

Invertible Matrix Theorem for $n \times n$ matrix A

- a. A is invertible.
- b. A is row equivalent to I .
- c. A has n pivot positions.
- d. $A\mathbf{x} = \mathbf{0}$ has only trivial solution.
- e. Columns of A lin. independent.
- f. Linear transf. $\mathbf{x} \rightarrow A\mathbf{x}$ 1-to-1.
- g. $A\mathbf{x} = \mathbf{b}$ has at least one solution for each \mathbf{b} .
- h. Columns of A span \mathbf{R}^n .
- i. Linear transf. $\mathbf{x} \rightarrow A\mathbf{x}$ onto.
- j. There is C such that $CA = I$.
- k. There is D such that $AD = I$.
- l. A^T is invertible.
- m. Columns of A form basis for \mathbf{R}^n .
- n. Column space of A is \mathbf{R}^n .
- o. $\dim \text{Col } A = n$, *i.e.* dimension of column space of A is n .
- p. $\text{rank } A = n$, *i.e.* rank of A is n .
- q. $\text{Nul } A = \{\mathbf{0}\}$, *i.e.* nullspace of A is $\{\mathbf{0}\}$.
- r. $\dim \text{Nul } A = 0$, the dimension of the null space of A is 0.
- s. A has n nonzero eigenvalues, *i.e.* 0 is not an eigenvalue of A .
- t. $\det A \neq 0$.
- u. $(\text{Col } A)^\perp = \{\mathbf{0}\}$, *i.e.* orthogonal complement of column space of A is $\{\mathbf{0}\}$.
- v. $(\text{Nul } A)^\perp = \mathbf{R}^n$, *i.e.* orthogonal complement of null space of A is \mathbf{R}^n .
- w. $\text{Row } A = \mathbf{R}^n$, row space of A is \mathbf{R}^n .