Math 316 Spring 2026 **Midterm Exam 1** **February 2, 2026**

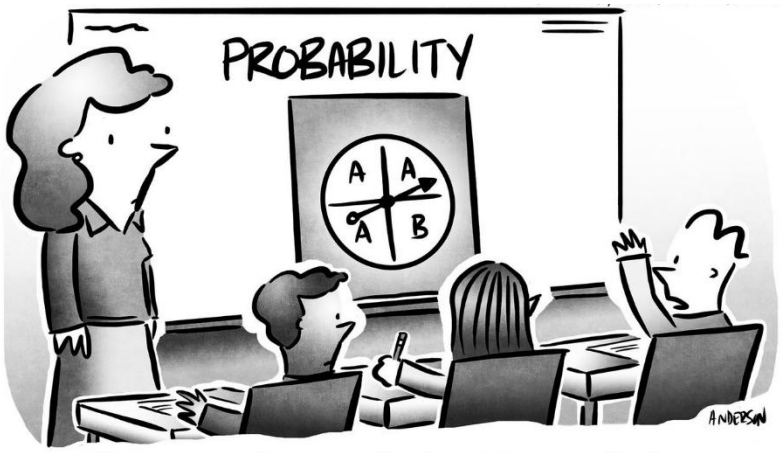
Name: Solutions

Problem	1	2	3 / 4	5 / 6	7 / 8	Total
Possible	20	22	23	14	21	100
Received						

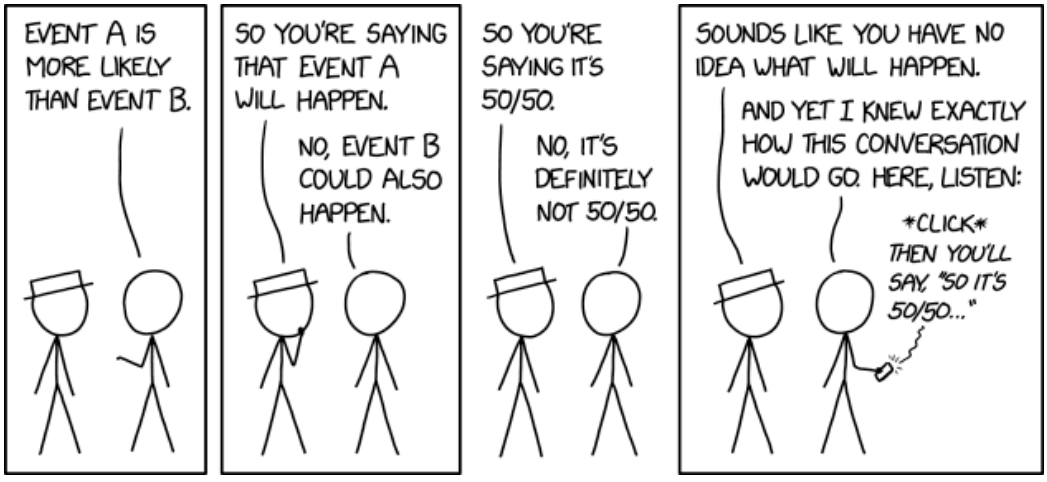
DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

YOU MAY USE A 3 X 5 CARD (BOTH SIDES) OF HANDWRITTEN NOTES AND A CALCULATOR.

FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.

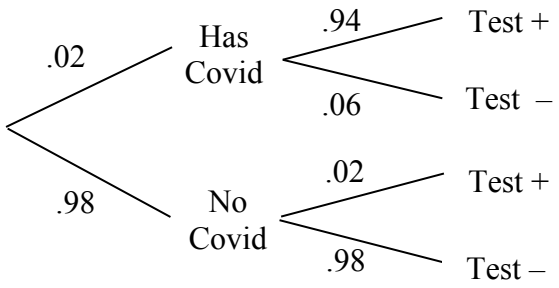


"I know mathematically that A is more likely, but I gotta say, I feel like B wants it more."



20 points 1. For the iHealth Covid-19 Antigen Rapid Test, a positive result is accurate 94% of the time, and a negative test is accurate 98% of the time. Suppose that 2% of the population currently has Covid.

Find the four values in the table below. Give values to four digits after the decimal (careful on rounding).



	Results of test: No Test	Positive	Negative
Probability person <u>has</u> Covid	.02	.4896	.0012
Probability person <u>does not</u> have Covid	.98	.5104	.9988

For example, this value is $\Pr\{No\ Covid \mid Test-\}$

Show all pertinent work below.

$$\Pr\{C \mid +\} = \frac{\Pr\{C \text{ and } +\}}{\Pr\{+\}} = \frac{(.02)(.94)}{(.02)(.94)+(.98)(.02)} = \frac{.0188}{.0384} = .4896$$

$$\Pr\{{}^{not}_{C} \mid +\} = \frac{\Pr\{{}^{not}_{C} \text{ and } +\}}{\Pr\{+\}} = \frac{(.98)(.02)}{.0384} = \frac{.0196}{.0384} = .5104 \quad (= 1 - .4896)$$

$$\Pr\{C \mid -\} = \frac{\Pr\{C \text{ and } -\}}{\Pr\{-\}} = \frac{(.02)(.06)}{(.02)(.06)+(.98)(.98)} = \frac{.0012}{.9616} = .0012$$

$$\Pr\{{}^{not}_{C} \mid -\} = \frac{\Pr\{{}^{not}_{C} \text{ and } -\}}{\Pr\{-\}} = \frac{(.98)(.98)}{.9616} = \frac{.9604}{.9616} = .9988 \quad (= 1 - .0012)$$

22 points **2**. Suppose that the average height of men in the United States is normally distributed with a mean of 70 inches and a standard deviation of 4 inches.

/4 (a) What fraction of men's heights are less than 68 inches?

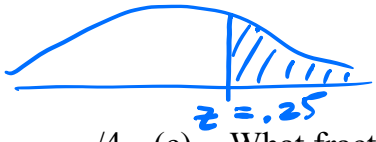
$Pr\{Y \le 68\} = Pr\{z \le \frac{68-70}{4}\} = Pr\{z \le -.5\}$

$= A(-.5) = .3085$

/4 (b) What fraction of men's heights are greater than 71 inches?

$Pr\{Y \ge 71\} = Pr\{z \ge \frac{71-70}{4}\} = Pr\{z \ge .25\} = 1 - Pr\{z < .25\}$

$= 1 - A(.25) = 1 - .5987 = .4013$



/4 (c) What fraction of men's heights are between 68 and 71 inches?

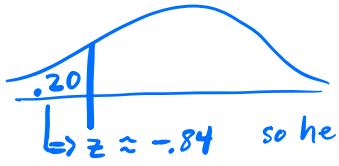
$1 - .3085 - .4013 = .2902$

OR $A(.25) - A(-.5)$

$= .5987 - .3085$

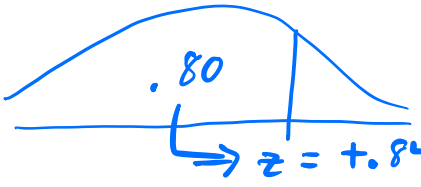
$= .2902$

/4 (d) What height is at the 20th percentile?



so height is $70 - .84(4) = 66.64$

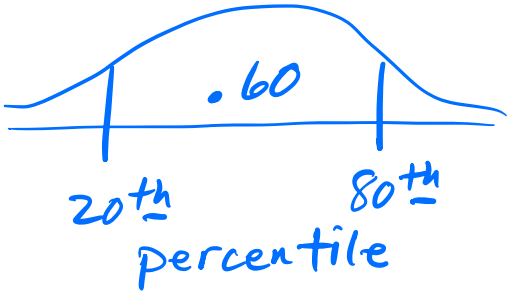
/4 (e) What height is at the 80th percentile?



so height is $70 + .84(4) = 73.36$

/2 (f) What fraction of men's heights are between the two heights you found in (d) and (e)?

60%



15 points **3**. Suppose that the average height of men in the United States is normally distributed with a mean of 70 inches and a standard deviation of 4 inches.

/6 (a) If you take a sample of 4 men, find $\Pr\{68 \le \bar{Y} \le 71\}$, the probability that the sample mean $\bar{Y}$ will be between 68 and 71 inches.

$= \Pr\left\{\frac{68-70}{\frac{4}{\sqrt{4}}} \le z \le \frac{71-70}{\frac{4}{\sqrt{4}}}\right\} = \Pr\{-1 \le z \le .5\} = A(.5) - A(-1)$

$= .6915 - .1587$

$= .5328$

/6 (b) If you take a sample of 25 men, find $\Pr\{68 \le \bar{Y} \le 71\}$, the probability that the sample mean $\bar{Y}$ will be between 68 and 71 inches.

$= \Pr\left\{\frac{68-70}{\frac{4}{\sqrt{25}}} \le z \le \frac{71-70}{\frac{4}{\sqrt{25}}}\right\} = \Pr\{-2.5 \le z \le 1.25\} = A(1.25) - A(-2.5)$

$= .8944 - .0062$

$= .8882$

/3 (c) If you take a sample of n men where $n > 25$, which one of the following is true? (Just circle one of the three statements.)

$\Pr\{68 \le \bar{Y} \le 71\} >$ the probability that you found in (b)

$\Pr\{68 \le \bar{Y} \le 71\} =$ the probability that you found in (b)

$\Pr\{68 \le \bar{Y} \le 71\} <$ the probability that you found in (b)

8 points **4**. Find the expected value μ_Y and standard deviation σ_Y given the following probability distribution for random variable Y. **Show all pertinent work**.

k	$\Pr\{Y = k\}$
0	.2
1	.6
2	.2

$\mu_Y = 0(.2) + 1(.6) + 2(.2) = 1$

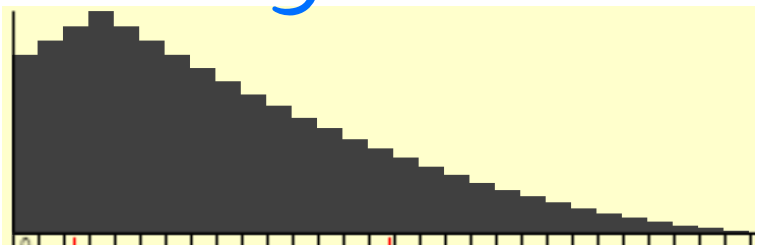
$\sigma_Y^2 = (0-1)^2(.2) + (1-1)^2(.6) + (2-1)^2(.2)$

$= .2 + 0 + .2 = .4$

so $\sigma_Y = \sqrt{.4} \approx .6325$

Larger sample size => less variation in sample (less spread out) and more normally distributed.

9 points **5**. The population distribution at right has a mean of 8 and a standard deviation of 6.



The sampling distributions using

$$n = 2,\ n = 5 \text{ and } n = 25$$

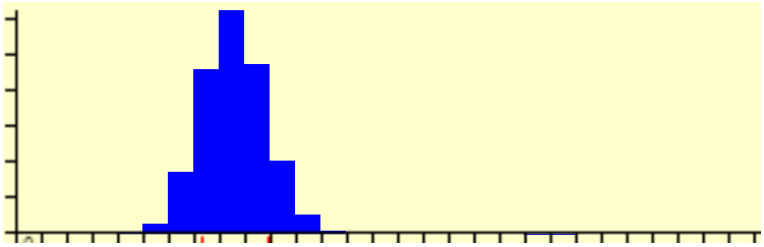
are shown below right (not in this order).

Determine the sample size in each case, and compute the sample mean and standard deviation for each sampling distribution.

$n =$ 25

$mean =$ 8

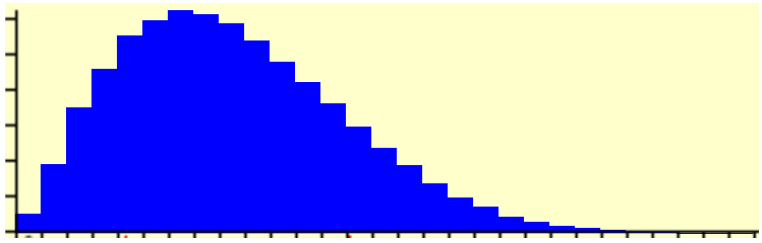
$SD =$ $6/\sqrt{25}$



$n =$ 2

$mean =$ 8

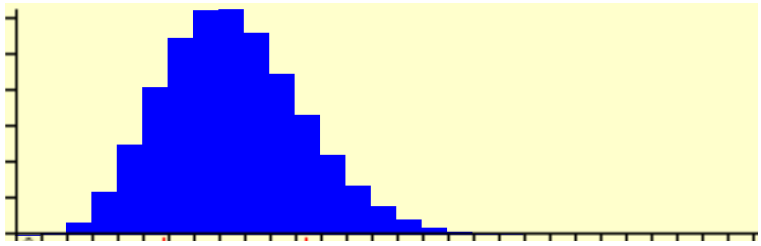
$SD =$ $6/\sqrt{2}$



$n =$ 5

$mean =$ 8

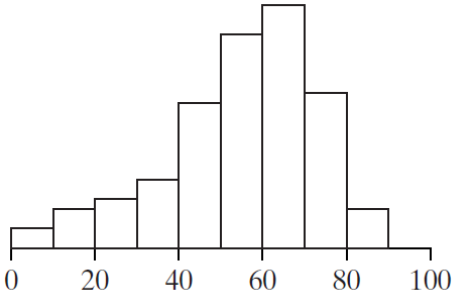
$SD =$ $6/\sqrt{5}$



5 points **6**. Estimate the mean and standard deviation of the data shown in the histogram at right.

mean $\approx$ 55 or 60

SD $\approx$ 15 or 20



For the next problem: ${}_5C_0 = 1,\ {}_5C_1 = 5,\ {}_5C_2 = 10,\ {}_5C_3 = 10,\ {}_5C_4 = 5,\ {}_5C_5 = 1$

13 points **7.** Suppose that approximately 30% of the population has Type A-positive blood. You take a sample of 5 persons. Let Y denote the number of persons in the sample with Type A-positive blood. Find each of the following.

/3 (a) $\Pr\{Y = 2\} =$ ${}_5C_2\,(.3)^2(.7)^3 = .3087$

/7 (b) $\Pr\{Y < 2\} =$ $\Pr\{Y=0\} + \Pr\{Y=1\}$

$= {}_5C_0\,(.3)^0(.7)^5 + {}_5C_1(.3)^1(.7)^4$

$= .16807 + .36015 = .52822$

/3 (c) $\Pr\{Y \geq 2\} =$ $1 - .52822 = .47178$

or $= \Pr\{Y=2\} + \Pr\{Y=3\} + \Pr\{Y=4\} + \Pr\{Y=5\}$

$= \ldots$

8 points **8.** We are interested in hair color vs. eye color.

Hair color

Eye Color	Brown	Black	Red	TOTAL
Brown	500	300	20	820
Blue	800	200	50	1,050
TOTAL	1,300	500	70	1,870

/2 (a) Find $Pr\{Black\ Hair\}$.

$\frac{500}{1870} \approx .2674$

/3 (b) Find $\Pr\{Black\ Hair \mid Brown\ Eyes\}$.

$\frac{300}{820} \approx .3659$

/3 (c) Are Black Hair and Brown Eyes independent traits or not? Explain/show work.

No. $\Pr\{\text{Black Hair} \mid \text{Brown Eyes}\} > \Pr\{\text{Black Hair}\}$

Brown-eyed people are more likely to have black hair.

(Similarly, $\Pr\{\text{Brown Eyes} \mid \text{Black Hair}\} > \Pr\{\text{Brown Eyes}\}$.)