Name: _____

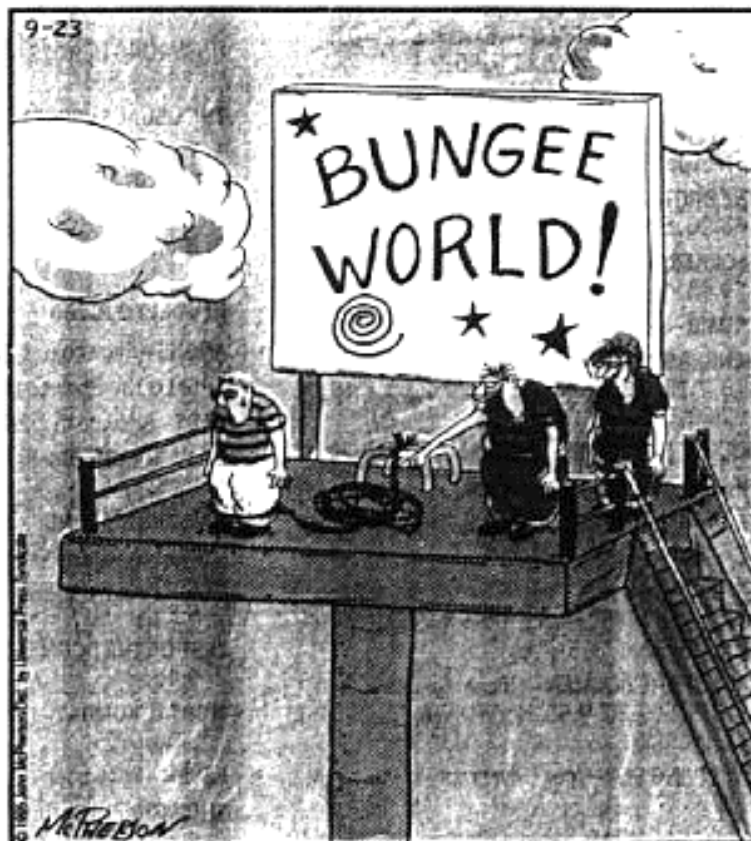| Problem	T / F	1 / 2	3 / 4	5 / 6	7 / 8	Total
Possible	34	20	15	16	15	100
Received						

DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

You may use a 3 x 5 card (both sides) of notes, but no calculator.

FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.

Close To Home   John McPherson

"Okee-doke! Let's just double-check. We're 130 feet up and we've got 45 yards of bungee cord, that's uh ... 90 feet. Allow for 30 feet of stretching, that gives us a total of ...120 feet. Perfect!"

34 points T/F. Answer the following 18 True/False questions. Each question is worth 2 points. Note: "True" means *always true* or *necessarily true*. "False" means that it may sometimes be true, but not always or not necessarily. **No work or explanation or justification is needed for these questions—just circle either True or False.**

True False   If the columns of a matrix are linearly independent, then it is possible for the problem $A\vec{x} = \vec{b}$ to have more than one solution $\vec{x}$ for some right-hand side $\vec{b}$.

True False   If the columns of an $m \times n$ matrix span R^m and are linearly independent, then it must be that $m = n$.

True False   If $A\vec{x} = \vec{0}$ for some $\vec{x} \neq \vec{0}$, then $A\vec{x} = \vec{b}$ will have multiple (infinite actually) solutions for all $\vec{b}$.

True False   If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, then $A^{-1} = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$.

True False   If $A = \begin{bmatrix} 1 & 4 & 1 \\ 4 & 0 & 4 \\ 1 & 4 & 1 \end{bmatrix}$, then $A\vec{x} = \vec{0}$ <u>must</u> have more than one solution.

True False   The columns of a 2×3 matrix (so 2 rows, 3 columns) <u>could</u> span R^3.

True False   The columns of a 2×3 <u>must</u> span R^2.

True False   The columns of a 2×3 matrix <u>must</u> be linearly dependent.

True False   If A is a 4×3 matrix and B is a 3×2 matrix, then $(AB)^T(AB)$ is a 2×2 matrix.

True False   The problem of solving for x_1 and x_2 in the system of equations

$$3x_1 + 2x_2 = 1$$
$$6x_1 + 4x_2 = 3$$

is equivalent of solving for x_1 and x_2 in the vector equation

$$x_1 \begin{bmatrix} 3 \\ 6 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

True False   If vector $\vec{b} = A\vec{x}$ for some vector $\vec{x}$, then $\vec{b}$ is a linear combination of the columns of A.

True False   If the columns of $n \times n$ A do not span R^n, then the columns are linearly dependent.

True False   If $Span\{\vec{v}_1, \vec{v}_2\} = Span\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$, then $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ is linearly dependent.

True False   In R^3, if $\{\vec{v}_1, \vec{v}_2\}$ is linearly independent and $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent, then $Span\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is a plane in R^3.

True False $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$ can be written as a linear combination of $\{\begin{bmatrix} 2.41 \\ -3.15 \end{bmatrix}, \begin{bmatrix} 5.6 \\ 6.5 \end{bmatrix}\}$.

True False   If the columns of $n \times n$ matrix A are linearly dependent, then $A\vec{x} = \vec{b}$ might have a solution, depending on what $\vec{b}$ is.

True False   For $A\vec{x} = \vec{b}$, it is possible that $A\vec{x} = \vec{b}_1$ has an infinite number of solutions for some $\vec{b}_1$ while $A\vec{x} = \vec{b}_2$ has one solution for some other $\vec{b}_2$.

12 points   **1.** Suppose I have some nickels (5 cents each) and dimes (10 cents each). I have 13 coins total, I have 3 more nickels than dimes (so $n = d + 3$), and I have 90 cents total. How many of each type of coin do I have? Solve this by coming up with the three equations that correspond to these three conditions (13 coins total, 3 more nickels than dimes, and 90 cents total), then doing row reduction (i.e. Gaussian Elimination) to find the solution(s) to this system of equations. <u>Don't just guess the solution</u>. Or show that there is no solution, if that is the case.

8 points   **2.** Find the 2×2 matrix which first rotates <u>clockwise</u> ↺ by 45 degrees, and then reflects across the y-axis.
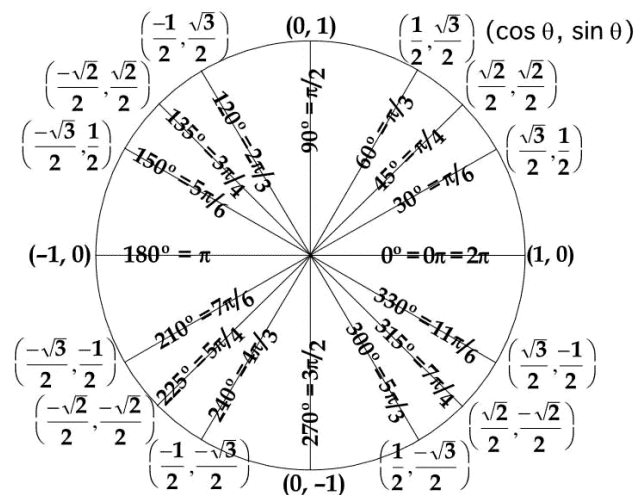
10 points **3.** Solve for x, y and z in the following system by <u>finding the inverse</u> of its coefficient matrix

$$
\begin{aligned}
x + y + \ z &= \ \ 1 \\
2x + y - \ z &= \ \ 5 \\
x + y + 2z &= -1
\end{aligned}
\qquad
\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & 2 \end{bmatrix}
$$

and using it to find the values of x, y and z. Use the method $[\,A\,|\,I\,] \to [\,I\,|\,A^{-1}]$ for finding the inverse. You should not encounter any fractions in finding it. Show work. Don't just guess answers.

5 points **4.** Prove that if the columns of A are linearly independent, then the columns of A^2 are linearly independent. (Recall that a matrix M has linearly independent columns if $M\vec{x} = \vec{0} \Rightarrow \vec{x} = \vec{0}$.)

5

6 points   **5.** We are interested in solving the following system of equations,

$$2x + 3y = 7$$
$$8x + ay = b$$

where a and b are some constants whose values have not yet been decided. Give an example of values of a and b that result in the system having:

No solution: $a =$ $b =$

One solution: $a =$ $b =$

Infinite solutions: $a =$ $b =$

10 points 6. Find the solution(s) to each of the following linear systems. If a system has more than one solution, give the general solution and then give *at least two* specific solutions. If a system has no solution, state that. Notice the left hand side is the same in both.

$$x + y - z + 2w = 5$$
$$-x - y + 3z \qquad = 7$$

$$x + y - z + 2w = 0$$
$$-x - y + 3z \qquad = 0$$

10 points   **7.** A company produces two items, but uses up some of each product in the production process, as described by the input-output (consumption) matrix

$$C = \begin{bmatrix} .5 & 0 \\ .2 & .6 \end{bmatrix}.$$

Note for this problem that $(.5)(.4) = .2$, and that $\frac{.5}{.2} = \frac{5}{2}$ and $\frac{.4}{.2} = 2.$

How much would you need to produce in order to *end up* with 10 units of each product? (Use the **formula** for finding the 2×2 matrix in this problem.) What is one thing about your solution that makes you think it is reasonable, i.e. that it could be the correct answer?

5 points   **8.** For what value(s) of a are the vectors $\begin{bmatrix} a \\ 1 \end{bmatrix}, \begin{bmatrix} a^2 \\ a^3 \end{bmatrix}$ linearly **dependent**?

Invertible Matrix Theorem for $n \times n$ matrix A

a. A is invertible.

b. A is row equivalent to I.

c. A has n pivot positions.

d. $A\mathbf{x} = \mathbf{0}$ has only trivial solution.

e. Columns of A lin. independent.

f. Linear transf. $\mathbf{x} \rightarrow A\mathbf{x}$ 1-to-1.

g. $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$.

h. Columns of A span $\mathbf{R}^n$.

i. Linear transf. $\mathbf{x} \rightarrow A\mathbf{x}$ onto.

j. There is C such that CA = I.

k. There is D such that AD = I.

l. A^T is invertible.

m. Columns of A form basis for $\mathbf{R}^n$.

n. Column space of A is $\mathbf{R}^n$.

o. dim Col A = n, i.e. dimension of column space of A is n.

p. rank A = n, i.e. rank of A is n.

q. Nul A = $\{\mathbf{0}\}$, i.e. nullspace of A is $\{\mathbf{0}\}$.

r. dim Nul A = 0, the dimension of the null space of A is 0.

s. A has n nonzero eigenvalues, i.e. 0 is not an eigenvalue of A.

t. det A $\neq$ 0.

u. $(\text{Col A})^\perp = \{\mathbf{0}\}$, i.e. orthogonal complement of column space of A is $\{\mathbf{0}\}$.

v. $(\text{Nul A})^\perp = \mathbf{R}^n$, i.e. orthogonal complement of null space of A is $\mathbf{R}^n$.

w. Row A = $\mathbf{R}^n$, row space of A is $\mathbf{R}^n$.