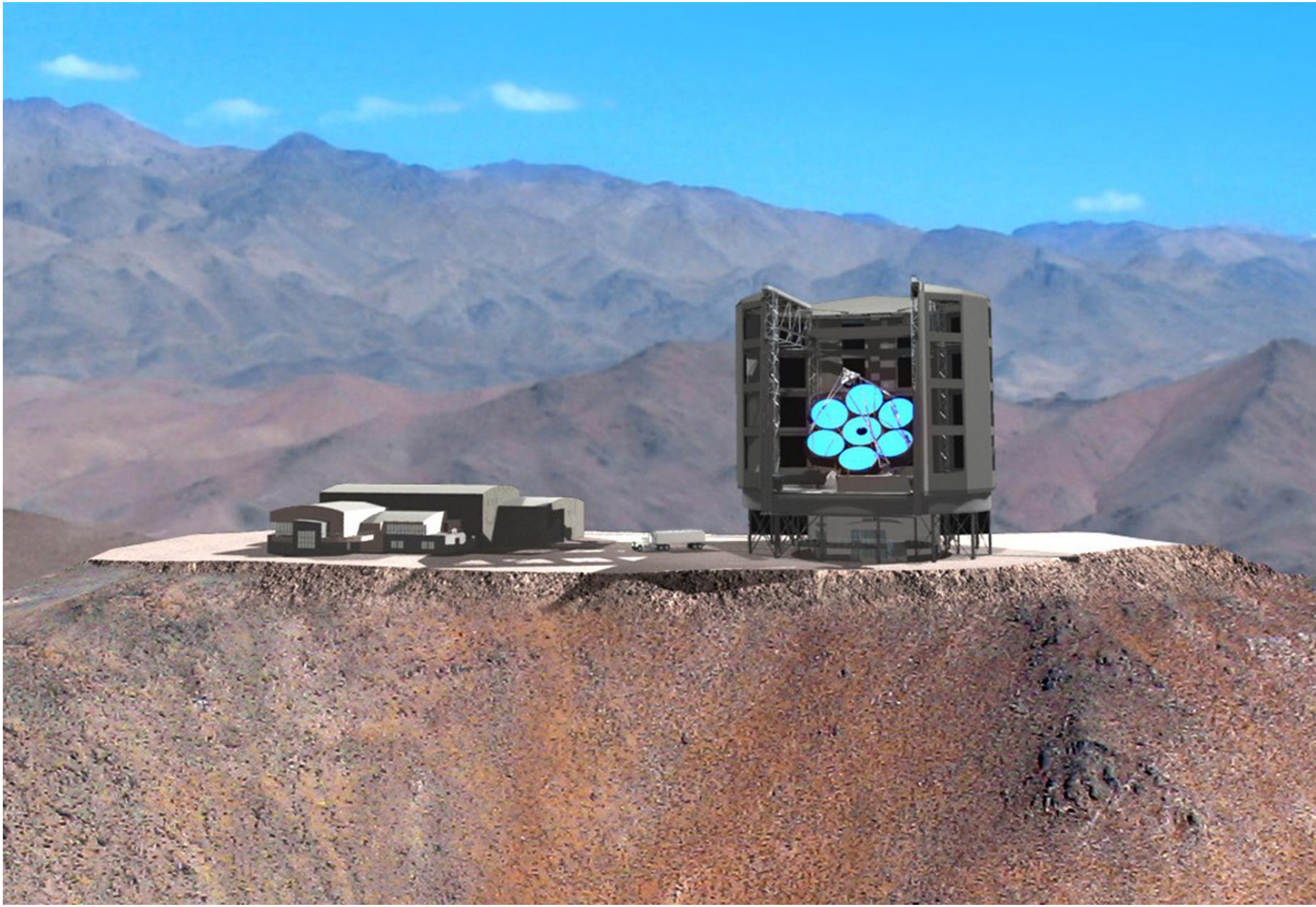


Show that the equation of This ellipse is given by

$$\frac{(x-a)^2}{a^2} + \frac{(y-0)^2}{2a-1} = 1$$

Solve this for y^2 and take the limit of this as a approaches infinity to show that a parabola can be thought of as an ellipse, one of whose foci is "at infinity". Explain how this gives us the reflective property of the parabola.

5. What is the diameter of each of the primary mirrors in the Giant Magellan Telescope and who casts these? Why do they use spin casting? Where will the Giant Magellan Telescope be located and when is it scheduled for completion.



2. What do the various digits in a ten-digit International Standard Book Number (ISBN) represent? Who developed the ISBN-10 system and when? When was this finally converted to the ISBN-13 system and why was this done? What does this have to do with an EAN and how does one convert an ISBN-10 number to an ISBN-13 number?

3. Referring to the old ISBN-10 system, I saw in a book that the number $d_1 d_2 \ldots d_{10}$ is a valid ISBN provided

$$11|(1 \cdot d_1 + 2 \cdot d_2 + \cdots + 9 \cdot d_9 + 10 \cdot d_{10})$$

I saw in another book that it is a valid ISBN provided

$$11|(10 \cdot d_1 + 9 \cdot d_2 + \cdots + 2 \cdot d_9 + 1 \cdot d_{10})$$

Which book is correct? Explain.

4. The Universal Product Code (UPC) is a 12-digit code number on each product manufactured. The first digit represents the kind of product. The next five digits represent the manufacturer. The next five digits represent the product. The last digit is a check digit. The number $d_1 d_2 d_3 \ldots d_{12}$ is a valid UPC number provided

$$10 \mid (3 \cdot d_1 + d_2 + 3 \cdot d_3 + d_4 + \cdots + d_{10} + 3 \cdot d_{11} + d_{12})$$

Show that this scheme will detect all single-digit errors and will detect approximately 88.89% of errors where two consecutive digits are switched.

1.a. Who are the R, S, and A in the RSA public key encryption scheme and when did they develop RSA encryption? What were their occupations at the time? What is the original name of the company they formed? Who owns the company now, when did they acquire it, and how much did they pay for it?

b. What is the RSA factor challenge? When and why was it formed? When and why did it end? What was the largest cash prize ever offered and what was the largest prize ever paid out?

2. Find the largest prime number known to date. When was it discovered and how many digits long is it?

The rest of the homework deals with the mathematics behind public key encryption.

3. Prove the following lemma.

Lemma. Let p be a prime number and let k be an integer with $0 < k < p$. Then

$$p | C(p, k)$$

where $C(p, k) = \frac{p!}{k!(p-k)!}$.

Show that this is not necessarily true if p is not prime.

4. Use the above lemma to prove the following theorem due to Pierre de Fermat (1640).

Fermat's Little Theorem. Let p be a prime number and let N be any integer. Then

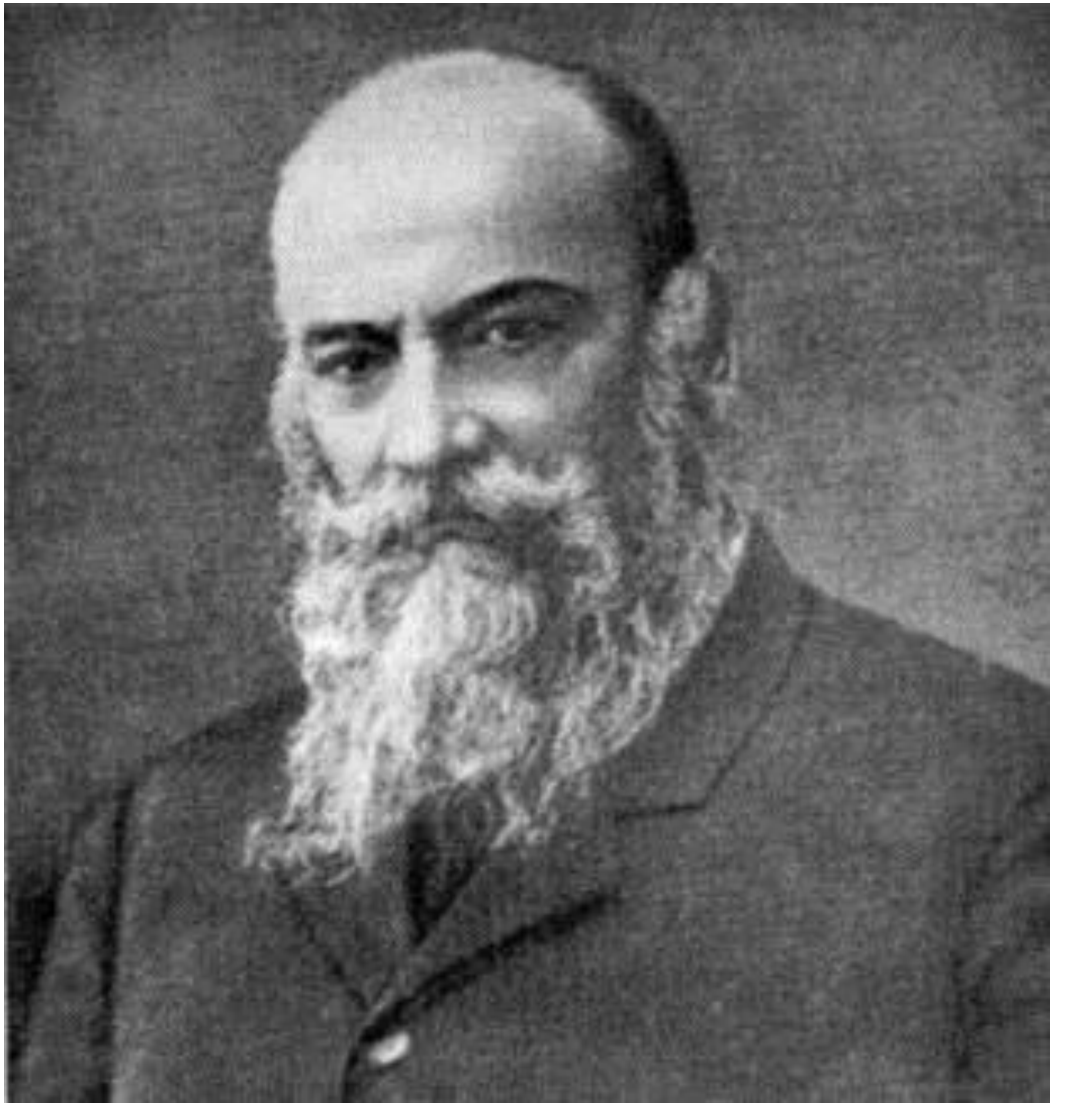
$$p|(N^p - N)$$

In particular, if p does not divide N then $p|(N^{p-1} - 1)$.

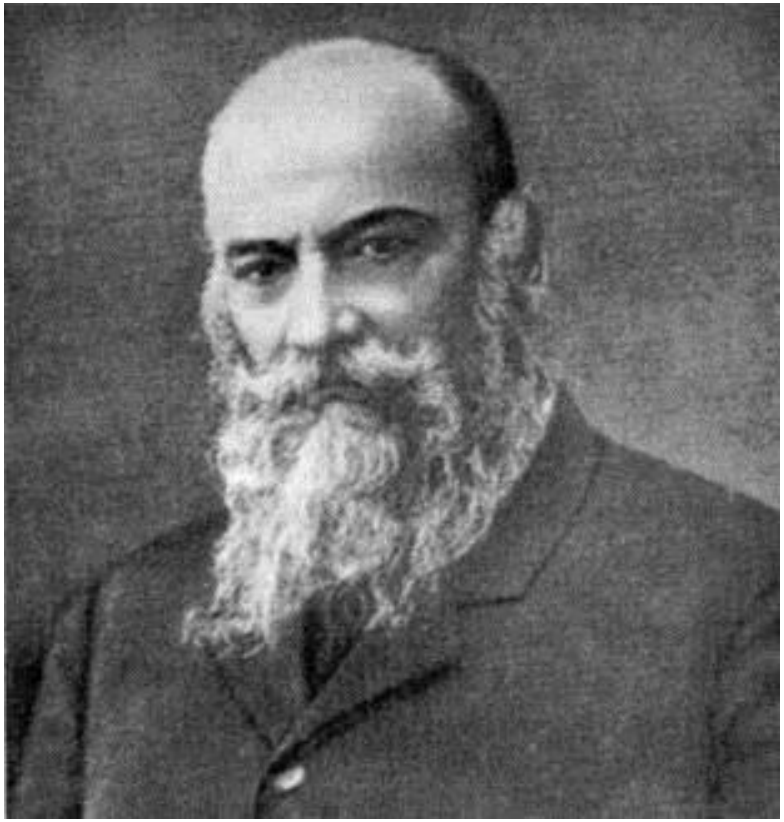
5. Let p and q be distinct prime numbers. Let $a = p \cdot q$ and $b = (p-1) \cdot (q-1)$. Then

$$a|(N^{1+bf} - N)$$

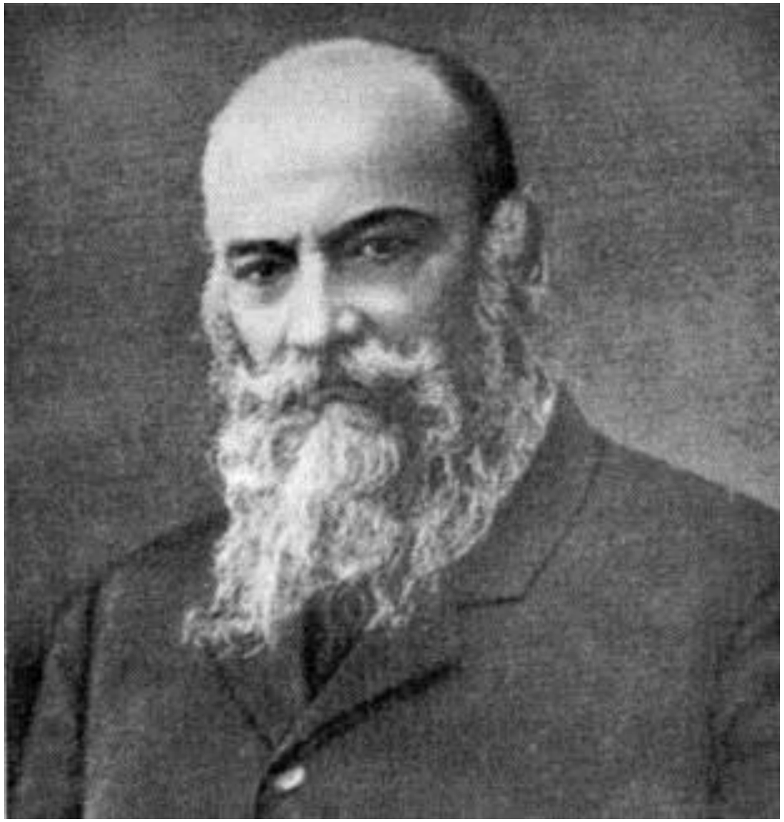
for any positive integers N and f.



Nikolai Egorovich Joukowski (1847-1941)

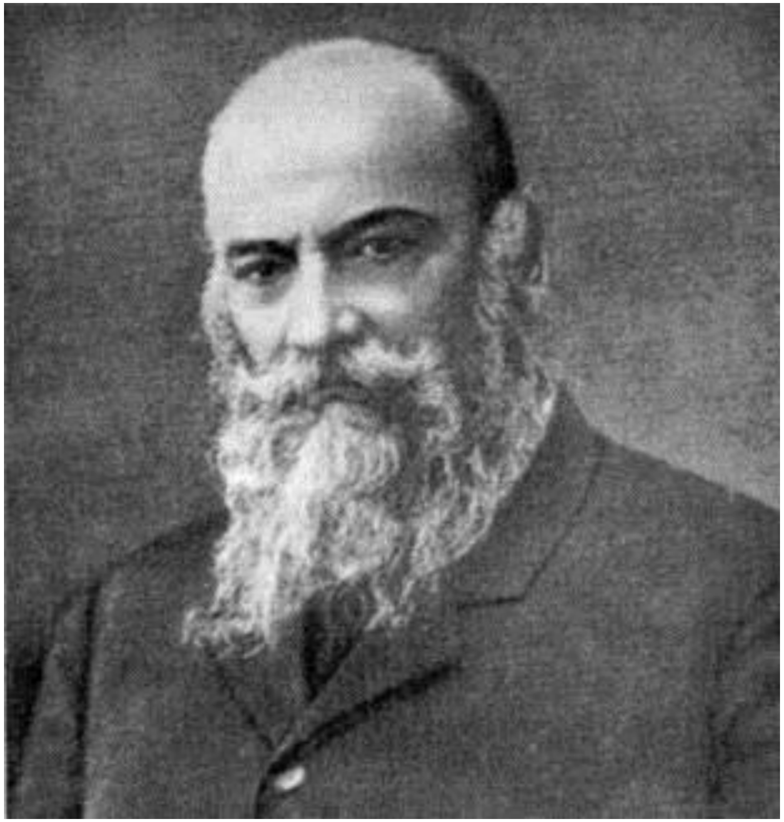


Nikolai Egorovich Joukowski was:



Nikolai Egorovich Joukowski was:

a. Minister of Defense under Stalin.



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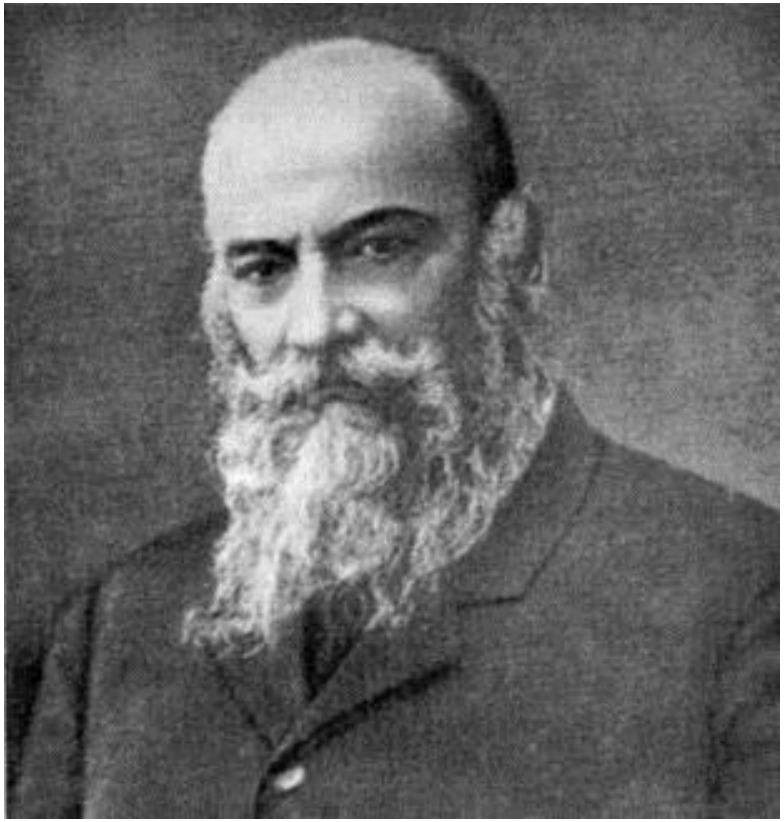

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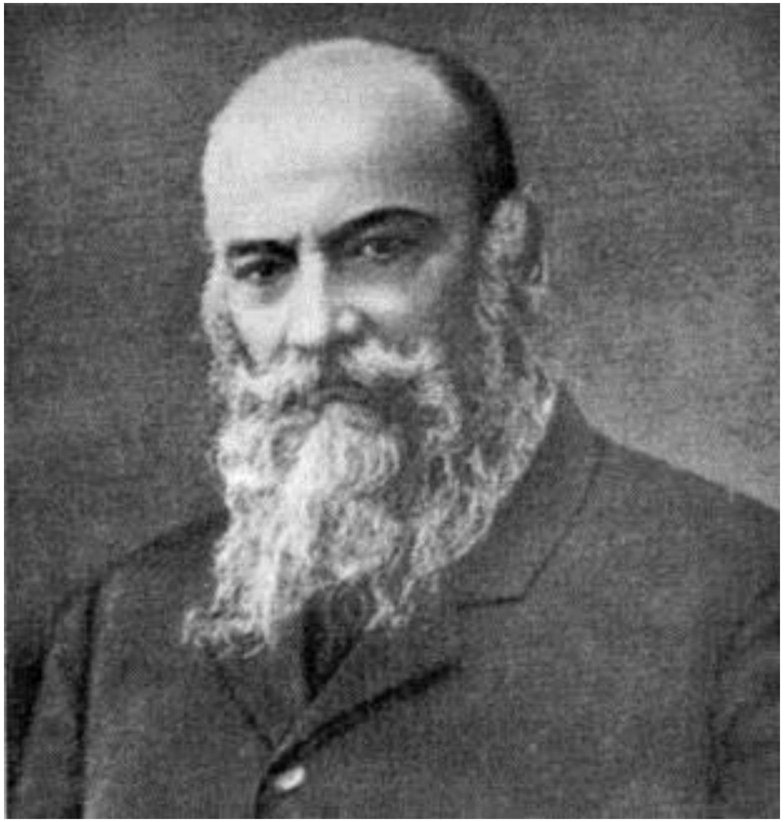


Nikolai Egorovich Joukowski was:

a. Minister of Defense under Stalin.

b. Minister of Science under Czar Nicholas II.

c. the “father of Russian Aviation.”

d. the head of the Moscow Conservatory of Music, 1910 – 1917.

Write the following complex number in the form

$$x + yi$$

$$\frac{1}{2 + i}$$

Write the following complex number in the form

$$x + yi$$

$$\frac{1}{2+i}$$

Solution:

$$\frac{1}{2+i} = \frac{1}{2+i} \cdot \frac{2-i}{2-i} = \frac{2-i}{5} = \frac{2}{5} - \frac{1}{5}i$$

$$z = x + iy$$

$$z = x + iy$$

$$\frac{1}{z} = \left(\frac{1}{x + iy} \right)$$

$$z = x + iy$$

$$\frac{1}{z} = \left(\frac{1}{x + iy} \right)$$

$$= \left(\frac{1}{x + iy} \right) \left(\frac{x - iy}{x - iy} \right)$$

$$= \frac{x - iy}{x^2 + y^2}$$

$$z = x + iy$$

$$\frac{1}{z} = \left(\frac{1}{x+iy}\right)$$

$$= \left(\frac{1}{x+iy}\right)\left(\frac{x-iy}{x-iy}\right)$$

$$= \frac{x-iy}{x^2+y^2}$$

$$\mathrm{Im}\left(z+\frac{1}{z}\right) = \left(y - \frac{y}{x^2+y^2}\right)$$

Plot

$$\operatorname{Im}\left(z + \frac{1}{z}\right) = \left(y - \frac{y}{x^2 + y^2}\right) = c$$

for various constants c

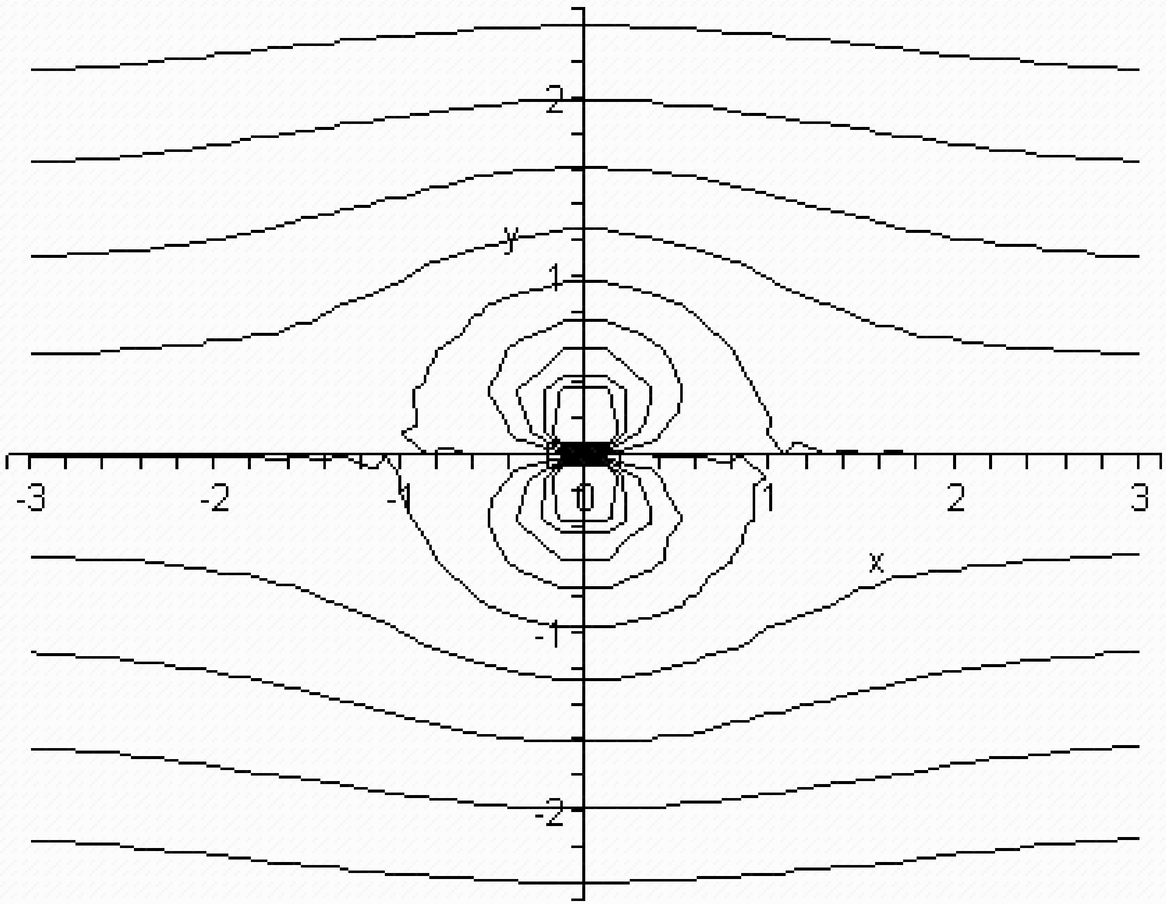
$$y - \frac{y}{x^2 + y^2} = -2 \qquad y - \frac{y}{x^2 + y^2} = -1.5$$

$$y - \frac{y}{x^2 + y^2} = -1 \qquad y - \frac{y}{x^2 + y^2} = -.5$$

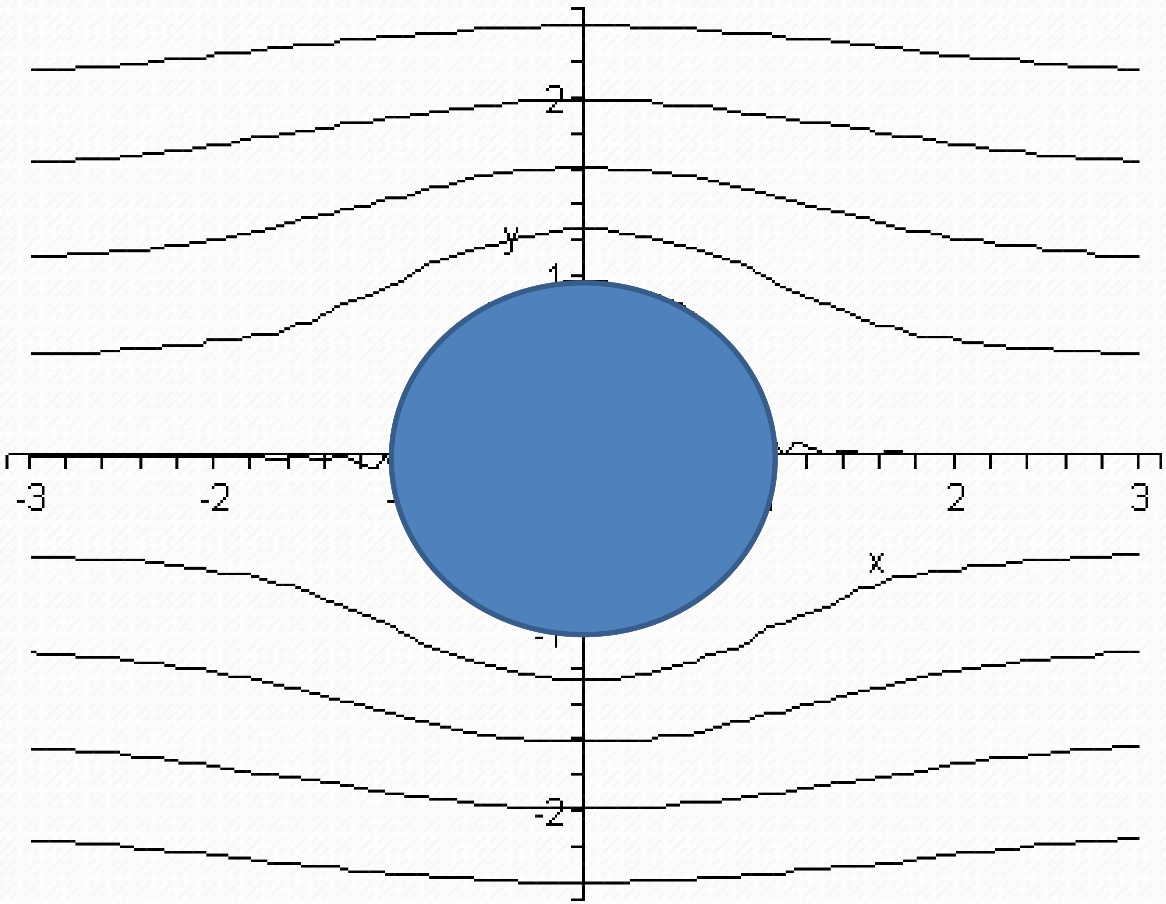
$$y - \frac{y}{x^2 + y^2} = 0 \qquad y - \frac{y}{x^2 + y^2} = .5$$

$$y - \frac{y}{x^2 + y^2} = 1 \qquad y - \frac{y}{x^2 + y^2} = 1.5$$

$$y - \frac{y}{x^2 + y^2} = 2$$

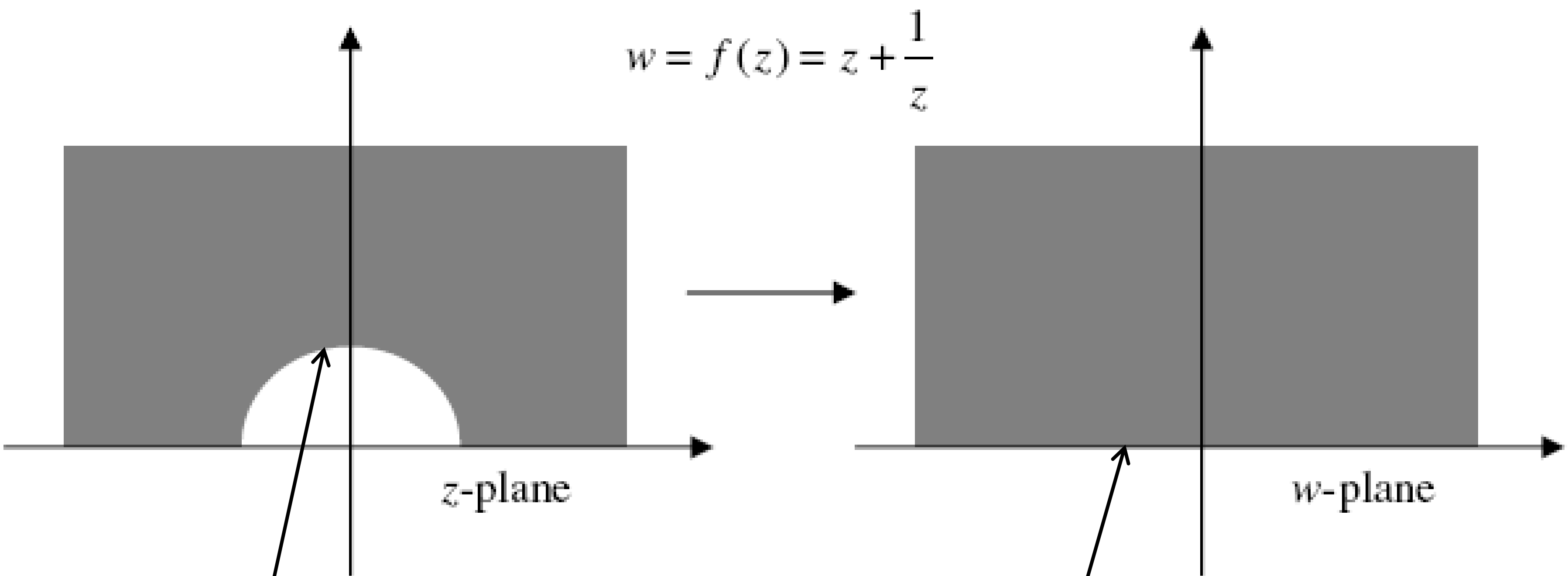


y
2
1
-1
-2
-3
-2
-1
0
1
2
3
x



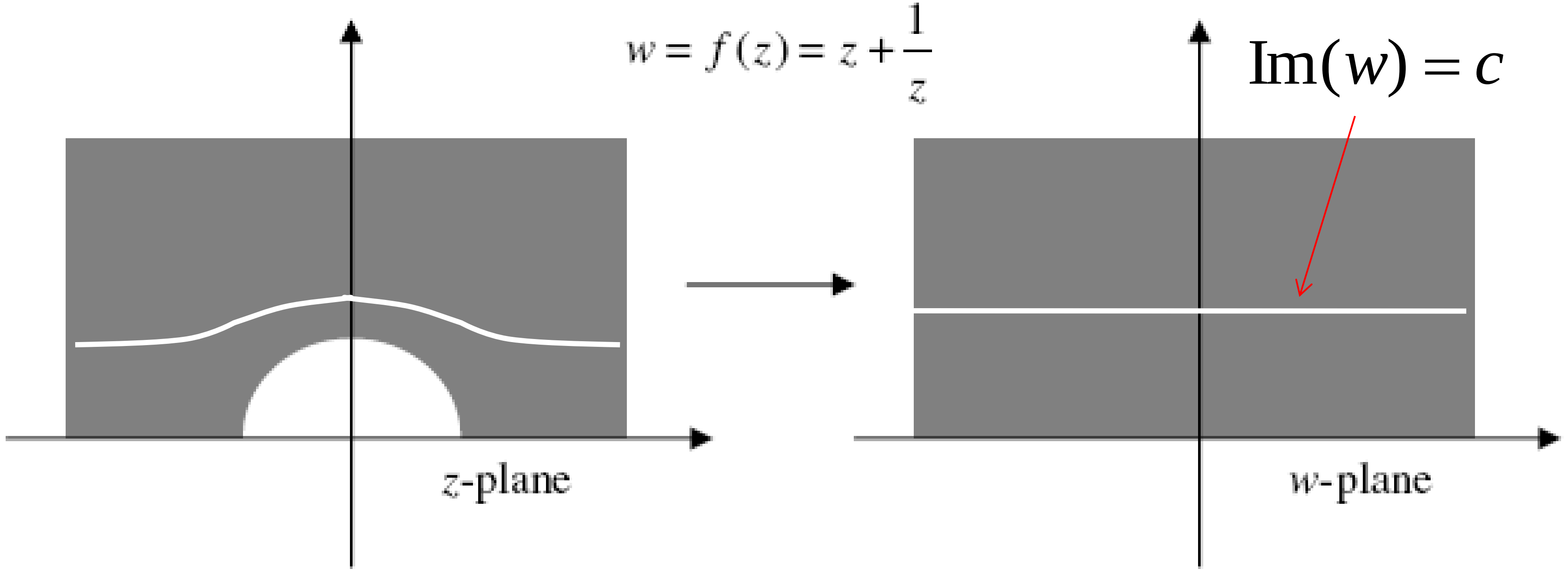
y
2
1
-3
-2
2
3
x
-1
-2

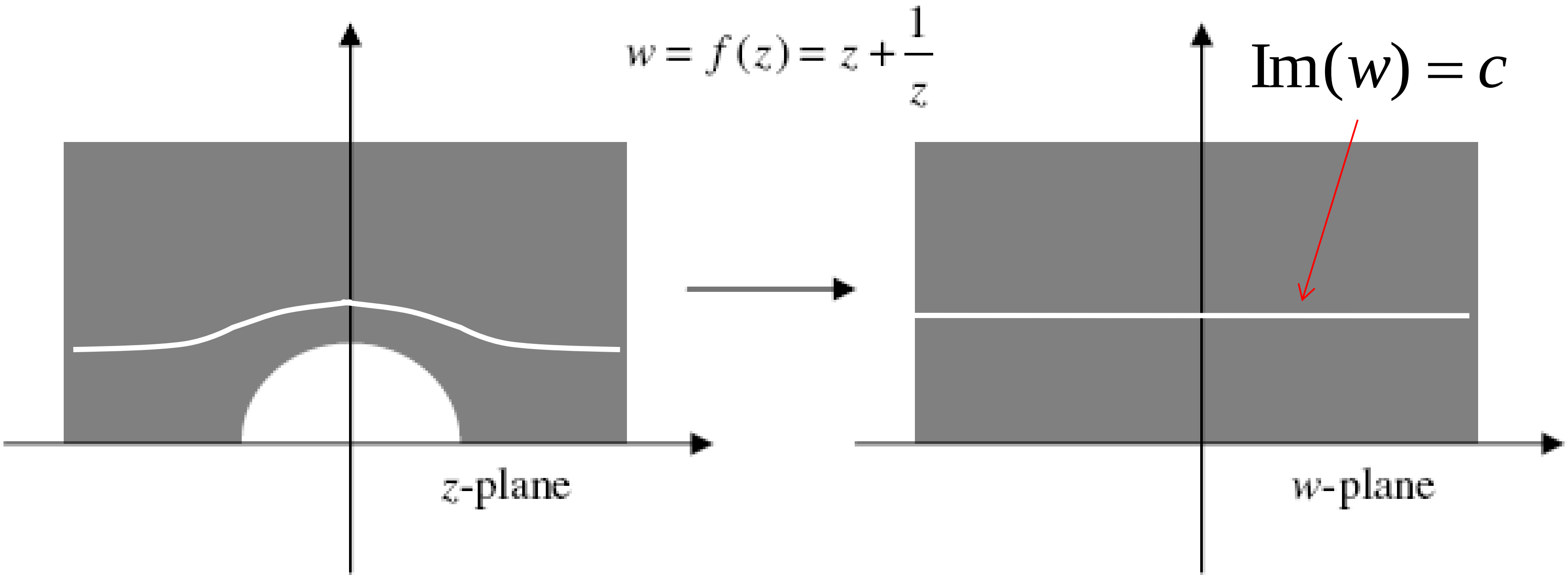
What happened?



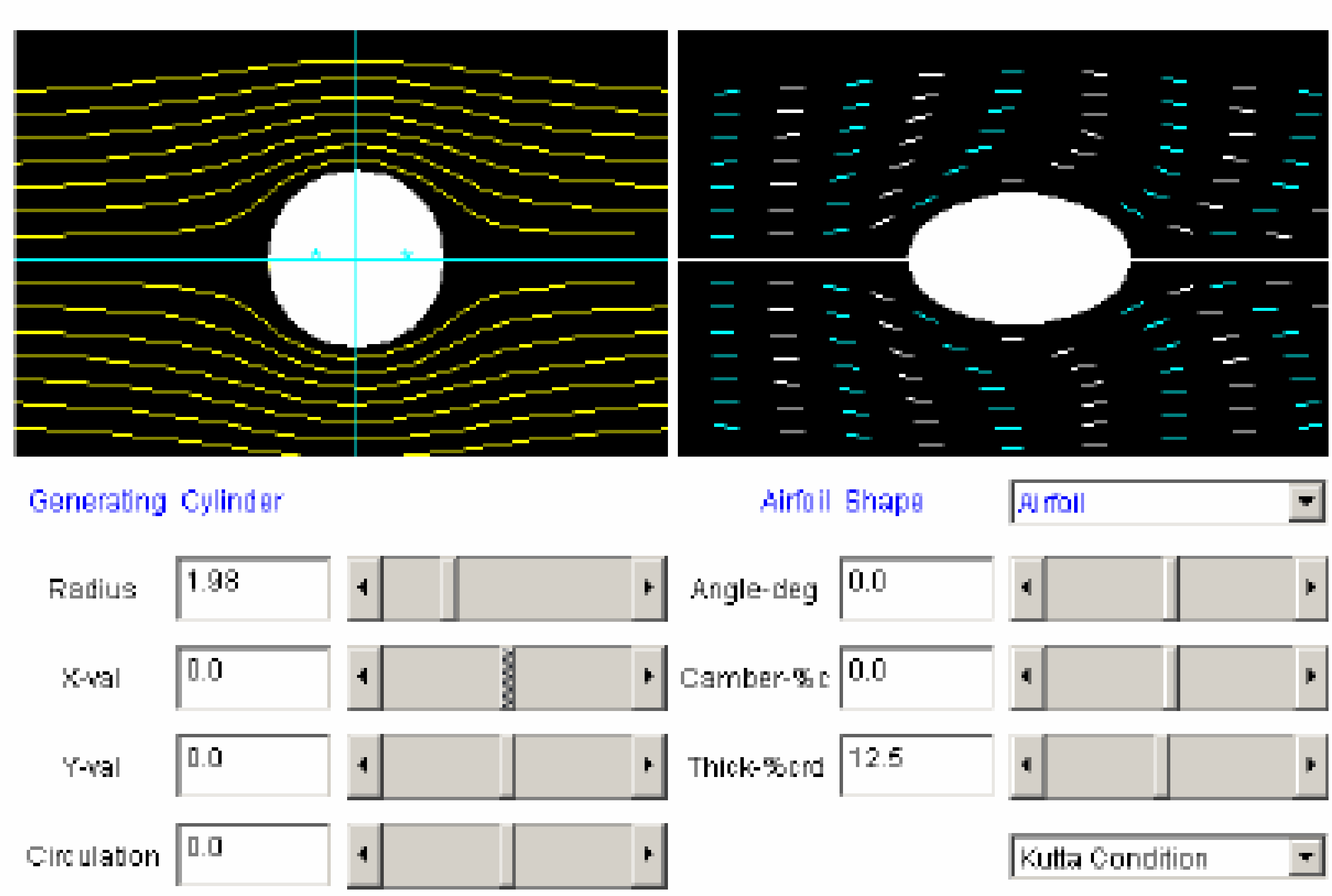
$$z = x + iy,\quad x^2 + y^2 = 1$$

$$f(z) = x + \frac{x}{x^2 + y^2} + i\left(y - \frac{y}{x^2 + y^2}\right) = 2x$$

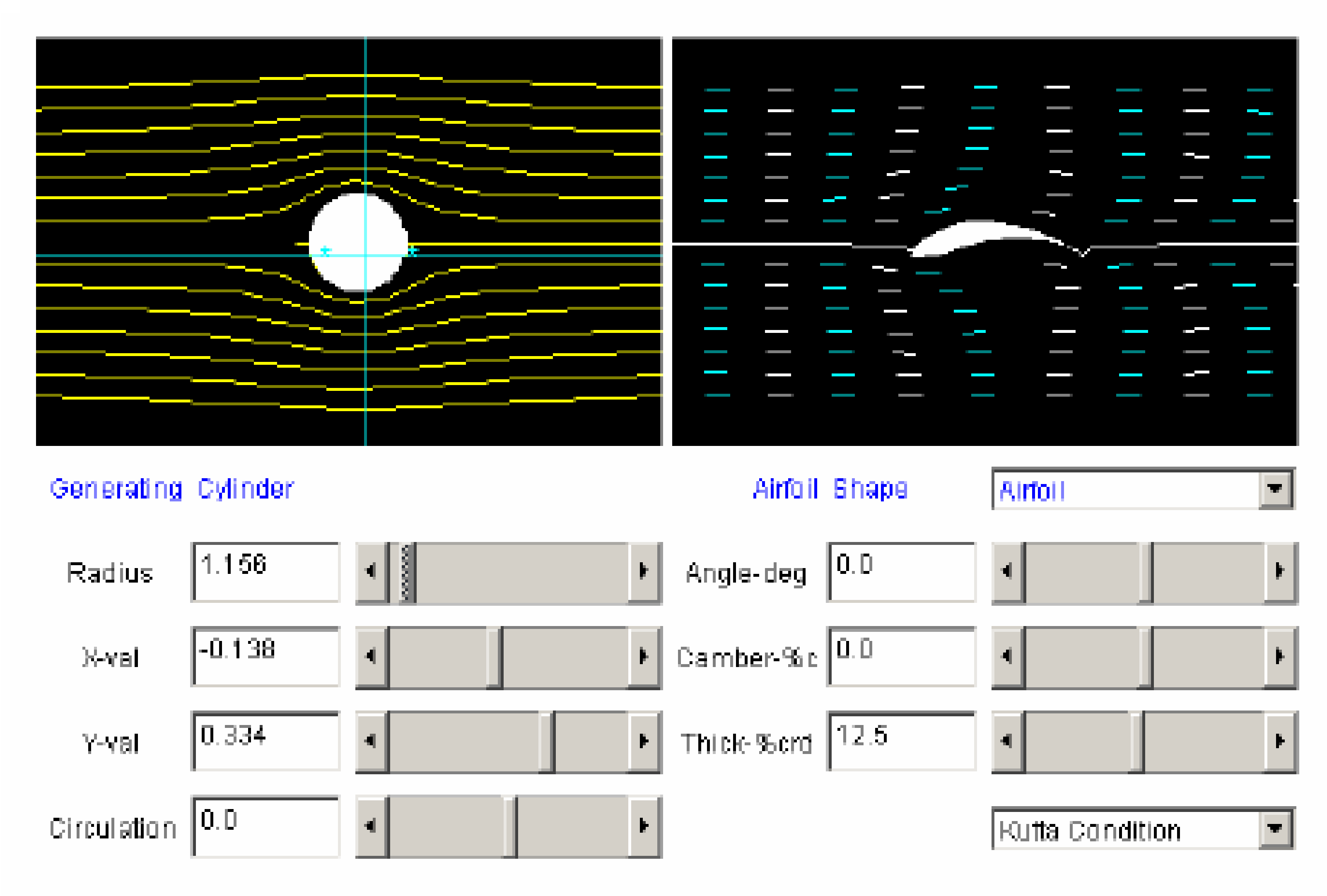

$w = f(z) = z + \frac{1}{z}$
$\mathrm{Im}(w) = c$
z-plane
w-plane



The function *f* is called the Joukowski Transformation



http://www.lerc.nasa.gov/WWW/K-12/airplane/map.html



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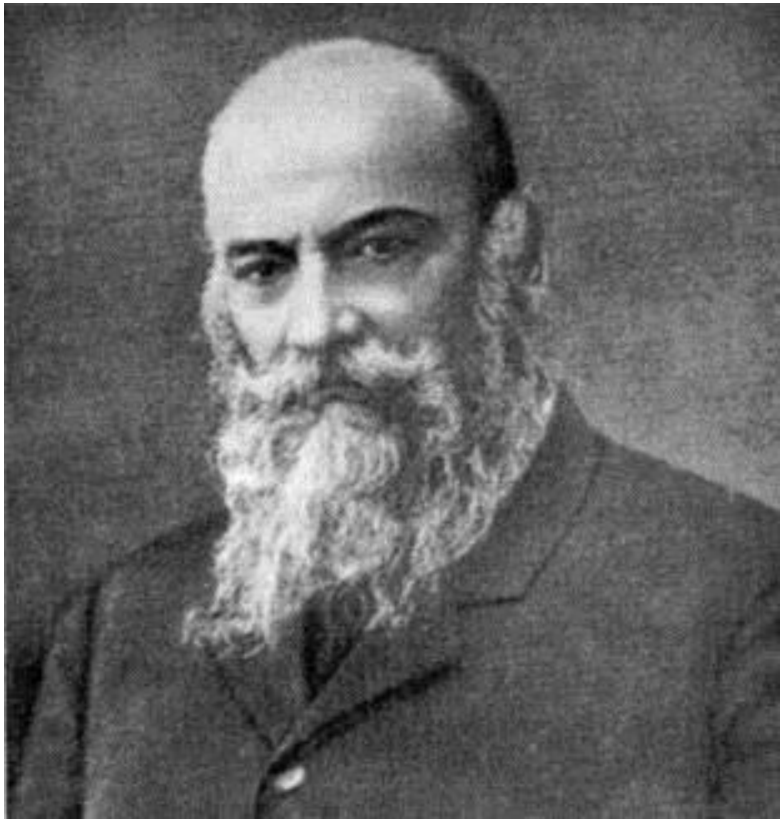


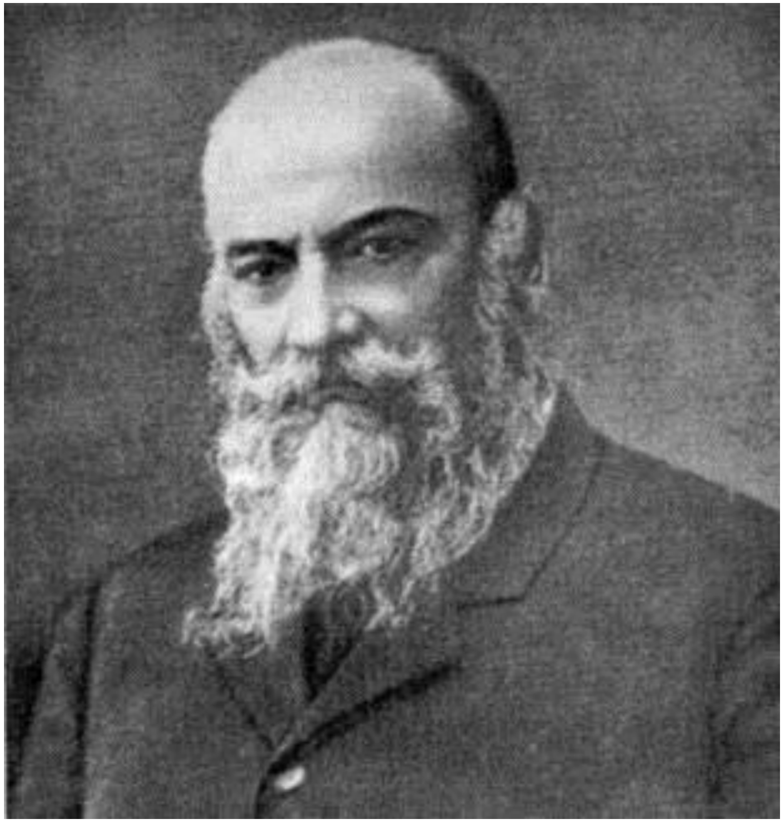
Nikolai Egorovich Joukowski was:

a. Minister of Defense under Stalin.

b. Minister of Science under Czar Nicholas II.

c. the “father of Russian Aviation.”

d. the head of the Moscow Conservatory of Music, 1910 – 1917.



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Example

A tease for next time:

What was the first movie to use fractals to computer generate its special effects?

Thank you!

robert.rogers@fredonia.edu