Main ideas

Multiplication principle: if there are t tasks with m_1, m_2, ..., m_t choices, then there are $m_1 \cdot m_2 \cdot \cdots \cdot m_t$ ways to accomplish the t tasks.

Permutation: $P(n,r) = \dfrac{n!}{(n-r)!} = n \cdot (n-1) \cdot \cdots \cdot (n-r+1)$.

Combination—choose r items from n: $C(n,r) = \dbinom{n}{r} = \dfrac{n!}{r! \cdot (n-r)!}$.

Alternative view of combinations: divide n items into two groups (for example, those we keep, those we don't) of sizes r and $n-r$, that is, $\dbinom{n}{r,\ n-r} = \dfrac{n!}{r! \cdot (n-r)!}$.

Multinomial partition: the number of ways to divide n items into m groups of sizes $n_1, n_2, ..., n_m$ (where $n_1 + n_2 + \cdots + n_m = n$) is $\dbinom{n}{n_1, n_2, ..., n_m} = \dfrac{n!}{n_1! \cdot n_2! \cdot \cdots \cdot n_m!}$.

There are $r! = r \cdot (r-1) \cdot (r-2) \cdot \cdots \cdot 2 \cdot 1$ ways to order (i.e. put in a particular order) r items.

Problems

1. A team plays 10 games. How many ways can these games result in:

    3 wins:

    3 wins and 7 losses:

    3 wins, 5 losses and 2 ties:

2. How many ways can a group of 100 students be assigned to dorms A, B and C:

    25 to Dorm A   40 to Dorm B   35 to Dorm C.

3. Of 14 applicants to a software company, 3 will be hired to work on programming languages, 4 will work on word processing software, and 5 will work on spreadsheet software. In how many ways can the company hire and assign the 12 new employees?

4. How many ways to divide 10 players into two 5-player groups that will play each other?

5. How many ways to divide 10 persons into two 5-player groups: the starting 5 and the 5 who will be subs?

6. How many ways to divide 20 persons into 4 groups of 5 each?

7. How many ways to divide 20 persons into groups of sizes 2, 2, 2 and 14?

8. How many ways to divide 20 persons into groups of sizes 2, 2, 8 and 8?

9. Suppose you can only move south (down) or east (right). How many ways are there to get from A to B? List them.
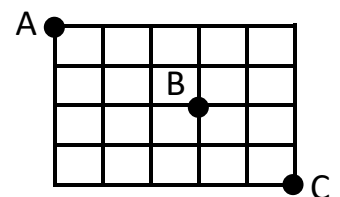
How many ways are there to get from B to C?

How many ways are there to get from A to C?

How many ways are there to get from A to C that pass through B?

10. A bag of 10 apples contains 2 rotten apples and 8 good apples.
A shopper selects a sample of 3 apples from the bag.
How many different samples are possible?

How many samples contain all good apples?

How many samples contain at least 1 rotten apple?

11. In how many ways can 6 married couples sit next to each other if
Anyone can sit next to anyone else:

Each couple must sit together:

Men and women must alternate (not necessarily as couples):

12. An urn contains 8 red and 4 white balls. You select 4 of the 12.   First note that the
total number of ways to choose 4 balls from 12 is:

Red balls	White balls	Number of possible samples
4	0	
3	1	
2	2	
1	3	
0	4	

13. In how many ways can a residence director assign six students to four dorm rooms if
two rooms are doubles, two rooms are singles, and two of the students cannot be
placed together?