Math 260 Homework 5.6.5/6

The eigenvalues of $A = \begin{bmatrix} 0.4 & 0.3 \\ -p & 1.2 \end{bmatrix}$ are the roots to $(0.4-\lambda)(1.2-\lambda)-(-p)(0.3) = \cdots = \lambda^2 - 1.6\lambda + .48 + 0.3p = 0$

$$\Rightarrow \lambda = \frac{1.6 \pm \sqrt{1.6^2 - 4(.48+0.3p)}}{2} = .8 \pm \sqrt{0.16 - 0.3p}\,.$$

Note that $0.16 - 0.3p \geq 0 \;\Leftrightarrow\; p \leq \frac{0.16}{0.30} = \frac{8}{15}$.

If $p > \frac{8}{15}$, then $0.16 - 0.3p < 0$, thus $0.3p - 0.16 > 0$, and the eigenvalues will be complex:

$$\lambda = 0.8 \pm \sqrt{0.16 - 0.3p} = 0.8 \pm i\sqrt{0.3p - 0.16}\,.$$

Case	p	$.16 - .3p$	$\sqrt{.16 - .3p}$	Eigenvalues		Oscillation in population?	Long term behavior
1	0	$.16$	$.4$	1.2	0.4	No	Growth
2	$0 < \quad < 0.4$	$0.4 < \quad < 0.16$	$.2 < \quad < .4$	$1 < \quad < 1.2$	$0.4 < \quad < 0.6$	No	Growth
3	0.4	0.04	$.2$	1	$.6$	No	Equilibrium
4	$0.4 < \quad < 8/15$	$0 < \quad < 0.4$	$0 < \quad < .2$	$0.8 < \quad < 1$	$0.6 < \quad < 0.8$	No	Decay to 0
5	$8/15$	0	0	0.8	0.8	No	Decay to 0
6	$8/15 < \quad < 26/15$	< 0	(Complex)	$0.8 \pm \square\, i$	$\lvert \square \rvert < 0.6$	Yes	Decay to 0
7	$26/15$	< 0	(Complex)	$0.8 + \square\, i$	$\lvert \square \rvert = 0.6$	Yes	"Equilibrium"
8	$26/15 <$	< 0	(Complex)	$0.8 + \square\, i$	$\lvert \square \rvert > 0.6$	Yes	Growth

The magnitude of a complex number $a + bi$ is $\sqrt{a^2 + b^2}$. Consequently, in Cases 6 – 8, the magnitude of both eigenvalues will be $0.8^2 + \left(\sqrt{0.3p - 0.16}\right)^2 = 0.64 + 0.3p - 0.16 = 0.48 + 0.3p$. Notice that

$$0.48 + 0.3p = 1 \Leftrightarrow 0.3p = 0.52 \Leftrightarrow p = \frac{0.52}{0.30} = \frac{26}{15},$$

with $\sqrt{0.3p - 0.16} = \sqrt{0.3\left(\frac{0.52}{0.3}\right) - 0.16} = \sqrt{0.36} = 0.6 \Rightarrow$ eigenvalues are $0.8 \pm 0.6\,i$, with magnitudes $0.8^2 + 0.6^2 = 1$.