Section 5.4 The Mean

Math 141

Main ideas

Sample mean $\bar{x} = x_1\left(\frac{f_1}{n}\right) + x_2\left(\frac{f_2}{n}\right) + \cdots + x_r\left(\frac{f_r}{n}\right)$ where $\frac{f_i}{n}$ is the fraction of the time each outcome x_i did occur.

Population mean $\mu = x_1\left(\frac{f_1}{N}\right) + x_2\left(\frac{f_2}{N}\right) + \cdots + x_r\left(\frac{f_r}{N}\right)$ where $\frac{f_i}{N}$ is the fraction of the time each outcome x_i did occur.

Expected value is $E(X) = x_1p_1 + x_2p_2 + \cdots + x_Np_N$ where p_i is the fraction of the time each outcome x_i should occur, and:

- Is the average outcome that will occur if the experiment is repeated multiple times and what happens follows what should happen, i.e. it matches the probability distribution.
- Is generally not actually one of the possible values that could occur.
- Is always between the minimum and maximum possible values that could occur.

Given n binomial trials with probability of success p, where X is the number of successes, then the expected value of X is $\boldsymbol{E(X) = np}$**.**

Problems

1. Suppose I am interested in the number of years each student in class has been at Pepperdine, and I get the following results:

   Sample mean. The average number of years each student in our class has been at Pepperdine is:

   $\bar{x} =$

2. Suppose you flip four coins ______ times:

Number of heads	Number of times outcome occurred	Fraction of time outcome *did* occur
0		
1		
2		
3		
4		

What should have happened:

Number of heads	Fraction of time outcome *should* occur
0	
1	
2	
3	
4	

Sample mean: the average number of heads that actually occured was:

$\bar{x} =$

$=$

Expected value: The average number of heads that should have occured is:

$E(X) =$

3. Pay $1 to play a game: flip a coin until you get heads or until you flip the coin four times. You win $.50 for each flip (you are guaranteed at least one flip: the first flip).

Suppose the following did occur:

Outcome	Winnings x_i	Occurrences f_i
H	-.50	
TH	0	
TTH	.50	
TTTH	1.00	
TTTT	1.00	

Sample mean: the average winnings was

$\bar{x} =$

$=$

What should occur, on average:

Outcome	Winnings x_i	Probability p_i
H	-.50	
TH	0	
TTH	.50	
TTTH	1.00	
TTTT	1.00	

Expected value: the expected average winnings

$E(X) =$

$=$

4. Expected return on investments.

Let X_A = return for investment A

k	$\Pr(X_A = k)$
$ 1000	.20
$ 2000	.50
$ 3000	.30

Let X_B= return for investment B

k	$\Pr(X_B = k)$
-$ 1000	.30
$ 0	.10
$ 4000	.60

$E(X_A) =$

$E(X_B) =$

5. Life insurance for couple. Policy is for 5 years.
Payout: $ 0 if both are still alive.
$ 10,000 if one dies, the other lives.
$ 15,000 if both die.
Pr(Man lives ≥ 5 years) = .90.
Pr(Woman lives ≥ 5 years) = .95.
Let X be payout to couple.

Man	Woman	Payout	Probability
Live	Live		
Live	Not		
Not	Live		
Not	Not		

$E(X) =$

$=$

6. If you are a 70% free throw shooter ($p = .70$) free throw shooter and you shoot 10 shots ($n = 10$), and where X is the number of shots made, then (using our work from Class Handout 5.3) we have

$E(X) =$

Or (much more simply) $E(X) =$