Section 4.2 Assignments of Probabilities

Math 141

Main ideas

For event E and sample space S:

n(E) is the number of outcomes in event E

n(S) is the number of outcomes in the sample space, i.e. the total number of outcomes

Pr(E) is the probability of event E.

Properties of probabilities:

$\Pr(E) = \frac{\text{n(E)}}{\text{n(S)}}$

$0 \leq \Pr(E) \leq 1$ (why decimal values rather than percentages? "per cent" means "per 100")

If E can occur in multiple ways {s, t, u, …, z}, then Pr(E) = Pr(s) + Pr(t) + Pr(u) + … + Pr(z)

For $S = \{s_1, s_2, \ldots, s_N\}$ with probabilities $\{p_1, p_2, \ldots, p_N\}$, then $p_1 + p_2 + \cdots + p_N = 1$.

Pr(E) + Pr(E') = 1 (more on this in Section 4.3).

Inclusion-exclusion principle:

Sets: $n(E \cup F) = n(E) + n(F) - n(E \cap F)$

Probabilities: $\Pr(E \cup F) = \Pr(E) + \Pr(F) - \Pr(E \cap F)$

Mutually exclusive:

Sets: $E \cap F = \varnothing$, $n(E \cap F) = 0$

Probabilities: $\Pr(E \cap F) = 0$

Probability distribution: the possible outcomes and the probability of each.

Problems

1. Roll 2 dice. We are interested in the sum.
   Event E is "sum is ≥ 7"
   Event F is "sum is odd"
   Event G is "doubles"

Sum of 2 dice

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Pr(E) =

Pr(F) =

$\Pr(E \cap F) =$

$\Pr(E \cup F) =$

Pr(E') =

Pr(G) =

$\Pr(F \cup G) =$

Probability distributions:

Sum of 2 dice

Outcome	Probability
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

Difference of 2 dice

Outcome	Probability

Sum of 2 dice

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Difference between 2 dice

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

2. Use the probability distribution at right.
   Let E = $\{s_1, s_2\}$, F = $\{s_2, s_3, s_4\}$.

Outcome	Probability
s_1	.10
s_2	.20
s_3	.40
s_4	.05
s_5	

   Pr(E) =

   Pr(F) =

   Pr(E $\cap$ F) =

   Pr(E $\cup$ F) =

   Pr(E $\cap$ F') =

   Notice:

3. Flip 6 coins. We are interested in how many are heads. Find the probability distribution.

# of heads	Probability
0	
1	
2	
3	
4	
5	
6	

<u>Preview of how probabilities are affected by additional information</u>

4. Two coins are tossed. What is the probability both coins are heads?
5. Two coins are tossed. We are told that at least one of the coins is heads.
   What is the probability both coins are heads?
6. Two coins are tossed. We can see one of the coins, and we see that it is heads.
   What is the probability both coins are heads?