Math 260 Homework 2.4

1. Apply the row-column rule as if the matrix entries were numbers, but for each product always write the entry of the left block-matrix on the *left*.

$$\begin{bmatrix} I & 0 \\ E & I \end{bmatrix}\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} IA+0C & IB+0D \\ EA+IC & EB+ID \end{bmatrix} = \begin{bmatrix} A & B \\ EA+C & EB+D \end{bmatrix}$$

2. Apply the row-column rule as if the matrix entries were numbers, but for each product always write the entry of the left block-matrix on the *left*.

$$\begin{bmatrix} E & 0 \\ 0 & F \end{bmatrix}\begin{bmatrix} P & Q \\ R & S \end{bmatrix} = \begin{bmatrix} EP+0R & EQ+0S \\ 0P+FR & 0Q+FS \end{bmatrix} = \begin{bmatrix} EP & EQ \\ FR & FS \end{bmatrix}$$

3. Apply the row-column rule as if the matrix entries were numbers, but for each product always write the entry of the left block-matrix on the *left*.

$$\begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0A+IC & 0B+ID \\ IA+0C & IB+0D \end{bmatrix} = \begin{bmatrix} C & D \\ A & B \end{bmatrix}$$

4. Apply the row-column rule as if the matrix entries were numbers, but for each product always write the entry of the left block-matrix on the *left*.

$$\begin{bmatrix} I & 0 \\ -E & I \end{bmatrix}\begin{bmatrix} W & X \\ Y & Z \end{bmatrix} = \begin{bmatrix} IW+0Y & IX+0Z \\ -EW+IY & -EX+IZ \end{bmatrix} = \begin{bmatrix} W & X \\ -EW+Y & -EX+Z \end{bmatrix}$$

5. Compute the left side of the equation:

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}\begin{bmatrix} I & 0 \\ X & Y \end{bmatrix} = \begin{bmatrix} AI+BX & A0+BY \\ CI+0X & C0+0Y \end{bmatrix}$$

Set this equal to the right side of the equation:

$$\begin{bmatrix} A+BX & BY \\ C & 0 \end{bmatrix} = \begin{bmatrix} 0 & I \\ Z & 0 \end{bmatrix} \quad \text{so that} \quad \begin{matrix} A+BX=0 & BY=I \\ C=Z & 0=0 \end{matrix}$$

Since the (2, 1) blocks are equal, $Z = C$. Since the (1, 2) blocks are equal, $BY = I$. To proceed further, assume that B and Y are square. Then the equation $BY = I$ implies that B is invertible, by the IMT, and $Y = B^{-1}$. (See the boxed remark that follows the IMT.) Finally, from the equality of the (1, 1) blocks,

$$BX = -A, \quad B^{-1}BX = B^{-1}(-A), \quad \text{and} \quad X = -B^{-1}A.$$

The order of the factors for X is crucial.

Note: For simplicity, statements (j) and (k) in the Invertible Matrix Theorem involve square matrices C and D. Actually, if A is $n \times n$ and if C is any matrix such that AC is the $n \times n$ identity matrix, then C must be $n \times n$, too. (For AC to be defined, C must have n rows, and the equation $AC = I$ implies that C has n columns.) Similarly, $DA = I$ implies that D is $n \times n$. Rather than discuss this in class, I expect that in Exercises 5–8, when students see an equation such as $BY = I$, they will decide that *both* B and Y should be square in order to use the IMT.

6. Compute the left side of the equation:

$$\begin{bmatrix} X & 0 \\ Y & Z \end{bmatrix}\begin{bmatrix} A & 0 \\ B & C \end{bmatrix} = \begin{bmatrix} XA+0B & X0+0C \\ YA+ZB & Y0+ZC \end{bmatrix} = \begin{bmatrix} XA & 0 \\ YA+ZB & ZC \end{bmatrix}$$

Set this equal to the right side of the equation:

$$\begin{bmatrix} XA & 0 \\ YA+ZB & ZC \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \quad \text{so that} \quad \begin{matrix} XA=I & 0=0 \\ YA+ZB=0 & ZC=I \end{matrix}$$

To use the equality of the (1, 1) blocks, assume that A and X are square. By the IMT, the equation $XA = I$ implies that A is invertible and $X = A^{-1}$. (See the boxed remark that follows the IMT.) Similarly, if C and Z are assumed to be square, then the equation $ZC = I$ implies that C is invertible, by the IMT, and $Z = C^{-1}$. Finally, use the (2, 1) blocks and right-multiplication by A^{-1}:

$$YA = -ZB = -C^{-1}B, \quad YAA^{-1} = (-C^{-1}B)A^{-1}, \quad \text{and} \quad Y = -C^{-1}BA^{-1}$$

The order of the factors for Y is crucial.

7. Compute the left side of the equation:

$$\begin{bmatrix} X & 0 & 0 \\ Y & 0 & I \end{bmatrix}\begin{bmatrix} A & Z \\ 0 & 0 \\ B & I \end{bmatrix} = \begin{bmatrix} XA+0+0B & XZ+0+0I \\ YA+0+IB & YZ+0+II \end{bmatrix}$$

Set this equal to the right side of the equation:

$$\begin{bmatrix} XA & XZ \\ YA+B & YZ+I \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \quad \text{so that} \quad \begin{matrix} XA=I & XZ=0 \\ YA+B=0 & YZ+I=I \end{matrix}$$

To use the equality of the (1, 1) blocks, assume that A and X are square. By the IMT, the equation $XA = I$ implies that A is invertible and $X = A^{-1}$. (See the boxed remark that follows the IMT) Also, X is invertible. Since $XZ = 0$, $X^{-1}XZ = X^{-1}0 = 0$, so Z must be 0. Finally, from the equality of the (2, 1) blocks, $YA = -B$. Right-multiplication by A^{-1} shows that $YAA^{-1} = -BA^{-1}$ and $Y = -BA^{-1}$. The order of the factors for Y is crucial.

8. Compute the left side of the equation:

$$\begin{bmatrix} A & B \\ 0 & I \end{bmatrix}\begin{bmatrix} X & Y & Z \\ 0 & 0 & I \end{bmatrix} = \begin{bmatrix} AX+B0 & AY+B0 & AZ+BI \\ 0X+I0 & 0Y+I0 & 0Z+II \end{bmatrix}$$

Set this equal to the right side of the equation:

$$\begin{bmatrix} AX & AY & AZ+B \\ 0 & 0 & I \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & I \end{bmatrix}$$

To use the equality of the (1, 1) blocks, assume that A and X are square. By the IMT, the equation $XA = I$ implies that A is invertible and $X = A^{-1}$. (See the boxed remark that follows the IMT. Since $AY = 0$, from the equality of the (1, 2) blocks, left-multiplication by A^{-1} gives $A^{-1}AY = A^{-1}0 = 0$, so $Y = 0$. Finally, from the (1, 3) blocks, $AZ = -B$. Left-multiplication by A^{-1} gives $A^{-1}AZ = A^{-1}(-B)$, and $Z = -A^{-1}B$. The order of the factors for Z is crucial.

Note: The *Study Guide* tells students, "Problems such as 5–10 make good exam questions. Remember to mention the IMT when appropriate, and remember that matrix multiplication is generally not commutative." When a problem statement includes a condition that a matrix is square, I expect my students to mention this fact when they apply the IMT.

9. Compute the left side of the equation:

$$\begin{bmatrix} I & 0 & 0 \\ A_{21} & I & 0 \\ A_{31} & 0 & I \end{bmatrix}\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \\ B_{31} & B_{32} \end{bmatrix} = \begin{bmatrix} IB_{11}+0B_{21}+0B_{31} & IB_{12}+0B_{22}+0B_{32} \\ A_{21}B_{11}+IB_{21}+0B_{31} & A_{21}B_{12}+IB_{22}+0B_{32} \\ A_{31}B_{11}+0B_{21}+IB_{31} & A_{31}B_{12}+0B_{22}+IB_{32} \end{bmatrix}$$

Set this equal to the right side of the equation:

$$\begin{bmatrix} B_{11} & B_{12} \\ A_{21}B_{11}+B_{21} & A_{21}B_{12}+B_{22} \\ A_{31}B_{11}+B_{31} & A_{31}B_{12}+B_{32} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ 0 & C_{22} \\ 0 & C_{32} \end{bmatrix}$$

$$\begin{aligned} & B_{11}=C_{11} && B_{12}=C_{12} \\ \text{so that } & A_{21}B_{11}+B_{21}=0 && A_{21}B_{12}+B_{22}=C_{22} \\ & A_{31}B_{11}+B_{31}=0 && A_{31}B_{12}+B_{32}=C_{32} \end{aligned}$$

Since the (2,1) blocks are equal, $A_{21}B_{11}+B_{21}=0$ and $A_{21}B_{11}=-B_{21}$. Since B_{11} is invertible, right multiplication by B_{11}^{-1} gives $A_{21}=-B_{21}B_{11}^{-1}$. Likewise since the (3,1) blocks are equal, $A_{31}B_{11}+B_{31}=0$ and $A_{31}B_{11}=-B_{31}$. Since B_{11} is invertible, right multiplication by B_{11}^{-1} gives $A_{31}=-B_{31}B_{11}^{-1}$. Finally, from the (2,2) entries, $A_{21}B_{12}+B_{22}=C_{22}$. Since $A_{21}=-B_{21}B_{11}^{-1}, C_{22}=-B_{21}B_{11}^{-1}B_{12}+B_{22}$.

10. Since the two matrices are inverses,

$$\begin{bmatrix} I & 0 & 0 \\ A & I & 0 \\ B & D & I \end{bmatrix}\begin{bmatrix} I & 0 & 0 \\ P & I & 0 \\ Q & R & I \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$$

Compute the left side of the equation:

$$\begin{bmatrix} I & 0 & 0 \\ A & I & 0 \\ B & D & I \end{bmatrix}\begin{bmatrix} I & 0 & 0 \\ P & I & 0 \\ Q & R & I \end{bmatrix} = \begin{bmatrix} II+0P+0Q & I0+0I+0R & I0+00+0I \\ AI+IP+0Q & A0+II+0R & A0+I0+0I \\ BI+DP+IQ & B0+DI+IR & B0+D0+II \end{bmatrix}$$

Set this equal to the right side of the equation:

$$\begin{bmatrix} I & 0 & 0 \\ A+P & I & 0 \\ B+DP+Q & D+R & I \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$$

$$\begin{aligned} & I=I && 0=0 && 0=0 \\ \text{so that } & A+P=0 && I=I && 0=0 \\ & B+DP+Q=0 && D+R=0 && I=I \end{aligned}$$

Since the (2,1) blocks are equal, $A+P=0$ and $P=-A$. Likewise since the (3, 2) blocks are equal, $D+R=0$ and $R=-D$. Finally, from the (3,1) entries, $B+DP+Q=0$ and $Q=-B-DP$. Since $P=-A,\ Q=-B-D(-A)=-B+DA$.

11. **a.** True. See the subsection *Addition and Scalar Multiplication.*

b. False. See the paragraph before Example 3.

12. **a.** False. The both AB and BA are defined.

b. False. The R^T and Q^T also need to be switched.

13. You are asked to establish an *if and only if* statement. First, supose that A is invertible, and let $A^{-1}=\begin{bmatrix} D & E \\ F & G \end{bmatrix}$. Then

$$\begin{bmatrix} B & 0 \\ 0 & C \end{bmatrix}\begin{bmatrix} D & E \\ F & G \end{bmatrix} = \begin{bmatrix} BD & BE \\ CF & CG \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

Since B is square, the equation $BD=I$ implies that B is invertible, by the IMT. Similarly, $CG=I$ implies that C is invertible. Also, the equation $BE=0$ imples that $E=B^{-1}0=0$. Similarly $F=0$. Thus

$$A^{-1}=\begin{bmatrix} B & 0 \\ 0 & C \end{bmatrix}^{-1} = \begin{bmatrix} D & E \\ E & G \end{bmatrix} = \begin{bmatrix} B^{-1} & 0 \\ 0 & C^{-1} \end{bmatrix} \qquad (*)$$

This proves that A is invertible *only if* B and C are invertible. For the "*if*" part of the statement, suppose that B and C are invertible. Then (*) provides a likely candidate for A^{-1} which can be used to show that A is invertible. Compute:

$$\begin{bmatrix} B & 0 \\ 0 & C \end{bmatrix}\begin{bmatrix} B^{-1} & 0 \\ 0 & C^{-1} \end{bmatrix} = \begin{bmatrix} BB^{-1} & 0 \\ 0 & CC^{-1} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

Since A is square, this calculation and the IMT imply that A is invertible. (Don't forget this final sentence. Without it, the argument is incomplete.) Instead of that sentence, you could add the equation:

$$\begin{bmatrix} B^{-1} & 0 \\ 0 & C^{-1} \end{bmatrix}\begin{bmatrix} B & 0 \\ 0 & C \end{bmatrix} = \begin{bmatrix} B^{-1}B & 0 \\ 0 & C^{-1}C \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

14. You are asked to establish an *if and only if* statement. First suppose that A is invertible. Example 5 shows that A_{11} and A_{22} are invertible. This proves that A is invertible *only if* A_{11} A_{22} are invertible. For the *if* part of this statement, suppose that A_{11} and A_{22} are invertible. Then the formula in Example 5 provides a likely candidate for A^{-1} which can be used to show that A is invertible . Compute:

$$\begin{aligned} \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}\begin{bmatrix} A_{11}^{-1} & -A_{11}^{-1}A_{12}A_{22}^{-1} \\ 0 & A_{22}^{-1} \end{bmatrix} &= \begin{bmatrix} A_{11}A_{11}^{-1}+A_{12}0 & A_{11}(-A_{11}^{-1})A_{12}A_{22}^{-1}+A_{12}A_{22}^{-1} \\ 0A_{11}^{-1}+A_{22}0 & 0(-A_{11}^{-1})A_{12}A_{22}^{-1}+A_{22}A_{22}^{-1} \end{bmatrix} \\ &= \begin{bmatrix} I & -(A_{11}A_{11}^{-1})A_{12}A_{22}^{-1}+A_{12}A_{22}^{-1} \\ 0 & I \end{bmatrix} \\ &= \begin{bmatrix} I & -A_{12}A_{22}^{-1}+A_{12}A_{22}^{-1} \\ 0 & I \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \end{aligned}$$

Since A is square, this calculation and the IMT imply that A is invertible.

15. The column-row expansions of G_k and G_{k+1} are:

$$\begin{aligned} G_k &= X_k X_k^T \\ &= \text{col}_1(X_k)\text{row}_1(X_k^T)+\cdots+\text{col}_k(X_k)\text{row}_k(X_k^T) \end{aligned}$$

and

$$G_{k+1} = X_{k+1}X_{k+1}^T$$
$$= \text{col}_1(X_{k+1})\text{row}_1(X_{k+1}^T) + \cdots + \text{col}_k(X_{k+1})\text{row}_k(X_{k+1}^T) + \text{col}_{k+1}(X_{k+1})\text{row}_{k+1}(X_{k+1}^T)$$
$$= \text{col}_1(X_k)\text{row}_1(X_k^T) + \cdots + \text{col}_k(X_k)\text{row}_k(X_k^T) + \text{col}_{k+1}(X_{k+1})\text{row}_{k+1}(X_k^T)$$
$$= G_k + \text{col}_{k+1}(X_{k+1})\text{row}_{k+1}(X_k^T)$$

since the first k columns of X_{k+1} are identical to the first k columns of X_k. Thus to update G_k to produce G_{k+1}, the matrix $\text{col}_{k+1}(X_{k+1})\ \text{row}_{k+1}(X_k^T)$ should be added to G_k.

16. Compute the right side of the equation:

$$\begin{bmatrix} I & 0 \\ X & I \end{bmatrix}\begin{bmatrix} A_{11} & 0 \\ 0 & S \end{bmatrix}\begin{bmatrix} I & Y \\ 0 & I \end{bmatrix} = \begin{bmatrix} A_{11} & 0 \\ XA_{11} & S \end{bmatrix}\begin{bmatrix} I & Y \\ 0 & I \end{bmatrix} = \begin{bmatrix} A_{11} & A_{11}Y \\ XA_{11} & XA_{11}Y + S \end{bmatrix}$$

Set this equal to the left side of the equation:

$$\begin{bmatrix} A_{11} & A_{11}Y \\ XA_{11} & XA_{11}Y + S \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \text{ so that } \begin{matrix} A_{11} = A_{11} & A_{11}Y = A_{12} \\ XA_{11} = A_{21} & XA_{11}Y + S = A_{22} \end{matrix}$$

Since the (1, 2) blocks are equal, $A_{11}Y = A_{12}$. Since A_{11} is invertible, left multiplication by A_{11}^{-1} gives $Y = A_{11}^{-1}A_{12}$. Likewise since the (2,1) blocks are equal, $XA_{11} = A_{21}$. Since A_{11} is invertible, right multiplication by A_{11}^{-1} gives that $X = A_{21}A_{11}^{-1}$. One can check that the matrix S as given in the exercise satisfies the equation $XA_{11}Y + S = A_{22}$ with the calculated values of X and Y given above.

17. Suppose that A and A_{11} are invertible. First note that

$$\begin{bmatrix} I & 0 \\ X & I \end{bmatrix}\begin{bmatrix} I & 0 \\ -X & I \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

and

$$\begin{bmatrix} I & Y \\ 0 & I \end{bmatrix}\begin{bmatrix} I & -Y \\ 0 & I \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

Since the matrices $\begin{bmatrix} I & 0 \\ X & I \end{bmatrix}$ and $\begin{bmatrix} I & Y \\ 0 & I \end{bmatrix}$ are square, they are both invertible by the IMT. Equation (7) may be left multipled by $\begin{bmatrix} I & 0 \\ X & I \end{bmatrix}^{-1}$ and right multipled by $\begin{bmatrix} I & Y \\ 0 & I \end{bmatrix}^{-1}$ to find

$$\begin{bmatrix} A_{11} & 0 \\ 0 & S \end{bmatrix} = \begin{bmatrix} I & 0 \\ X & I \end{bmatrix}^{-1} A \begin{bmatrix} I & Y \\ 0 & I \end{bmatrix}^{-1}$$

Thus by Theorem 6, the matrix $\begin{bmatrix} A_{11} & 0 \\ 0 & S \end{bmatrix}$ is invertible as the product of invertible matrices. Finally, Exercise 13 above may be used to show that S is invertible.

18. Since $W = [X\ \mathbf{x}_0]$,

$$W^TW = \begin{bmatrix} X^T \\ \mathbf{x}_0^T \end{bmatrix}[X\ \ \mathbf{x}_0] = \begin{bmatrix} X^TX & X^T\mathbf{x}_0 \\ \mathbf{x}_0^TX & \mathbf{x}_0^T\mathbf{x}_0 \end{bmatrix}$$

By applying the formula for S from Exercise 15, S may be computed:

$$S = \mathbf{x}_0^T\mathbf{x}_0 - \mathbf{x}_0^TX(X^TX)^{-1}X^T\mathbf{x}_0$$
$$= \mathbf{x}_0^T(I_m - X(X^TX)^{-1}X^T)\mathbf{x}_0$$
$$= \mathbf{x}_0^TM\mathbf{x}_0$$

19. The matrix equation (8) in the text is equivalent to

$$(A - sI_n)\mathbf{x} + B\mathbf{u} = \mathbf{0} \text{ and } C\mathbf{x} + \mathbf{u} = \mathbf{y}$$

Rewrite the first equation as $(A - sI_n)\mathbf{x} = -B\mathbf{u}$. When $A - sI_n$ is invertible,

$$\mathbf{x} = (A - sI_n)^{-1}(-B\mathbf{u}) = -(A - sI_n)^{-1}B\mathbf{u}$$

Substitute this formula for $\mathbf{x}$ into the second equation above:

$$C(-(A - sI_n)^{-1}B\mathbf{u}) + \mathbf{u} = \mathbf{y}\text{, so that } I_m\mathbf{u} - C(A - sI_n)^{-1}B\mathbf{u} = \mathbf{y}$$

Thus $\mathbf{y} = (I_m - C(A - sI_n)^{-1}B)\mathbf{u}$. If $W(s) = I_m - C(A - sI_n)^{-1}B$, then $\mathbf{y} = W(s)\mathbf{u}$. The matrix $W(s)$ is the Schur complement of the matrix $A - sI_n$ in the system matrix in equation (8)

20. The matrix in question is

$$\begin{bmatrix} A - BC - sI_n & B \\ -C & I_m \end{bmatrix}$$

By applying the formula for S from Exercise 16, S may be computed:

$$S = I_m - (-C)(A - BC - sI_n)^{-1}B$$
$$= I_m + C(A - BC - sI_n)^{-1}B$$

21. a. $A^2 = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1+0 & 0+0 \\ 2-2 & 0+(-1)^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

b. $M^2 = \begin{bmatrix} A & 0 \\ I & -A \end{bmatrix}\begin{bmatrix} A & 0 \\ I & -A \end{bmatrix} = \begin{bmatrix} A^2+0 & 0+0 \\ A-A & 0+(-A)^2 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$

22. Let C be any nonzero 2×2 matrix. Define $M = \begin{bmatrix} I_2 & 0 & 0 \\ 0 & I_2 & 0 \\ C & 0 & -I_2 \end{bmatrix}$. Then

$$M^2 = \begin{bmatrix} I_2 & 0 & 0 \\ 0 & I_2 & 0 \\ C & 0 & -I_2 \end{bmatrix}\begin{bmatrix} I_2 & 0 & 0 \\ 0 & I_2 & 0 \\ C & 0 & -I_2 \end{bmatrix} = \begin{bmatrix} I_2 & 0 & 0 \\ 0 & I_2 & 0 \\ C-C & 0 & I_2 \end{bmatrix} = \begin{bmatrix} I_2 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_2 \end{bmatrix}$$

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$$\sim\begin{bmatrix}1&0&0&-7\\0&1&0&-2\\0&0&1&6\end{bmatrix},\ \text{so } \mathbf{y}=\begin{bmatrix}-7\\-2\\6\end{bmatrix}.$$

Next, solve $U\mathbf{x}=\mathbf{y}$, using back-substitution (with matrix notation).

$$[U\ \ \mathbf{y}]=\begin{bmatrix}3&-7&-2&-7\\0&-2&-1&-2\\0&0&-1&6\end{bmatrix}\sim\begin{bmatrix}3&-7&-2&-7\\0&-2&-1&-2\\0&0&1&-6\end{bmatrix}\sim\begin{bmatrix}3&-7&0&-19\\0&-2&0&-8\\0&0&1&-6\end{bmatrix}$$

$$\sim\begin{bmatrix}3&-7&0&-19\\0&1&0&4\\0&0&1&-6\end{bmatrix}\sim\begin{bmatrix}3&0&0&9\\0&1&0&4\\0&0&1&-6\end{bmatrix}\sim\begin{bmatrix}1&0&0&3\\0&1&0&4\\0&0&1&-6\end{bmatrix},\ \text{So } \mathbf{x}=\begin{bmatrix}3\\4\\-6\end{bmatrix}.$$

To confirm this result, row reduce the matrix $[A\ \mathbf{b}]$:

$$[A\ \ \mathbf{b}]=\begin{bmatrix}3&-7&-2&-7\\-3&5&1&5\\6&-4&0&2\end{bmatrix}\sim\begin{bmatrix}3&-7&-2&-7\\0&-2&-1&-2\\0&10&4&16\end{bmatrix}\sim\begin{bmatrix}3&-7&-2&-7\\0&-2&-1&-2\\0&0&-1&6\end{bmatrix}$$

From this point the row reduction follows that of $[U\ \ \mathbf{y}]$ above, yielding the same result.

2. $L=\begin{bmatrix}1&0&0\\-2&1&0\\0&1&1\end{bmatrix}, U=\begin{bmatrix}2&-6&4\\0&-4&8\\0&0&-2\end{bmatrix}, \mathbf{b}=\begin{bmatrix}2\\-4\\6\end{bmatrix}$. First, solve $L\mathbf{y}=\mathbf{b}$:

$$[L\ \ \mathbf{b}]=\begin{bmatrix}1&0&0&2\\-2&1&0&-4\\0&1&1&6\end{bmatrix}\sim\begin{bmatrix}1&0&0&2\\0&1&0&0\\0&0&1&6\end{bmatrix},\ \text{so } \mathbf{y}=\begin{bmatrix}2\\0\\6\end{bmatrix}.$$

Next solve $U\mathbf{x}=\mathbf{y}$, using back-substitution (with matrix notation):

$$[U\ \ \mathbf{y}]=\begin{bmatrix}2&-6&4&2\\0&-4&8&0\\0&0&-2&6\end{bmatrix}\sim\begin{bmatrix}2&-6&0&14\\0&-4&0&24\\0&0&1&-3\end{bmatrix}$$

$$\sim\begin{bmatrix}2&0&0&-22\\0&1&0&-6\\0&0&1&-3\end{bmatrix}\sim\begin{bmatrix}1&0&0&-11\\0&1&0&-6\\0&0&1&-3\end{bmatrix},\ \text{so } \mathbf{x}=\begin{bmatrix}-11\\-6\\-3\end{bmatrix}.$$

To confirm this result, row reduce the matrix $[A\ \ \mathbf{b}]$:

$$[A\ \ \mathbf{b}]=\begin{bmatrix}2&-6&4&2\\-4&8&0&-4\\0&-4&6&6\end{bmatrix}\sim\begin{bmatrix}2&-6&4&2\\0&-4&8&0\\0&0&-2&6\end{bmatrix}$$

From this point the row reduction follows that of $[U\ \ \mathbf{y}]$ above, yielding the same result.

3. $L=\begin{bmatrix}1&0&0\\-2&1&0\\3&-1&1\end{bmatrix}, U=\begin{bmatrix}2&-4&2\\0&-3&6\\0&0&1\end{bmatrix}, \mathbf{b}=\begin{bmatrix}6\\0\\6\end{bmatrix}$. First, solve $L\mathbf{y}=\mathbf{b}$:

$$[L\ \ \mathbf{b}]=\begin{bmatrix}1&0&0&6\\-2&1&0&0\\3&-1&1&6\end{bmatrix}\sim\begin{bmatrix}1&0&0&6\\0&1&0&12\\0&-1&1&-12\end{bmatrix}\sim\begin{bmatrix}1&0&0&6\\0&1&0&12\\0&0&1&0\end{bmatrix},\ \text{so } \mathbf{y}=\begin{bmatrix}6\\12\\0\end{bmatrix}.$$

Next solve $U\mathbf{x}=\mathbf{y}$, using back-substitution (with matrix notation):

$$[U\ \ \mathbf{y}]=\begin{bmatrix}2&-4&2&6\\0&-3&6&12\\0&0&1&0\end{bmatrix}\sim\begin{bmatrix}2&-4&0&6\\0&-3&0&12\\0&0&1&0\end{bmatrix}\sim\begin{bmatrix}2&0&0&-10\\0&1&0&-4\\0&0&1&0\end{bmatrix}\sim\begin{bmatrix}1&0&0&-5\\0&1&0&-4\\0&0&1&0\end{bmatrix},$$

$$\text{so } \mathbf{x}=\begin{bmatrix}-5\\-4\\0\end{bmatrix}.$$

4. $L=\begin{bmatrix}1&0&0\\1&1&0\\3&-5&1\end{bmatrix}, U=\begin{bmatrix}1&-1&2\\0&-2&-1\\0&0&-6\end{bmatrix}, \mathbf{b}=\begin{bmatrix}0\\-5\\7\end{bmatrix}$. First, solve $L\mathbf{y}=\mathbf{b}$:

$$[L\ \ \mathbf{b}]=\begin{bmatrix}1&0&0&0\\1&1&0&-5\\3&-5&1&7\end{bmatrix}\sim\begin{bmatrix}1&0&0&0\\0&1&0&-5\\0&-5&1&7\end{bmatrix}\sim\begin{bmatrix}1&0&0&0\\0&1&0&-5\\0&0&1&-18\end{bmatrix},\ \text{so } \mathbf{y}=\begin{bmatrix}0\\-5\\-18\end{bmatrix}.$$

Next solve $U\mathbf{x}=\mathbf{y}$, using back-substitution (with matrix notation):

$$[U\ \ \mathbf{y}]=\begin{bmatrix}1&-1&2&0\\0&-2&-1&-5\\0&0&-6&-18\end{bmatrix}\sim\begin{bmatrix}1&-1&2&0\\0&-2&-1&-5\\0&0&1&3\end{bmatrix}\sim\begin{bmatrix}1&-1&0&-6\\0&-2&0&-2\\0&0&1&3\end{bmatrix}$$

$$\sim\begin{bmatrix}1&-1&0&-6\\0&1&0&1\\0&0&1&3\end{bmatrix}\sim\begin{bmatrix}1&0&0&-5\\0&1&0&1\\0&0&1&3\end{bmatrix},\ \text{so } \mathbf{x}=\begin{bmatrix}-5\\1\\3\end{bmatrix}.$$

5. $L=\begin{bmatrix}1&0&0&0\\3&1&0&0\\-1&0&1&0\\-3&4&-2&1\end{bmatrix}, U=\begin{bmatrix}1&-2&-2&-3\\0&-3&6&0\\0&0&2&4\\0&0&0&1\end{bmatrix}, \mathbf{b}=\begin{bmatrix}1\\6\\0\\3\end{bmatrix}$. First solve $L\mathbf{y}=\mathbf{b}$:

$$[L\ \ \mathbf{b}]=\begin{bmatrix}1&0&0&0&1\\3&1&0&0&6\\-1&0&1&0&0\\-3&4&-2&1&3\end{bmatrix}\sim\begin{bmatrix}1&0&0&0&1\\0&1&0&0&3\\0&0&1&0&1\\0&4&-2&1&6\end{bmatrix}$$

$$\sim\begin{bmatrix}1&0&0&0&1\\0&1&0&0&3\\0&0&1&0&1\\0&0&-2&1&-6\end{bmatrix}\sim\begin{bmatrix}1&0&0&0&1\\0&1&0&0&3\\0&0&1&0&1\\0&0&0&1&-4\end{bmatrix},\ \text{so } \mathbf{y}=\begin{bmatrix}1\\3\\1\\-4\end{bmatrix}.$$

Next solve $U\mathbf{x}=\mathbf{y}$, using back-substitution (with matrix notation):

$$[U\ \ \mathbf{y}]=\begin{bmatrix}1&-2&-2&-3&1\\0&-3&6&0&3\\0&0&2&4&1\\0&0&0&1&-4\end{bmatrix}\sim\begin{bmatrix}1&-2&-2&0&-11\\0&-3&6&0&3\\0&0&2&0&17\\0&0&0&1&-4\end{bmatrix}\sim\begin{bmatrix}1&-2&-2&0&-11\\0&-3&6&0&3\\0&0&1&0&17/2\\0&0&0&1&-4\end{bmatrix}$$

$$\sim\begin{bmatrix}1&-2&0&0&6\\0&-3&0&0&-48\\0&0&1&0&17/2\\0&0&0&1&-4\end{bmatrix}\sim\begin{bmatrix}1&-2&0&0&6\\0&1&0&0&16\\0&0&1&0&17/2\\0&0&0&1&-4\end{bmatrix}\sim\begin{bmatrix}1&0&0&0&38\\0&1&0&0&16\\0&0&1&0&17/2\\0&0&0&1&-4\end{bmatrix},\ \text{so } \mathbf{x}=\begin{bmatrix}38\\16\\17/2\\-4\end{bmatrix}.$$

6. $L=\begin{bmatrix}1&0&0&0\\-2&1&0&0\\3&-3&1&0\\-5&4&-1&1\end{bmatrix}, U=\begin{bmatrix}1&3&2&0\\0&3&0&12\\0&0&-2&0\\0&0&0&1\end{bmatrix}, \mathbf{b}=\begin{bmatrix}1\\-2\\-1\\2\end{bmatrix}$. First, solve $L\mathbf{y}=\mathbf{b}$:

$$[L\ \ \mathbf{b}]=\begin{bmatrix}1&0&0&0&1\\-2&1&0&0&-2\\3&-3&1&0&-1\\-5&4&-1&1&2\end{bmatrix}\sim\begin{bmatrix}1&0&0&0&1\\0&1&0&0&0\\0&-3&1&0&-4\\0&4&-1&1&7\end{bmatrix}\sim\begin{bmatrix}1&0&0&0&1\\0&1&0&0&0\\0&0&1&0&-4\\0&0&-1&1&7\end{bmatrix}$$

$$\sim\begin{bmatrix}1&0&0&0&1\\0&1&0&0&0\\0&0&1&0&-4\\0&0&0&1&3\end{bmatrix},\ \text{so } \mathbf{y}=\begin{bmatrix}1\\0\\-4\\3\end{bmatrix}.$$ Next solve $U\mathbf{x}=\mathbf{y}$, using back-substitution (with matrix notation):

$$[U\ \ \mathbf{y}]=\begin{bmatrix}1&3&2&0&1\\0&3&0&12&0\\0&0&-2&0&-4\\0&0&0&1&3\end{bmatrix}\sim\begin{bmatrix}1&3&2&0&1\\0&3&0&0&-36\\0&0&-2&0&-4\\0&0&0&1&3\end{bmatrix}\sim\begin{bmatrix}1&3&0&0&-3\\0&3&0&0&-36\\0&0&1&0&2\\0&0&0&1&3\end{bmatrix}$$

$$\sim\begin{bmatrix}1&0&0&0&33\\0&1&0&0&-12\\0&0&1&0&2\\0&0&0&1&3\end{bmatrix},\ \text{so } \mathbf{x}=\begin{bmatrix}33\\-12\\2\\3\end{bmatrix}.$$

7. Place the first pivot column of $\begin{bmatrix}2&5\\-3&-4\end{bmatrix}$ into L, after dividing the column by 2 (the pivot), then add 3/2 times row 1 to row 2, yielding U.

$$A=\begin{bmatrix}2&5\\-3&-4\end{bmatrix}\sim\begin{bmatrix}2&5\\0&7/2\end{bmatrix}=U$$

$$\begin{bmatrix}2\\-3\end{bmatrix}\quad\begin{bmatrix}7/2\end{bmatrix}$$

$$\div 2\quad \div 7/2$$

$$\begin{bmatrix}1&\\-3/2&1\end{bmatrix},\ L=\begin{bmatrix}1&0\\-3/2&1\end{bmatrix}$$

8. Row reduce A to echelon form using only row replacement operations. Then follow the algorithm in Example 2 to find L.

$$A=\begin{bmatrix}6&4\\12&5\end{bmatrix}\sim\begin{bmatrix}6&4\\0&-3\end{bmatrix}=U$$

$$\begin{bmatrix}6\\12\end{bmatrix}\quad\begin{bmatrix}-3\end{bmatrix}$$

$$\div 6\quad \div -3$$

$$\begin{bmatrix}1&\\2&1\end{bmatrix},\ L=\begin{bmatrix}1&0\\2&1\end{bmatrix}$$

9. $$A=\begin{bmatrix}3&1&2\\-9&0&-4\\9&9&14\end{bmatrix}\sim\begin{bmatrix}3&1&2\\0&3&2\\0&6&8\end{bmatrix}\sim\begin{bmatrix}3&1&2\\0&3&2\\0&0&4\end{bmatrix}=U$$

$$\begin{bmatrix}3\\-9\\9\end{bmatrix}\quad\begin{bmatrix}3\\6\end{bmatrix}\quad\begin{bmatrix}4\end{bmatrix}$$

$$\div 3\quad \div 3\quad \div 4$$

$$\begin{bmatrix} 1 & & \\ -3 & 1 & \\ 3 & 2 & 1 \end{bmatrix}, \; L = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

10. $$A = \begin{bmatrix} -5 & 0 & 4 \\ 10 & 2 & -5 \\ 10 & 10 & 16 \end{bmatrix} \sim \begin{bmatrix} -5 & 0 & 4 \\ 0 & 2 & 3 \\ 0 & 10 & 24 \end{bmatrix} \sim \begin{bmatrix} -5 & 0 & 4 \\ 0 & 2 & 3 \\ 0 & 0 & 9 \end{bmatrix} = U$$

$$\begin{bmatrix} -5 \\ 10 \\ 10 \end{bmatrix} \; \begin{bmatrix} 2 \\ 10 \end{bmatrix} \; \begin{bmatrix} 9 \end{bmatrix}$$

$\div -5 \;\; \div 2 \;\; \div 9$

$$\begin{bmatrix} 1 & & \\ -2 & 1 & \\ -2 & 5 & 1 \end{bmatrix}, \; L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -2 & 5 & 1 \end{bmatrix}$$

11. $$A = \begin{bmatrix} 3 & 7 & 2 \\ 6 & 19 & 4 \\ -3 & -2 & 3 \end{bmatrix} \sim \begin{bmatrix} 3 & 7 & 2 \\ 0 & 5 & 0 \\ 0 & 5 & 5 \end{bmatrix} \sim \begin{bmatrix} 3 & 7 & 2 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = U$$

$$\begin{bmatrix} 3 \\ 6 \\ -3 \end{bmatrix} \; \begin{bmatrix} 5 \\ 5 \end{bmatrix} \; \begin{bmatrix} 5 \end{bmatrix}$$

$\div 3 \;\; \div 5 \;\; \div 5$

$$\begin{bmatrix} 1 & & \\ 2 & 1 & \\ -1 & 1 & 1 \end{bmatrix}, \; L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

12. Row reduce A to echelon form using only row replacement operations. Then follow the algorithm in Example 2 to find L. Use the last column of I_3 to make L unit lower triangular.

$$A = \begin{bmatrix} 2 & 3 & 2 \\ 4 & 13 & 9 \\ -6 & 5 & 4 \end{bmatrix} \sim \begin{bmatrix} 2 & 3 & 2 \\ 0 & 7 & 5 \\ 0 & 14 & 10 \end{bmatrix} \sim \begin{bmatrix} 2 & 3 & 2 \\ 0 & 7 & 5 \\ 0 & 0 & 0 \end{bmatrix} = U$$

$$\begin{bmatrix} 2 \\ 4 \\ -6 \end{bmatrix} \; \begin{bmatrix} 7 \\ 14 \end{bmatrix}$$

$\div 2 \;\; \div 7$

$$\begin{bmatrix} 1 & & \\ 2 & 1 & \\ -3 & 2 & 1 \end{bmatrix}, \; L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & 2 & 1 \end{bmatrix}$$

13. $$\begin{bmatrix} 1 & 3 & -5 & -3 \\ -1 & -5 & 8 & 4 \\ 4 & 2 & -5 & -7 \\ -2 & -4 & 7 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & -5 & -3 \\ 0 & -2 & 3 & 1 \\ 0 & -10 & 15 & 5 \\ 0 & 2 & -3 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & -5 & -3 \\ 0 & -2 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = U$$ No more pivots!

$$\begin{bmatrix} 1 \\ -1 \\ 4 \\ -2 \end{bmatrix} \; \begin{bmatrix} -2 \\ -10 \\ 2 \end{bmatrix}$$ Use the last two columns of I_4 to make L unit lower triangular.

$\div 1 \;\; \div -2$

$$\begin{bmatrix} 1 & & & \\ -1 & 1 & & \\ 4 & 5 & 1 & \\ -2 & -1 & 0 & 1 \end{bmatrix}, \; L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 4 & 5 & 1 & 0 \\ -2 & -1 & 0 & 1 \end{bmatrix}$$

14. $$A = \begin{bmatrix} 1 & 3 & 1 & 5 \\ 5 & 20 & 6 & 31 \\ -2 & -1 & -1 & -4 \\ -1 & 7 & 1 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 1 & 5 \\ 0 & 5 & 1 & 6 \\ 0 & 5 & 1 & 6 \\ 0 & 10 & 2 & 12 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 1 & 5 \\ 0 & 5 & 1 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = U$$

$$\begin{bmatrix} 1 \\ 5 \\ -2 \\ -1 \end{bmatrix} \quad \begin{bmatrix} 5 \\ 5 \\ 10 \end{bmatrix}$$ Use the last two columns of I_4 to make L unit lower triangular.

$$\begin{matrix} \div 1 & \div 5 & & \\ \end{matrix}$$

$$\begin{bmatrix} 1 & & & \\ 5 & 1 & & \\ -2 & 1 & 1 & \\ -1 & 2 & 0 & 1 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 \\ -2 & 1 & 1 & 0 \\ -1 & 2 & 0 & 1 \end{bmatrix}$$

15. $A = \begin{bmatrix} 2 & 0 & 5 & 2 \\ -6 & 3 & -13 & -3 \\ 4 & 9 & 16 & 17 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & 5 & 2 \\ 0 & 3 & 2 & 3 \\ 0 & 9 & 6 & 13 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & 5 & 2 \\ 0 & 3 & 2 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix} = U$

$$\begin{bmatrix} 2 \\ -6 \\ 4 \end{bmatrix} \quad \begin{bmatrix} 3 \\ 9 \end{bmatrix} \quad \begin{bmatrix} 4 \end{bmatrix}$$

$$\begin{matrix} \div 2 & \div 3 & \div 4 \end{matrix}$$

$$\begin{bmatrix} 1 & & \\ -3 & 1 & \\ 2 & 3 & 1 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix}$$

16. $A = \begin{bmatrix} 2 & -3 & 4 \\ -4 & 8 & -7 \\ 6 & -5 & 14 \\ -6 & 9 & -12 \\ 8 & -6 & 19 \end{bmatrix} \sim \begin{bmatrix} 2 & -3 & 4 \\ 0 & 2 & 1 \\ 0 & 4 & 2 \\ 0 & 0 & 0 \\ 0 & 6 & 3 \end{bmatrix} \sim \begin{bmatrix} 2 & -3 & 4 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = U$

$$\begin{bmatrix} 2 \\ -4 \\ 6 \\ -6 \\ 8 \end{bmatrix} \quad \begin{bmatrix} 2 \\ 4 \\ 0 \\ 6 \end{bmatrix}$$ Use the last three columns of I_5 to make L unit lower triangular.

$$\begin{matrix} \div 2 & \div 2 \end{matrix}$$

$$\begin{bmatrix} 1 & & & & \\ -2 & 1 & & & \\ 3 & 2 & 1 & & \\ -3 & 0 & 0 & 1 & \\ 4 & 3 & 0 & 0 & 1 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 & 0 \\ 3 & 2 & 1 & 0 & 0 \\ -3 & 0 & 0 & 1 & 0 \\ 4 & 3 & 0 & 0 & 1 \end{bmatrix}$$

17. $L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}, U = \begin{bmatrix} 2 & -6 & 4 \\ 0 & -4 & 8 \\ 0 & 0 & -2 \end{bmatrix}$ To find L^{-1}, use the method of Section 2.2; that is, row reduce $[L \;\; I]$:

$$[L \;\; I] = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ -2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & -2 & -1 & 1 \end{bmatrix} = [I \;\; L^{-1}],$$

so $L^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -2 & -1 & 1 \end{bmatrix}$. Likewise to find U^{-1}, row reduce $[U \; I]$:

$$[U \;\; I] = \begin{bmatrix} 2 & -6 & 4 & 1 & 0 & 0 \\ 0 & -4 & 8 & 0 & 1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 2 & -6 & 0 & 1 & 0 & 2 \\ 0 & -4 & 0 & 0 & 1 & 4 \\ 0 & 0 & -2 & 0 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 0 & 0 & 1 & -3/2 & -4 \\ 0 & 1 & 0 & 0 & -1/4 & -1 \\ 0 & 0 & 1 & 0 & 0 & -1/2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1/2 & -3/4 & -2 \\ 0 & 1 & 0 & 0 & -1/4 & -1 \\ 0 & 0 & 1 & 0 & 0 & -1/2 \end{bmatrix} = [I \;\; U^{-1}],$$

so $U^{-1} = \begin{bmatrix} 1/2 & -3/4 & -2 \\ 0 & -1/4 & -1 \\ 0 & 0 & -1/2 \end{bmatrix}$. Thus

$$A^{-1} = U^{-1}L^{-1} = \begin{bmatrix} 1/2 & -3/4 & -2 \\ 0 & -1/4 & -1 \\ 0 & 0 & -1/2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -2 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 5/4 & -2 \\ 3/2 & 3/4 & -1 \\ 1 & 1/2 & -1/2 \end{bmatrix}$$

18. $L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -1 & 1 \end{bmatrix}, U = \begin{bmatrix} 2 & -4 & 2 \\ 0 & -3 & 6 \\ 0 & 0 & 1 \end{bmatrix}$ To find L^{-1}, row reduce $[L \;\; I]$:

$$[L \;\; I] = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ -2 & 1 & 0 & 0 & 1 & 0 \\ 3 & -1 & 1 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 1 & 0 \\ 0 & -1 & 1 & -3 & 0 & 1 \end{bmatrix}$$