

Section 1.3 Math 141
Maxima and Minima of Functions of Several Variables

Main ideas

For $f(x)$, if at a certain point $x = a$ we have $f'(a) = 0$ (that is, $\frac{df}{dx}(a) = 0$), then:

- $f''(a) > 0$ (that is, $\frac{d^2f}{dx^2}(a) > 0$) $\Rightarrow f(x)$ has a local/rel. min at $x = a$.
- $f''(a) < 0$ (that is, $\frac{d^2f}{dx^2}(a) < 0$) $\Rightarrow f(x)$ has a local/rel. max at $x = a$.
- $f''(a) = 0$ (that is, $\frac{d^2f}{dx^2}(a) = 0$) \Rightarrow we can't tell if there is min or max or neither at $x = a$.

For $f(x, y)$, if at $(x, y) = (a, b)$ we have $\frac{\partial f}{\partial x}(a, b) = 0$ and $\frac{\partial f}{\partial y}(a, b) = 0$, then where

$$D(x, y) = \begin{vmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{vmatrix} = \left(\frac{\partial^2 f}{\partial x^2}\right) \left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial x \partial y}\right) \left(\frac{\partial^2 f}{\partial y \partial x}\right) = \left(\frac{\partial^2 f}{\partial x^2}\right) \left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2$$

we have:

- $D(a, b) > 0$, $\frac{\partial^2 f}{\partial x^2}(a, b) > 0 \Rightarrow f(x, y)$ has a local/rel. min at $(x, y) = (a, b)$.
- $D(a, b) > 0$, $\frac{\partial^2 f}{\partial x^2}(a, b) < 0 \Rightarrow f(x, y)$ has a local/rel. max at $(x, y) = (a, b)$.
- $D(a, b) < 0 \Rightarrow f(x, y)$ has neither a min or max at $(x, y) = (a, b)$.
- $D(a, b) = 0 \Rightarrow$ we can't tell if there is a min or max at $(x, y) = (a, b)$.

Day 1, In Class

1. Let's quickly review (above) how to use derivatives to recognize where a function of one variable $f(x)$ has maxima and minima.
2. Let's look above to see how you find maxima and minima of function $f(x, y)$.
3. Let's find the maxima and minima of $f(x, y) = x^3 + y^2 - 3x - 8y + 12$.
We can plot functions of one or two variables. They exist in two and three dimensions.
We cannot plot function of three variables, since they exist in four dimensions.
Yet those functions can have maxima and minima that we can find. We won't in this class.

Day 1, In Groups

4. You will work HW 1.3.25.

Day 2, In Class

5. I will work a minimization word problem.

Day 2, In Groups

6. You will work HW 1.3.49.