Sections 4.5 and 4.6 Continued for Medical Testing
Math 141

Main ideas

Sensitivity is $\Pr(+|C)$.

Specificity is $\Pr(-|C')$.

Positive predictive value is $\Pr(C|+)$.

Negative predictive value is $\Pr(C'|-)$.

Problems

1. A medical test checks for a certain condition.
   95% of those with condition test positive
   5% of those with condition test negative
   2% of those without condition test positive
   98% of those without condition test negative.

   In the general population, from past experience:
   0.5% of the population has the condition
   99.5% of the population does not have the condition.

$$\Pr(C|+) = \frac{\Pr(C \text{ and } +)}{\Pr(+)} = \frac{\Pr(C \text{ and } +)}{\Pr(C \text{ and } +)+\Pr(\text{not } C \text{ and } +)} = \frac{(.005)(.95)}{(.005)(.95)+(.995)(.02)} = \frac{.00475}{.02465} \approx .1927$$

$$\Pr(\text{not } C|+) = \frac{\Pr(\text{not } C \text{ and } +)}{\Pr(+)} = \frac{\Pr(\text{not } C \text{ and } +)}{\Pr(C \text{ and } +)+\Pr(\text{not } C \text{ and } +)} = \frac{(.995)(.02)}{(.005)(.95)+(.995)(.02)} = \frac{.01990}{.02465} \approx .8073$$

$$\Pr(C|-) = \frac{\Pr(C \text{ and } -)}{\Pr(-)} = \frac{\Pr(C \text{ and } -)}{\Pr(C \text{ and } -)+\Pr(\text{not } C \text{ and } -)} = \frac{(.005)(.05)}{(.005)(.05)+(.995)(.98)} = \frac{.00025}{.97535} \approx .0003$$

$$\Pr(\text{not } C|-) = \frac{\Pr(\text{not } C \text{ and } -)}{\Pr(-)} = \frac{\Pr(\text{not } C \text{ and } -)}{\Pr(C \text{ and } -)+\Pr(\text{not } C \text{ and } -)} = \frac{(.995)(.98)}{(.005)(.05)+(.995)(.98)} = \frac{.97510}{.97535} \approx .9997$$

	Results of test		
	No Test	Positive +	Negative −
$Pr(C)$	.005	.1927	.0003
$Pr(not\ C)$	.995	.8073	.9997

Notice sum of $\Pr(\mathrm{C})$ and $\Pr(\text{not C})$ in each case.

Another view of why this occurs: "Natural Frequencies." Same info as before:

95% of those with the condition test positive
5% of those with condition test negative
2% of those not with the condition test positive
98% of those not with the condition test negative.

In the general population:

0.5% of the population has the condition
99.5% of the population does not have the condition.

Condition	Test	Persons of this type ("Natural Frequencies")
Yes	+	(.005)(1,000,000)(.95) =
Yes	–	(.005)(1,000,000)(.05) =
No	+	(.995)(1,000,000)(.02) =
No	–	(.995)(1,000,000)(.98) =

Pr(C) = ______________________ = ____________ =

Pr(C|+) = ____

= ____

= ______________________

= ______________________

=

2. Effects of changing values:

Pr(C) = .005
Pr(C') = .995
Pr(+|C) = .95
Pr(–|C) = .05
Pr(+|C') = .02
Pr(–|C') = .98

Pr(C|+) = ______________________

=

Pr(C) = .05
Pr(C') = .95
Pr(+|C) = .95
Pr(–|C) = .05
Pr(+|C') = .02
Pr(–|C') = .98

Pr(C|+) = ______________________

=

Pr(C) = .005
Pr(C') = .995
Pr(+|C) = .5
Pr(–|C) = .5
Pr(+|C') = .5
Pr(–|C') = .5

Pr(C|+) = ______________________

=

Pr(C) = .005
Pr(C') = .995
Pr(+|C) = 1
Pr(–|C) = 0
Pr(+|C') = .02
Pr(–|C') = .98

Pr(C|+) = ______________________

=

Pr(C) = .005
Pr(C') = .995
Pr(+|C) = .95
Pr(–|C) = .05
Pr(+|C') = 0
Pr(–|C') = 1

Pr(C|+) = ______________________

=

Pr(C) = .005
Pr(C') = .995
Pr(+|C) = .95
Pr(–|C) = .05
Pr(+|C') = .02
Pr(–|C') = .98

Pr(C|**+,–**) = ______________________

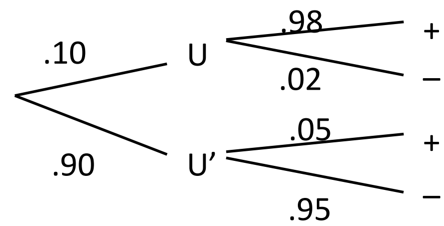
=

Pr(C) = .005
Pr(C') = .995
Pr(+|C) = .95
Pr(–|C) = .05
Pr(+|C') = .05
Pr(–|C') = .95

Pr(C|**+,–**) = ______________________

=

3. A drug-testing lab produces false negative results 2% of the time and false positives 5% of the time. Suppose that the laboratory has been hired by a company at which they estimate that 10% of the employees use drugs. Let U be "is drug user," + be "tests positive," and – be "tests negative."



$\Pr(+) =$

$\Pr(U \text{ and } ++) =$

$\Pr(++) =$

$\Pr(+++) =$

$\Pr(+\, n \text{ times}) = (.10)(.98)^n + (.90)(.05)^n \to \qquad as\ n \to \infty.$

$\Pr(U|+) =$

$\Pr(U|++) =$

$$\Pr(U|+\, n\ times) = \frac{\Pr(U \text{ and } +\, n \text{ times})}{\Pr(+\, n \text{ times})} = \frac{(.10)(.98)^n}{(.10)(.98)^n + (.90)(.05)^n} \to \qquad as\ n \to \infty.$$

$\Pr(U|+,-) =$

$\Pr(++|+) =$

$\Pr(+++|++) =$

$$\Pr(+\ again\ |+\ n \text{ times}) = \frac{\Pr(+\ \ n+1 \text{ times})}{\Pr(+\ \ n \text{ times})} = \frac{(.10)(.98)^{n+1} + (.90)(.05)^{n+1}}{(.10)(.98)^n + (.90)(.05)^n} \to \qquad as\ n \to \infty.$$