Most important ideas:

- **Echelon Form (or Row Echelon Form), Reduced Echelon Form, on page 14.**
  See Example 1 in particular.
- **Pivot, Pivoting, Pivot Column. Make sure you understand Example 3 on page 15.**
- **How to recognize when a linear system has 0 (no solution) or 1 (a unique solution) or ∞ (infinitely many) solutions.**
- **Use of technology in row reduction: links on are class homepage.**

Sections 1.1 and 1.2 are quite similar. There are two main differences.

1. In Section 1.1 every problem had a unique solution (which was a result of the fact that there were always the same number of equations as unknowns—e.g. two equations with two unknowns, or three equations with three unknowns—more on this later.) In Section 1.2, we will encounter problems with more equations than unknowns ("too many equations") or fewer equations than unknowns ("too few unknowns") which will lead to some problems should have no solutions and some with infinite solutions.
2. In Section 1.1, in row reducing the augmented matrix, we were allowed to do whatever we wanted, as long as each step we made the augmented matrix simpler in some way. In Section 1.2, a specific process is introduced and strongly encouraged. While it is a bit restrictive, it is simpler and much more reliable.

After reading through Book Example 1, take a look at Book Example 2.

Recall Book Example 1 on page 5. Of course we can't simply look at

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_2 - 8x_3 &= 8 \\ -4x_1 + 5x_2 + 9x_3 &= -9 \end{aligned}$$

and see the solution $x_1 = 29, x_2 = 16$ and $x_3 = 3$. However, by doing row reduction

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ -4 & 5 & 9 & -9 \end{array}\right] \sim \cdots \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

the solution is now staring us in the face:

$$\begin{aligned} \boldsymbol{x} \qquad\qquad &= \mathbf{29} \\ \boldsymbol{y} \qquad &= \mathbf{16} \\ \boldsymbol{z} &= \mathbf{3} \end{aligned}$$

We like the 0's and 1's in the augmented matrix $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 3 \end{array}\right]$. Why?

The 0's correspond to unknowns being eliminated from equations. We like that.

The 1's correspond to the unknowns that are still there—the unknowns we are solving for.

The process of row reduction <u>with pivoting</u> is a specific way of doing row reduction to get from

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ -4 & 5 & 9 & -9 \end{array}\right] \text{ to } \left[\begin{array}{ccc|c} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 3 \end{array}\right]$$