

Name: Solutions

	Total
Possible	100
Received	

Don't open exam until told to do so.

You will not use a calculator on this exam.
In answering the following questions, not simplify the answers.
For example, leave your answer in the form
 $P(5,3)$ or $12!$ or $C(4,3) \cdot C(7,4)$ or $2^5 - 2^3$ or $7 \cdot 6 \cdot 5$ or ...



"Who is putting all the Math books in the Horror section?"

For these problems, 5 History books and 4 Novels will be arranged on a shelf.

/2 How many ways can the books be arranged if there are no special restrictions on the books?

$$9!$$

/4 How many ways can they be arranged if all the History books must be to the left of all Novels?

$$5! \cdot 4!$$

History Novels

/4 How many ways can they be arranged if all the History books must be next to each other (but the Novels do not necessarily need to be together)?

$$5! \cdot 5! \text{ or } 5 \cdot 4! \cdot 5!$$

For these problems, 4 artists each have 10 paintings (40 total) to be arranged side by side.

/2 How many different arrangements are possible if there are no restrictions?

$$40!$$

/4 How many different arrangements are possible if each artist's paintings must be grouped together?

$$4! \cdot (10!)^4$$

Order the artists Order each of their 10 paintings

For these problems, 4 couples (4 men, 4 women; 8 persons total) go to a movie together.

/4 In how many ways can they be seated so that the four men are seated together (side by side) and the four women are seated together?

$$2! \cdot 4! \cdot 4!$$

Men or women first?

/4 In how many ways can they be seated so that they are seated as couples?

$$4! \cdot 2^4 \text{ OR } \underline{8} \cdot \underline{1} \cdot \underline{6} \cdot \underline{1} \cdot \underline{4} \cdot \underline{1} \cdot \underline{2} \cdot \underline{1}$$

/4 In how many ways can they be seated man/women/man/woman/etc., not necessarily as couples?

$$4! \cdot 4!$$

Men Women

/2 In how many ways can they be seated if there are no restrictions on who sits where?

$$8!$$

For these problems, consider the numbers from (and including) 0000 to 9999.

/2 How many numbers are there from 0000 to 9999?

$$\underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} = 10000$$

/3 How many numbers are there from 0000 to 9999 in which all four digits are different?

$$10 \cdot 9 \cdot 8 \cdot 7$$

/2 How many numbers are there from 0000 to 9999 in which all four digits are the same?

$$10$$

/5 How many numbers are there from 0000 to 9999 in which three of the digits are the same and the fourth digit is different (e.g. 5535 or ...)

$$\underbrace{C(4,3)}_{\text{or } C(4,1)} \cdot 10 \cdot 9$$

For these problems, you toss a coin eight times.

/2 How many possible outcomes are there?

$$2^8$$

/3 How many possible outcomes are there in which exactly 2 of the 8 tosses are heads?

$$C(8,2)$$

/2 How many possible outcomes are there in which none of the coins are heads?

$$1 \text{ (all tails)}$$

/4 How many possible outcomes are there in which the first and last coins are heads?

$$1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 1$$

/3 Compute $C(20,2)$. $= \frac{20!}{2!18!} = \frac{20 \cdot 19}{2 \cdot 1} = 10 \cdot 19 = 190$.

/5 How many even numbers are there between 10,000 and 99,000? *If* were 99,999, then it would be $9 \cdot 10 \cdot 10 \cdot 10 \cdot 5$ ← Last digit must be even. *As is*, it's a trickier problem, so I was lenient in grading.

/4 How many three-letter (from A to Z) airport codes are possible? For example: LAX.
Note: there can be repetition.

$$26 \cdot 26 \cdot 26$$

/5 In how many ways could we divide 15 persons into four groups:

3 groups of 2 persons

1 group of 9 persons.

$$\binom{15}{2, 2, 2, 9} / 3!$$

Three groups of 2

/4 How many ways can you select 5 stocks from 10 and 3 bonds from 8?

$$C(10, 5) \cdot C(8, 3)$$

/4 How many ways can you select 4 friends from 10 to give \$1, \$2, \$5 and \$10 to (one bill for each friend)?

$$P(10, 4) = C(10, 4) \cdot 4! = 10 \cdot 9 \cdot 8 \cdot 7$$

For these problems, there are 31 flavors of ice cream. You'll get three scoops.

/2 How many ways to have all three scoops the same flavor?

$$31$$

/3 How many ways to have all three scoops different?

$$C(31, 3) = \frac{31 \cdot 30 \cdot 29}{3 \cdot 2 \cdot 1}$$

/4 How many ways to have two scoops of one flavor, and one scoop of a different flavor?

$$P(31, 2) = C(31, 2) \cdot 2 = 31 \cdot 30$$

For these problems, there is a deck of 90 cards of 9 different colors of cards which are numbered 1 to 10. We will choose 8 cards.

- /5 In how many ways can you choose the 8 cards so that we have:
 4 of one number
 3 of another number
 1 of another number?

One example: 2 2 2 2 5 5 5 9

$$C(10, 3) \cdot 3! \cdot C(9, 4) \cdot C(9, 3) \cdot C(9, 1)$$

$P(10, 3)$ } Choose the 3 numbers and which will be 4 and 3 and 1

- /4 In how many ways can you choose 8 cards so that we have a straight?

One example: 2 3 4 5 6 7 8 9

$$3 \cdot [C(9, 1)]^8$$

Choose the color of each number.

1 to 8 or 2 to 9 or 3 to 10

- /4 In how many ways could you have all 8 cards be the same number?

One example: 2 2 2 2 2 2 2 2

$$C(10, 1) \cdot C(9, 8)$$

Choose the number of the 8 cards of the desired number.

Extra credit:

- /2 In how many ways can you choose 8 cards so that:

4 of them are an even number of one color

4 of them are an odd number of a second color?

$$C(9, 2) \cdot 2 \cdot C(5, 4) \cdot C(5, 4)$$

$P(9, 2) = 9 \cdot 8$

Choose the 2 colors and decide which is even and which is odd.

For the even color, choose 4 of the 5 even cards.

Similarly for the odd cards.