Math 260 Spring 2025 **Exam 2** **March 18, 2025**

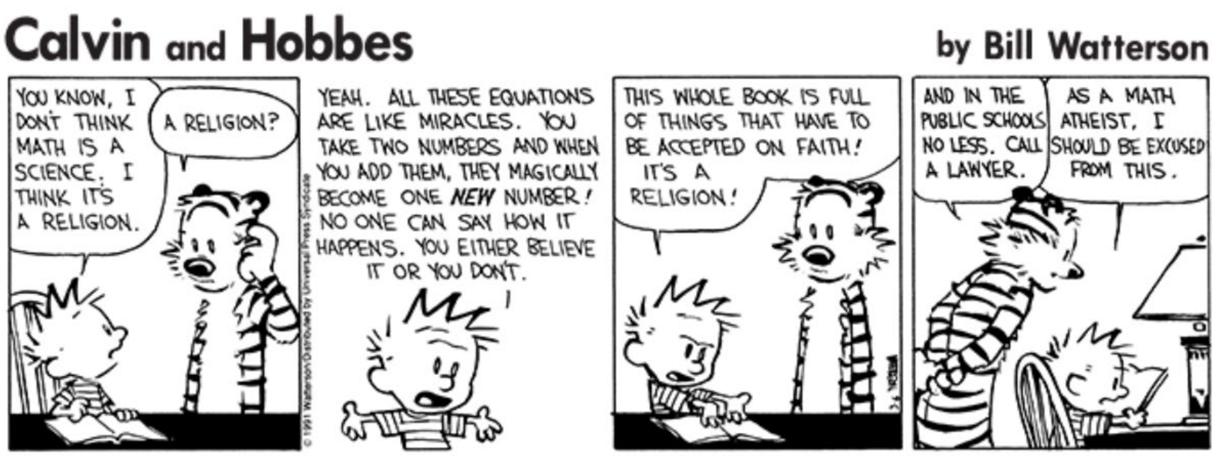
Name: ____

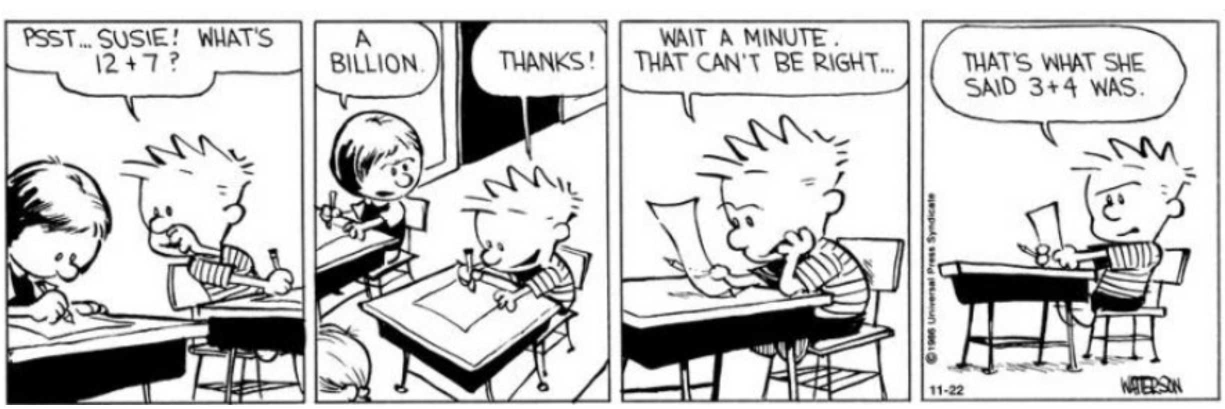
Problem	T/F	1	2 / 3	4 / 5	Total
Possible	46	25	15	14	100
Received					

DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

You may use a 3 × 5 card of notes (no calculator).

FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.





46 points Answer the following True/False questions. Each question is worth 2 points. **"True" means *always true* or *necessarily true*. "False" means that it may be true sometimes or under some circumstances, but not *always* or not *necessarily*.** No explanation is necessary whether true or false.

True False Assuming A^{-1} exists, $\det(A^{-1}B) = \frac{\det(B)}{\det(A)}$.

True False The nullspace of A is the set of all vectors $\vec{x}$ for which $A\vec{0} = \vec{x}$.

True False The columns of any invertible 3×3 matrix form a basis for R^3.

True False If S is a linearly independent set in V, then S is a basis for V.

True False If A is a 4×7 matrix (so 4 rows and 7 columns), then $\dim Row\ A < \dim Col\ A$.

True False If A is a 4×7 matrix, then $4 \leq rank(A) \leq 7$.

True False If A is a 4×7 matrix, then $\dim Nul\ A \leq 3$.

True False If $A = PDP^{-1}$, where the columns of P are the eigenvectors of A and the entries of diagonal matrix D are the corresponding eigenvalues, then the eigenvectors of A^T are simply the columns of $(P^T)^{-1}$.

True False If $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ is an eigenvector of a matrix, then $\begin{bmatrix} -15 \\ -5 \end{bmatrix}$ is also an eigenvector.

True False The largest possible dimension of $Nul\ A$ for a 3×7 matrix A is 3.

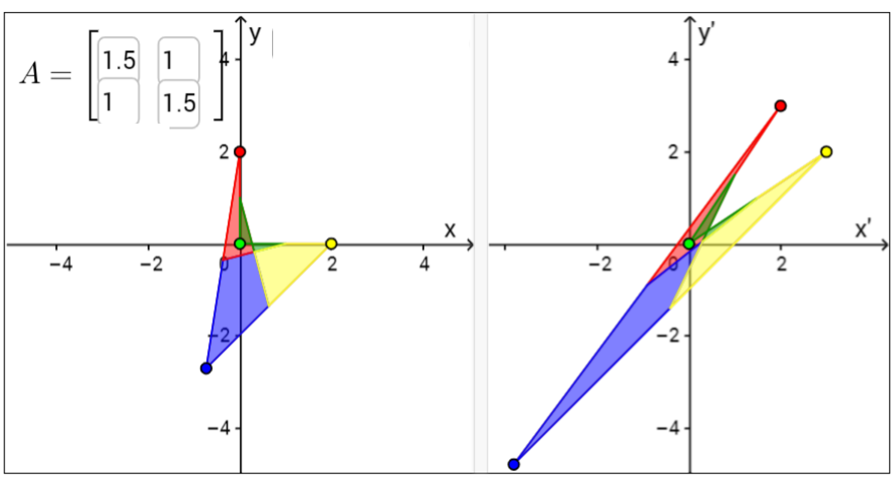
True False If $A\vec{v} = \lambda\vec{v}$ and if A has an inverse, then $A^{-1}\vec{v} = \frac{1}{\lambda}\vec{v}$.

True False The set of vectors of the form $\begin{bmatrix} a \\ b \\ 1 \end{bmatrix}$ (for some a and b) is a subspace of R^3.

True False Where H is a vector space and $T(H)$ is the image of linear transformation T, then $\dim T(H) \leq \dim H$.

True False If $A\vec{v}_1 = \lambda_1\vec{v}_1$ and $A\vec{v}_2 = \lambda_2\vec{v}_2$ where $\vec{v}_1$ and $\vec{v}_2$ are linearly independent eigenvectors of some matrix A, then it must be that $\lambda_1 \neq \lambda_2$.

True False If the area in the figure at left below is 4, and if the area at right is the area at left transformed by the matrix $\begin{bmatrix} 1.5 & 1 \\ 1 & 1.5 \end{bmatrix}$, then the area at right is 8.



True False If a 4×4 matrix A has rank 2, then $A\vec{x} = \vec{b}$ has an infinite number of solutions for some right hand side $\vec{b}$, no solution for some other right hand side $\vec{b}$, and a unique solution for yet another right hand side $\vec{b}$.

True False For $\vec{v}_1, \vec{v}_2 \in R^4$, $Span\{v_1, v_2\}$ is a subspace of R^4.

True False For $\vec{v}_1, \vec{v}_2 \in R^4$, $\{\vec{v}_1, \vec{v}_2\}$ is a basis for $Span\{\vec{v}_1, \vec{v}_2\}$.

True False The sum of three eigenvectors of a matrix is an eigenvector of that same matrix.

True False There is a value of k for which $\begin{bmatrix} 1 & 2 \\ k & 4 \end{bmatrix}$ has an eigenvalue of 0.

True False $\det(-A) = -\det A$.

True False If the determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 3$, then $\begin{vmatrix} a + 2c & b + 2d \\ c & d \end{vmatrix} = 6$.

True False Let A be a 5×5 matrix with just three different eigenvalues. It is possible for A to have a complete set of 5 linearly independent eigenvectors.

25 points 1. Consider the matrix $A = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$.

/12 Find (and show your work) the eigenvalues λ_1 and λ_2 and corresponding eigenvectors $\vec{v}_1$ and $\vec{v}_2$ of A.

/2 Give two different diagonalizations $PDP^{-1} = [\vec{v}_1\ \vec{v}_2]\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}[\vec{v}_1\ \vec{v}_2]^{-1}$ of A.

$$\begin{bmatrix} & \end{bmatrix}\begin{bmatrix} & \end{bmatrix}\begin{bmatrix} & \end{bmatrix}^{-1} \quad \text{and} \quad \begin{bmatrix} & \end{bmatrix}\begin{bmatrix} & \end{bmatrix}\begin{bmatrix} & \end{bmatrix}^{-1}$$

/4 Let B be the R^2 basis with eigenvectors $\vec{v}_1$ and $\vec{v}_2$. Where $\vec{x} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$, find $[\vec{x}]_B$, the co-ordinates of $\vec{x}$ with respect to B. (That is, find c_1 and c_2 so that $\vec{x} = c_1\vec{v}_1 + c_2\vec{v}_2$.)

/7 Where $\vec{x} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$, find $A^5\vec{x}$. (Note that $2^5 \cdot 3 = 96$.)

5 points 2. Use Cramer's Rule to find x_3 (not x_1 or x_2) in the linear system $\begin{array}{lcr} 3x_1 & = & 6 \\ 4x_1 + 2x_2 & = & 2 \\ 5x_1 \qquad + x_3 & = & 11 \end{array}$.

10 points 3. Consider probability matrix $P = \begin{bmatrix} .2 & .4 \\ .8 & .6 \end{bmatrix}$ and initial vector $\vec{x}_0 = \begin{bmatrix} 0 \\ 70 \end{bmatrix}$. Let $\vec{x}_k = P^k \vec{x}_0$.

/2 Find $\vec{x}_1$.

/6 Find $\vec{x}_\infty$.

/1 Find the eigenvector of P that corresponds to eigenvalue 1.

/1 Find P^∞.

8 points 4. Consider matrix A which is row equivalent to matrix B:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 4 \\ 1 & -2 & 4 & 2 & 5 \\ 1 & 1 & 1 & 3 & 6 \\ 1 & 3 & -1 & 4 & 7 \\ 1 & 2 & 0 & 5 & 8 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 2 & 0 & 3 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

/2 Find a basis for $Col\ A$:

/2 Find a basis for $Row\ A$:

/4 Find a basis for $Nul\ A$:

6 points 5. Prove that the nullspace of a $m \times n$ matrix is a subspace of R^n.

Invertible Matrix Theorem for $n \times n$ matrix A

a. A is invertible.
b. A is row equivalent to I.
c. A has n pivot positions.
d. A**x** = **0** has only trivial solution.
e. Columns of A lin. independent.
f. Linear transf. **x** → A**x** 1-to-1.
g. A**x** = **b** has at least one solution for each **b**.
h. Columns of A span $\mathbf{R}^n$.
i. Linear transf. **x** → A**x** onto.
j. There is C such that CA = I.
k. There is D such that AD = I.
l. A^T is invertible.
m. Columns of A form basis for $\mathbf{R}^n$.
n. Column space of A is $\mathbf{R}^n$.
o. dim Col A = n, *i.e.* dimension of column space of A is n.
p. rank A = n, *i.e.* rank of A is n.
q. Nul A = {**0**}, *i.e.* nullspace of A is {**0**}.
r. dim Nul A = 0, the dimension of the null space of A is 0.
s. A has n nonzero eigenvalues, *i.e.* 0 is not an eigenvalue of A.
t. det A ≠ 0.
u. $(\text{Col A})^{\perp} = \{\mathbf{0}\}$, *i.e.* orthogonal complement of column space of A is {**0**}.
v. $(\text{Nul A})^{\perp} = \mathbf{R}^n$, *i.e.* orthogonal complement of null space of A is $\mathbf{R}^n$.
w. Row A = $\mathbf{R}^n$, row space of A is $\mathbf{R}^n$.