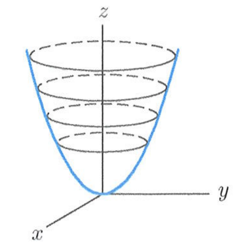
SOLUTION (a) $f(81, 16) = 60(81)^{3/4} \cdot (16)^{1/4} = 60 \cdot 27 \cdot 2 = 3240$. There will be 3240 units of goods produced.

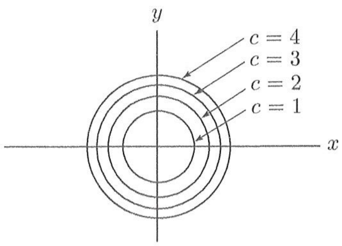
(b) Utilization of a units of labor and b units of capital results in the production of $f(a, b) = 60a^{3/4}b^{1/4}$ units of goods. Utilizing $2a$ and $2b$ units of labor and capital, respectively, results in $f(2a, 2b)$ units produced. Set $x = 2a$ and $y = 2b$. Then, we see that

$$\begin{aligned} f(2a, 2b) &= 60(2a)^{3/4}(2b)^{1/4} \\ &= 60 \cdot 2^{3/4} \cdot a^{3/4} \cdot 2^{1/4} \cdot b^{1/4} \\ &= 60 \cdot 2^{(3/4+1/4)} \cdot a^{3/4}b^{1/4} \\ &= 2^1 \cdot 60a^{3/4}b^{1/4} \\ &= 2f(a, b). \end{aligned}$$

▶ **Now Try Exercise 9**



Graph of $f(x, y) = x^2 + y^2$



Level curves of $f(x, y) = x^2 + y^2$

Figure 4 Level curves.

Level Curves

It is possible graphically to depict a function $f(x, y)$ of two variables using a family of curves called *level curves*. Let c be any number. Then, the graph of the equation $f(x, y) = c$ is a curve in the xy-plane called the *level curve of height c*. This curve describes all points of height c on the graph of the function $f(x, y)$. As c varies, we have a family of level curves indicating the sets of points on which $f(x, y)$ assumes various values c. In Fig. 4, we have drawn the graph and various level curves for the function $f(x, y) = x^2 + y^2$.

Level curves often have interesting physical interpretations. For example, surveyors draw *topographic maps* that use level curves to represent points having equal altitude. Here $f(x, y)$ = the altitude at point (x, y). Figure 5(a) shows the graph of $f(x, y)$ for a typical hilly region. Figure 5(b) shows the level curves corresponding to various altitudes. Note that when the level curves are closer together the surface is steeper.

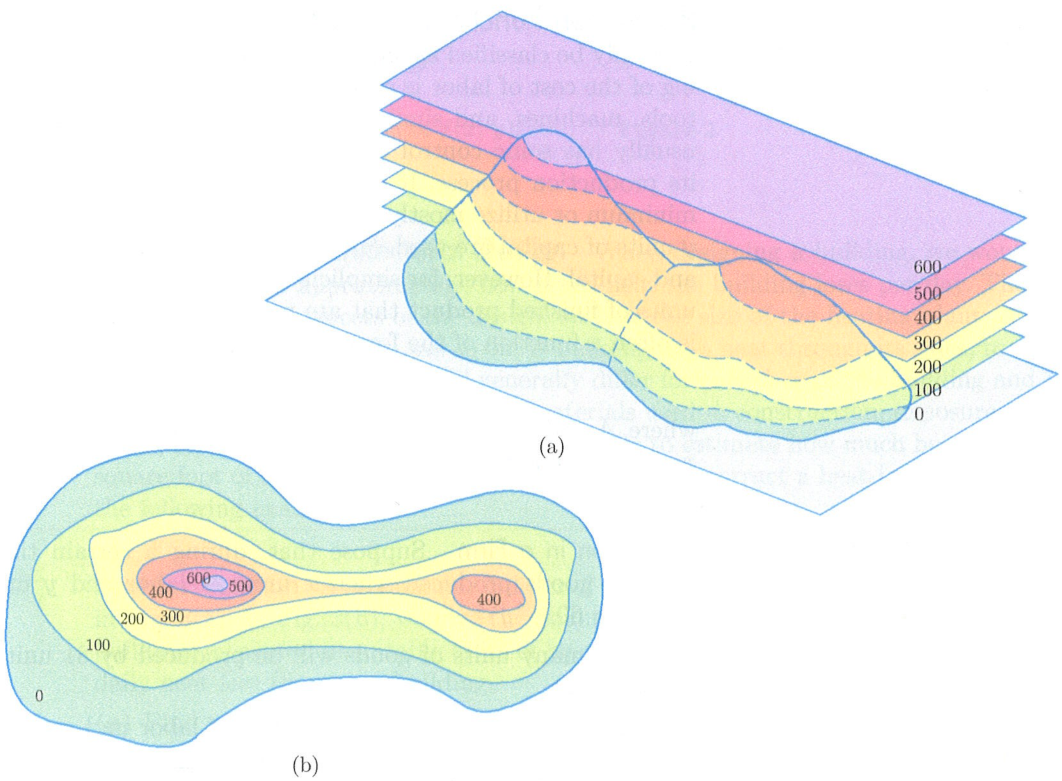


Figure 5 Topographic level curves show altitudes.