Math 260 Section 2.3

Most important ideas:

- **Finally: The Invertible Matrix Theorem for <u>square</u> matrices, given on page 112. There will eventually be 24 different "conditions" we consider for a matrix.**

Condition	Good version	Bad version
Example…	$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$	$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$
A invertible?	**Yes.**	**No.**
A row equivalent to I ($A \sim I$)?	**Yes.** $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	**No.** $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$
Pivot in every row and column?	**Yes.**	**No.**
Solution(s) to $A\vec{x} = \vec{0}$?	**Only $\vec{x} = \vec{0}$.**	**$\vec{x} = \vec{0}$, as well as one or more $\vec{x} \neq \vec{0}$ and its scalar multiples.**
Columns of A linearly independent?	**Yes.**	**No.**
Function $T(\vec{x}) = A\vec{x}$ (i.e. $\vec{x} \to A\vec{x}$) 1-to-1?	**Yes.**	**No.**
Solution exists to $A\vec{x} = \vec{b}$, regardless of $\vec{b}$?	**Yes.**	**No.**
Columns of A span R^n?	**Yes.**	**No.**
Function $T(\vec{x}) = A\vec{x}$ (i.e. $\vec{x} \to A\vec{x}$) onto?	**Yes.**	**No.**

If any one of the good things is true, then all of the good things are true.

If any one of the bad things is true, then all of the bad things are true.