Name: _____

Problem	T/F	1 / 2	3 / 4 / 5	6 / 7	8	Total
Possible	40	15	20	15	10	100
Received						

Be sure to show all pertinent work to receive full credit.

You may use a 3" x 5" card of handwritten notes, both sides.

You will not use a calculator on this exam.
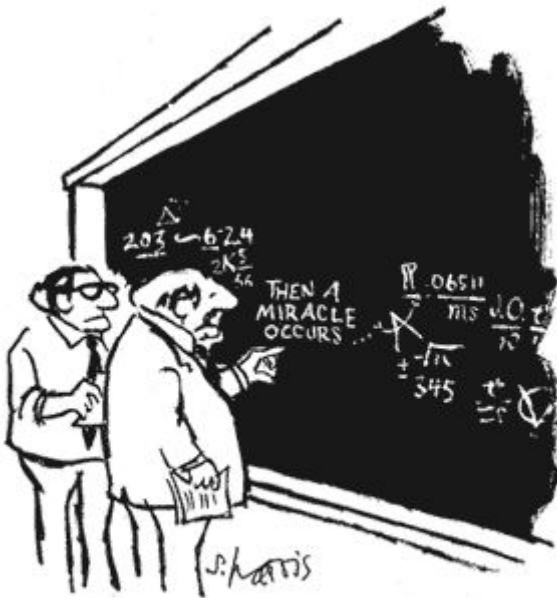
"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."
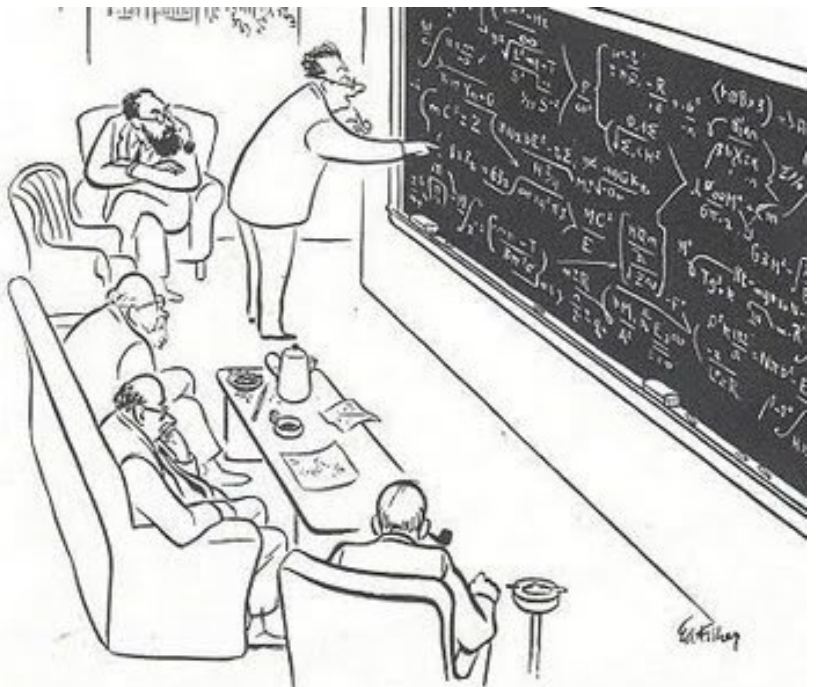
"Say, I think I see where we went off. Isn't eight times seven fifty-six?"

Mr. Jones lives 50 miles away from you. You both leave home at 5:00 and drive toward each other.

Mr. Jones travels at 35 mph., and you drive at 40 mph. At what time will you pass Mr. Jones on the road?

GIVEN THE TRAFFIC AROUND HERE at 5:00, WHO KNOWS?

I ALWAYS CATCH THESE TRICK QUESTIONS.

40 points T/F. Answer the following 20 True/False questions. Each question is worth 2 points.
Note: "True" means *always true* or *necessarily true*. "False" means that it may
sometimes be true, but not always or not necessarily. *No work or explanation or
justification is needed to be shown for these questions.*

True False   If the columns of 4×5 (4 rows, 5 columns) A span R^4, then the columns are linearly independent.

True False   If $Span\{\vec{v}_1, \vec{v}_2\} = Span\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$, then $\{\vec{v}_1, \vec{v}_2\}$ is linearly dependent.

True False   In R^3 if $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent, then $Span\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is a plane in R^3.

True False $\begin{bmatrix} 1.1 \\ -\sqrt{\pi} \end{bmatrix}$ can be written as a linear combination of $\{\begin{bmatrix} 1 \\ 3.14 \end{bmatrix}, \begin{bmatrix} 1/2 \\ 1/3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\}$.

True False   For 3×3 matrix A, the first column of A^{-1} is the same as the solution $\vec{x}$ to the problem $A\vec{x} = \vec{b}$ where $\vec{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

True False   If the columns of $n \times n$ matrix A are linearly <u>independent</u>, then $A\vec{x} = \vec{b}$ <u>will</u> have a solution, no matter what $\vec{b}$ is.

True False   If the columns of $n \times n$ matrix A are linearly <u>dependent</u>, then $A\vec{x} = \vec{b}$ will <u>not</u> have a solution, no matter what $\vec{b}$ is.

True False   For $A\vec{x} = \vec{b}$, it is possible that $A\vec{x} = \vec{b}_1$ has <u>exactly one</u> solution for some $\vec{b}_1$ while $A\vec{x} = \vec{b}_2$ has <u>no</u> solution for some other $\vec{b}_2$.

True False   If $T(x_1, x_2) = (2x_1 + 3x_2, 4x_1 + 5x_2)$, then $T(\vec{x}) = A\vec{x}$ where $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$.

True False   If $n \times n$ matrices A, B, C and D are invertible, then $(ABAB)^{-1} = A^{-1}B^{-1}A^{-1}B^{-1}$.

True False $\begin{bmatrix} a^2 \\ b \end{bmatrix}, \begin{bmatrix} a \\ 1 \end{bmatrix}$ are linearly dependent if $a = b$, and linearly independent otherwise.

True False If $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix}$, then $A^T = A$.

True False If $A = \begin{bmatrix} 1 & c & 2 \\ 1 & 0 & 2 \\ 1 & c & 2 \end{bmatrix}$, where c is some non-zero constant, then $A^{-1} = A$.

True False If $A = \begin{bmatrix} 1 & c & 2 \\ 1 & 0 & 2 \\ 1 & c & 2 \end{bmatrix}$, then $A\vec{x} = \vec{b}$ __must__ have more than one solution for some $\vec{b} \neq \vec{0}$.

True False The columns of a 3×2 matrix <u>could</u> be linearly dependent.

True False The columns of a 3×2 matrix <u>could</u> span R^2.

True False The columns of a 2×3 matrix <u>must</u> span R^2.

True False If A is a 2×3 matrix and B is a 3×4 matrix, then $(AB)^T(AB)$ is a 4×4 matrix.

True False The problem of solving for x_1 and x_2 in the system of equations
$$2x_1 + 3x_2 = 4$$
$$5x_1 + 6x_2 = 7$$
is equivalent of solving for x_1 and x_2 in the vector equation
$$x_1 \begin{bmatrix} 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}.$$

True False Where A and B are matrices, any vector that can be written as $AB\vec{x}$ for some vector $\vec{x}$ is a linear combination of the columns of A.

3

9 points   1. Find the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$.

6 points   2. Suppose $A\vec{x}_1 = \vec{b}$ and $A\vec{x}_2 = \vec{b}$ for $\vec{x}_1 \neq \vec{x}_2$. Show that $A\vec{x} = \vec{0}$ has a non-trivial (i.e. non-zero) solution $\vec{x}$, and show that $A\vec{x} = \vec{b}$ for the same right-hand side $\vec{b}$ has an infinite number of solutions.

4

6 points 3. Suppose for $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, and $E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$

you have $E_3 E_2 E_1 A = I$. Find A.

6 points 4. Find the inverse of $\begin{bmatrix} 1 & -2 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & -2 & 1 \end{bmatrix}$. Hint: think partitioned/block matrices.

Show your work.

8 points 5. Find the 2×2 matrix which first rotates <u>clockwise</u> ↻ by 120 degrees, and then reflects across the x-axis.

5
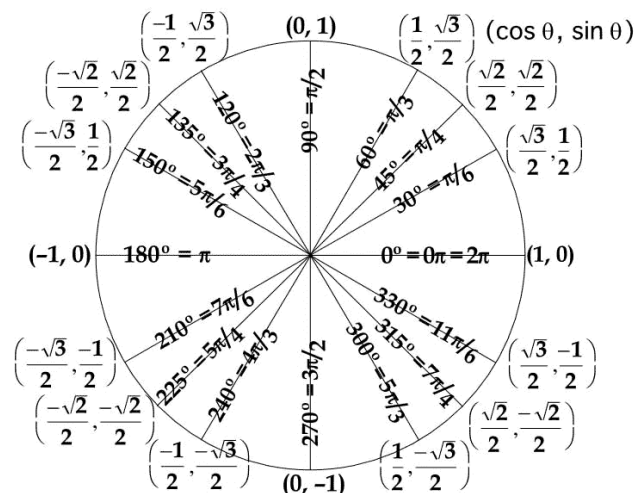
5 points   6. Suppose that you have some nickels (5 cents) and dimes (10 cents) and you want $4.00 using exactly 46 coins.

Set up a system of two equations and two unknowns which corresponds to this information. (Let n be the number nickels and d be the number of dimes.)

Before solving for n and d, why do you expect there to be exactly one solution?

Now find the needed number of nickels and dimes by solving this system using row reduction or using an appropriate matrix inverse.

10 points   7. A company produces two items, but some of each product is consumed during the production process as described by the consumption matrix

$$C = \begin{bmatrix} .6 & .2 \\ 0 & .5 \end{bmatrix}$$

How much should they produce if they want to end up with 10 units of each product? What is one thing about your solution that makes it seem reasonable Note for this problem that $(.5)(.4) = .2$ and that $\frac{.5}{.2} = \frac{5}{2}$ and $\frac{.4}{.2} = 2$.

How much more of each product needs to be produced if demand for Product Two were to increase by 1 unit (and demand for Product One does not change)?

10 points 8. Consider $A = \begin{bmatrix} 6 & -7 & -27 & -3 & 17 \\ -6 & 9 & 33 & 4 & -19 \\ -1 & 2 & 7 & 1 & -4 \end{bmatrix}$ which is the coefficient matrix for

the system of 3 equations and 5 unknowns $A\vec{x} = \vec{b}$ with augmented matrix

$$\begin{bmatrix} 6 & -7 & -27 & -3 & 17 & 48 \\ -6 & 9 & 33 & 4 & -19 & -53 \\ -1 & 2 & 7 & 1 & -4 & -10 \end{bmatrix}.$$

I've done all of the work to find that

$$\begin{bmatrix} 6 & -7 & -27 & -3 & 17 & 48 \\ -6 & 9 & 33 & 4 & -19 & -53 \\ -1 & 2 & 7 & 1 & -4 & -10 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 0 & 2 & 5 \\ 0 & 1 & 3 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 & -4 & 1 \end{bmatrix}.$$

/6 Give the <u>general</u> solution to the above problem $A\vec{x} = \vec{b}$ and give <u>two specific</u> solutions. Which part of your solution is the solution to the <u>homogeneous</u> problem $A\vec{x} = \vec{0}$?

/2 What is it about the reduced row echelon matrix $\begin{bmatrix} 1 & 0 & -1 & 0 & 2 & 5 \\ 0 & 1 & 3 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 & -4 & 1 \end{bmatrix}$ that

tells you that there is a solution to $A\vec{x} = \vec{b}$ no matter what the right hand side $\vec{b}$ is?

/2 Which columns of $\begin{bmatrix} 6 & -7 & -27 & -3 & 17 \\ -6 & 9 & 33 & 4 & -19 \\ -1 & 2 & 7 & 1 & -4 \end{bmatrix}$ form a linearly independent set?

Just circle the columns in this matrix.

/2 Extra-credit: write each of the dependent columns of A as linear combinations of the independent ones.

Invertible Matrix Theorem for $n \times n$ matrix A

a. A is invertible.

b. A is row equivalent to I.

c. A has n pivot positions.

d. $A\mathbf{x} = \mathbf{0}$ has only trivial solution.

e. Columns of A lin. independent.

f. Linear transf. $\mathbf{x} \rightarrow A\mathbf{x}$ 1-to-1.

g. $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$.

h. Columns of A span $\mathbf{R}^n$.

i. Linear transf. $\mathbf{x} \rightarrow A\mathbf{x}$ onto.

j. There is C such that $CA = I$.

k. There is D such that $AD = I$.

l. A^T is invertible.

m. Columns of A form basis for $\mathbf{R}^n$.

n. Column space of A is $\mathbf{R}^n$.

o. dim Col A = n, *i.e.* dimension of column space of A is n.

p. rank A = n, *i.e.* rank of A is n.

q. Nul A = $\{\mathbf{0}\}$, *i.e.* nullspace of A is $\{\mathbf{0}\}$.

r. dim Nul A = 0, the dimension of the null space of A is 0.

s. A has n nonzero eigenvalues, *i.e.* 0 is not an eigenvalue of A.

t. det A $\neq$ 0.

u. (Col A)$^\perp$ = $\{\mathbf{0}\}$, *i.e.* orthogonal complement of column space of A is $\{\mathbf{0}\}$.

v. (Nul A)$^\perp$ = $\mathbf{R}^n$, *i.e.* orthogonal complement of null space of A is $\mathbf{R}^n$.

w. Row A = $\mathbf{R}^n$, row space of A is $\mathbf{R}^n$.