

Examples of Elementary Row Operations
(also known as *Gaussian Elimination* or *Row Reduction*)

The original equations:

$$\begin{aligned} 2x + 3y &= 7 \\ -x + 4y &= 2 \end{aligned}$$

The original matrix:

$$\left[\begin{array}{cc|c} 2 & 3 & 7 \\ -1 & 4 & 2 \end{array} \right]$$

Multiply Eqn 2 by 2:

$$\begin{aligned} 2x + 3y &= 7 \\ -2x + 8y &= 4 \end{aligned}$$

Multiply Row 2 by 2:

$$\left[\begin{array}{cc|c} 2 & 3 & 7 \\ -2 & 8 & 4 \end{array} \right]$$

Add Eqn 1 to Eqn 2:

$$\begin{aligned} 2x + 3y &= 7 \\ 11y &= 11 \end{aligned}$$

Add Row 1 to Row 2:

$$\left[\begin{array}{cc|c} 2 & 3 & 7 \\ 0 & 11 & 11 \end{array} \right]$$

Divide Eqn 2 by 11 (multiply by $\frac{1}{11}$):

$$\begin{aligned} 2x + 3y &= 7 \\ y &= 1 \end{aligned}$$

Divide Row 2 by 11 (multiply by $\frac{1}{11}$):

$$\left[\begin{array}{cc|c} 2 & 3 & 7 \\ 0 & 1 & 1 \end{array} \right]$$

Multiply Eqn 2 by -3:

$$\begin{aligned} 2x + 3y &= 7 \\ -3y &= -3 \end{aligned}$$

Multiply Row 2 by -3:

$$\left[\begin{array}{cc|c} 2 & 3 & 7 \\ 0 & -3 & -3 \end{array} \right]$$

Add Eqn 2 to Eqn 1:

$$\begin{aligned} 2x &= 4 \\ -3y &= -3 \end{aligned}$$

Add Row 2 to Row 1:

$$\left[\begin{array}{cc|c} 2 & 0 & 4 \\ 0 & -3 & -3 \end{array} \right]$$

Divide Eqn 1 by 2 and Eqn 2 by -3:

$$\begin{aligned} x &= 2 \\ y &= 1 \end{aligned}$$

Divide Row 1 by 2 and Row 2 by -3:

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array} \right]$$

Example of Gaussian Elimination *with Pivoting*

Initial matrix

$$\left[\begin{array}{ccc|c} 2 & 4 & -2 & 12 \\ 3 & 5 & 0 & 13 \\ 1 & -2 & 1 & -4 \end{array} \right]$$

$\frac{1}{2} \cdot$ Row 1

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 6 \\ 3 & 5 & 0 & 13 \\ 1 & -2 & 1 & -4 \end{array} \right]$$

Row 2 + (-3) · Row 1

Row 3 + (-1) · Row 1

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 6 \\ 0 & -1 & 3 & -5 \\ 0 & -4 & 2 & -10 \end{array} \right]$$

(-1) · Row 2

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 6 \\ 0 & 1 & -3 & 5 \\ 0 & -4 & 2 & -10 \end{array} \right]$$

Row 1 + (-2) · Row 2

Row 3 + (+4) · Row 2

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & -4 \\ 0 & 1 & -3 & 5 \\ 0 & 0 & -10 & 10 \end{array} \right]$$

$-\frac{1}{10} \cdot$ Row 3

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & -4 \\ 0 & 1 & -3 & 5 \\ 0 & 0 & 1 & -1 \end{array} \right]$$

Row 1 + (-5) · Row 3

Row 2 + 3 · Row 3

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array} \right]$$

Initial equations

$$2x + 4y - 2z = 12$$

$$3x + 5y = 13$$

$$x - 2y + z = -4$$

$\frac{1}{2} \cdot$ Eqn 1

$$x + 2y - z = 6$$

$$3x + 5y = 13$$

$$x - 2y + z = -4$$

Eqn 2 + (-3) · Eqn 1

Eqn 3 + (-1) · Eqn 1

$$x + 2y - z = 6$$

$$-y + 3z = -5$$

$$-4y + 2z = -10$$

(-1) · Eqn 2

$$x + 2y - z = 6$$

$$y - 3z = 5$$

$$-4y + 2z = -10$$

Eqn 1 + (-2) · Eqn 2

Eqn 3 + (+4) · Eqn 2

$$x + 5z = -4$$

$$y - 3z = 5$$

$$-10z = 10$$

$-\frac{1}{10} \cdot$ Eqn 3

$$x + 5z = -4$$

$$y - 3z = 5$$

$$z = -1$$

Eqn 1 + (-5) · Eqn 3

Eqn 2 + 3 · Eqn 3

$$x = 1$$

$$y = 2$$

$$z = -1$$