Some forms of echelon matrices

For reduced row echelon form, also get 0s *above* the pivots (the 1s)

* means any number

Form of matrix	Solutions (regardless of RHS)	Every row has a pivot	Columns span R^m? (ONTO)	Every column has a pivot?	Columns lin. indep.? (1-TO-1)	Free variable in solution
$\left[\begin{array}{ccc\|c} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 1 & * \end{array}\right]$	1	Yes	Yes	Yes	Yes	
$\left[\begin{array}{cccc\|c} 1 & * & * & * & * \\ 0 & 1 & * & * & * \\ 0 & 0 & 1 & * & * \end{array}\right]$	∞	Yes	Yes	No	No	x_4
$\left[\begin{array}{cccc\|c} 1 & * & * & * & * \\ 0 & 1 & * & * & * \\ 0 & 0 & 0 & 1 & * \end{array}\right]$	∞	Yes	Yes	No	No	x_3
$\left[\begin{array}{ccccc\|c} 1 & * & * & * & * & * \\ 0 & 0 & 1 & * & * & * \\ 0 & 0 & 0 & 1 & * & * \end{array}\right]$	∞	Yes	Yes	No	No	x_2, x_5
$\left[\begin{array}{cc\|c} 1 & * & * \\ 0 & 1 & * \\ 0 & 0 & a \end{array}\right]$	1 if $a = 0$ 0 if $a \neq 0$	No	No	Yes	Yes	
$\left[\begin{array}{ccc\|c} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 0 & a \end{array}\right]$	∞ if $a = 0$ 0 if $a \neq 0$	No	No	No	No	x_3 if there is a solution
$\left[\begin{array}{ccc\|c} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 1 & * \\ 0 & 0 & 0 & a \end{array}\right]$	1 if $a = 0$ 0 if $a \neq 0$	No	No	Yes	Yes	
$\left[\begin{array}{cccc\|c} 1 & * & * & * & * \\ 0 & 1 & * & * & * \\ 0 & 0 & 1 & * & * \\ 0 & 0 & 0 & 0 & a \end{array}\right]$	∞ if $a = 0$ 0 if $a \neq 0$	No	No	No	No	x_4 if there is a solution
$\left[\begin{array}{ccc\|c} 1 & * & * & * \\ 0 & 0 & 0 & a \\ 0 & 0 & 0 & b \end{array}\right]$	∞ if $a = b = 0$ 0 otherwise	No	No	No	No	x_2, x_3 if there is a solution