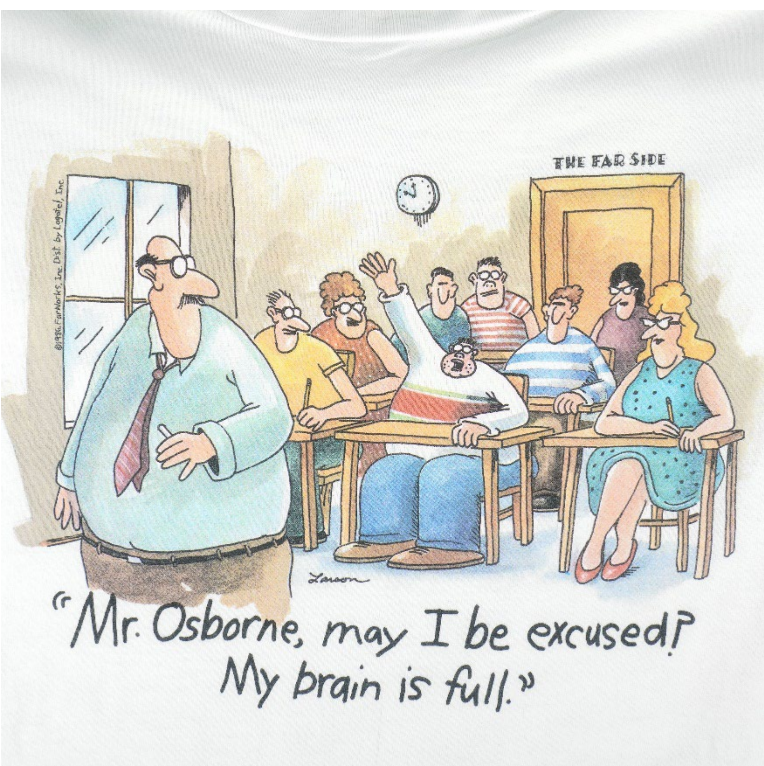
Math 260 Fall 2024 **Exam 2** **March 15, 2024**

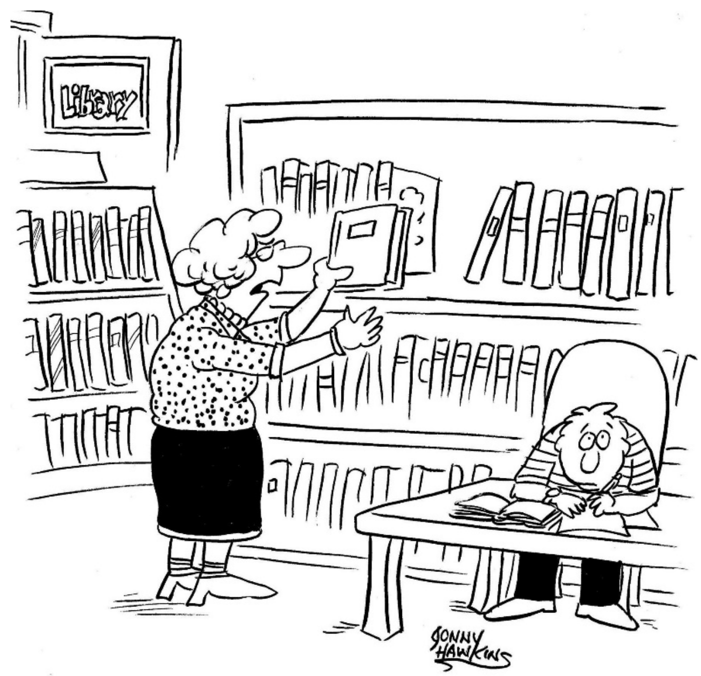
Name: ____

Problem	T/F	1 / 2	3	4 / 5	Total
Possible	50	15	15	20	100
Received					

YOU MAY USE A 3 X 5 CARD OF NOTES.

FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.





“Who is putting all the Math books in the Horror section?”

50 points T/F. Answer the following 25 True/False questions. Each question is worth 2 points. Note: "True" means *always true* or *necessarily true*. "False" means that it may be true sometimes or under some circumstances, but not always or not necessarily. No explanation is necessary whether true or false.

(1) T F If 4×5 matrix A has two pivot columns, then $Col\ A = R^2$.

(2) T F If $A\vec{v}_1 = \lambda_1 \vec{v}_1$ and $A\vec{v}_2 = \lambda_2 \vec{v}_2$ where $\vec{v}_1$ and $\vec{v}_2$ are eigenvectors of A and $\lambda_1 \neq \lambda_2$, then it must be that $\vec{v}_1$ and $\vec{v}_2$ are linearly independent.

(3) T F For matrix A, if $\vec{v}_1$ is an eigenvector corresponding to eigenvalue λ_1 and $\vec{v}_2$ is an eigenvector corresponding to eigenvalue λ_2, then $\vec{v}_1 - \vec{v}_2$ is an eigen-vector corresponding to eigenvalue $\lambda_1 - \lambda_2$.

(4) T F For 3×5 matrix A, $3 \leq rank\ A \leq 5$.

(5) T F If a 5×5 matrix A has rank 4, then $A\vec{x} = \vec{b}$ could have a unique solution for some right hand side $\vec{b}$.

(6) T F If a 5×5 matrix A has rank 4, then $A\vec{x} = \vec{b}$ will have no solution for some right hand side $\vec{b}$ and an infinite number of solutions for some other $\vec{b}$.

(7) T F If a 5×5 matrix A has rank 4, then $A\vec{x} = \vec{b}$ will have an infinite number of solutions for some right hand side $\vec{b}$, no solution for some other right hand side $\vec{b}$, and a unique solution for yet another right hand side $\vec{b}$.

(8) T F The area of the parallelogram with vertices of the origin, $(1,3)$ and $(2,1)$ is 5.

(9) T F For any $\vec{v}_1, \vec{v}_2, \vec{v}_3$ in R^5, $span\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is a subspace of R^5.

(10) T F $\begin{bmatrix} 5 \\ 5 \end{bmatrix}$ is in both the column space and the null space of $\begin{bmatrix} 3 & -3 \\ 3 & -3 \end{bmatrix}$.

(11) T F $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 6 & 0 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} 6 & 1 \\ 8 & 2 \end{bmatrix} \begin{bmatrix} 7 & 0 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} 6 & 1 \\ 8 & 2 \end{bmatrix}^{-1}$.

(12) T F For a 3×5 matrix, the dimensions of the row space and column space will be different.

(13) T F If $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is a basis for R^3, then the matrix $[\, \vec{v}_1 \ \vec{v}_2 \ \vec{v}_3 \,]$ is invertible.

(14) T F If $\det A = 2$ and $\det B = -3$, then $\det(A + B) = -1$.

(15) T F $\det(A^T) = -\det A$.

(16) T F If $A\vec{x} = \vec{b}$, then $\vec{x}$ is in the column space of A.

(17) T F $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector of matrix $\begin{bmatrix} a & b & c \\ c & a & b \\ b & c & a \end{bmatrix}$, no matter what a, b and c are.

(18) T F The determinant of $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 0 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 0 \end{bmatrix}^{-1}$ is 0.

(19) T F If the determinant of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 0 & 0 \end{bmatrix}$ were 5 (it's actually not), then the determinant of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 7 & 9 \end{bmatrix}$ would also be 5.

(20) T F If the determinant of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 0 & 0 \end{bmatrix}$ were 5 (it's actually not), then the determinant of $\begin{bmatrix} 0 & 0 & 0 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix}$ would also be 5.

(21) T F If the determinant of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 0 & 0 \end{bmatrix}$ were 5 (it's actually not), then the determinant of $\begin{bmatrix} 10 & 20 & 30 \\ 40 & 50 & 60 \\ 0 & 0 & 0 \end{bmatrix}$ would be 50.

For the final four problems, V is a vector space, and vectors $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$ come from V.

(22) T F If $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ form a basis for V, then $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ is linearly independent.

(23) T F If $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ is linearly independent, then $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ form a basis for V.

(24) T F If $span\,\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\} = V$, then some subset of $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ is a basis for V.

(25) T F If $\dim V = n$ and $span\,\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\} = V$, then $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ is linearly independent.

5 points 1. If a 5×9 matrix A has rank 3, then: $\dim Row\ A$ = ______

$rank\ A^T$ = ______

$\dim Nul\ A$ = ______

If a 9×5 matrix A has rank 3, then: $\dim Nul\ A$ = ______

If a 5×5 matrix A has rank 3, then: $\dim Nul\ A$ = ______

10 points 2. Consider the probability matrix $A = \begin{bmatrix} 1/3 & 1/2 \\ 2/3 & 1/2 \end{bmatrix}$, which has eigenvectors $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and corresponding eigenvalues $-1/6$ and 1.

First, suppose that $\vec{x}_0 = \begin{bmatrix} 500 \\ 500 \end{bmatrix}$. Find $\vec{x}_0$ as a linear combination of $\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}$. That is find $[\vec{x}_0]_B$ where $B = \{\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}\}$. Don't just guess—show your work.

Second, find $\vec{x}_\infty = A^\infty \vec{x}_0$ $\left(\text{that is, find } \lim_{k\to\infty} A^k \vec{x}_0\right)$. Again, show your work.

11 points 3. Consider matrix A which is row equivalent to matrix B:

$$A = \begin{bmatrix} 1 & -5 & 1 & 2 & -1 & 1 \\ 1 & -5 & 2 & 0 & -3 & 3 \\ 1 & -5 & 3 & -2 & 3 & 13 \\ 1 & -5 & 4 & -4 & 5 & 19 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -5 & 0 & 4 & 0 & -2 \\ 0 & 0 & 1 & -2 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

First, even though there are six columns, based on the dimensions of A, how do you know that dim $Col\ A \leq 4$?

Find the following:

$rank\ A =$

A basis for $Col\ A$:

A basis for $Row\ A$:

A basis for $Nul\ A$:

9 points 4. Use Cramer's Rule to find x_2 in the linear system
$$\begin{array}{rcr} -x_1 + 2x_2 + 3x_3 & = & 0 \\ 3x_2 + 5x_3 & = & -2 \\ x_1 + 4x_2 & = & 0 \end{array}.$$

15 points 5. Find the two eigenvalues and corresponding eigenvectors of $A = \begin{bmatrix} 2 & 2 \\ -3 & 7 \end{bmatrix}$.

Hint in factoring: $4 \cdot 5 = 20$

Find matrices X and D so that $A = XDX^{-1}$.

How do you know that X is invertible?