Name: _____Solutions_____

Problem	1 / 2	3 / 4 / 5	6 / 7	8 / 9	Total
Possible	25	34	25	16	100
Received					

DO NOT OPEN YOUR EXAM UNTIL TOLD TO DO SO.

You may use a 3 x 5 card of notes, both sides, and a calculator.

FOR FULL CREDIT, SHOW ALL WORK RELATED TO FINDING EACH SOLUTION.
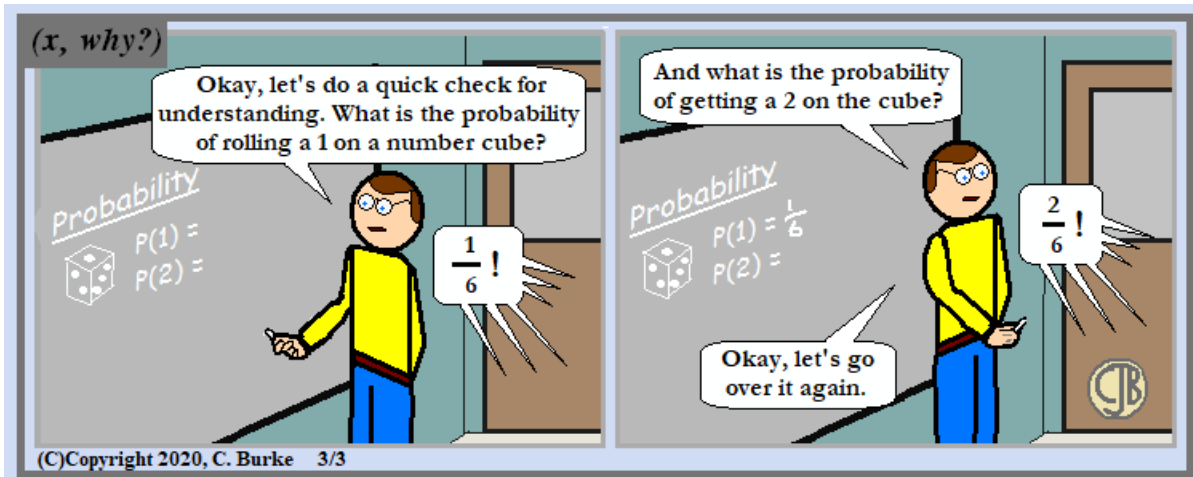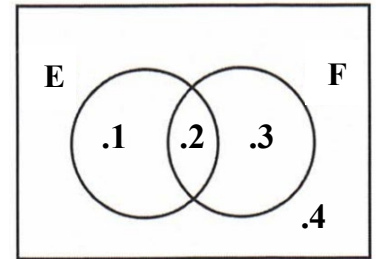
10 points   1. Suppose that $\Pr(E) = .3, \Pr(F) = .5, \Pr(E \cap F) = .2$.

Find each of the following.

/2 $\Pr(E \cup F) = .1 + .2 + .3 = .6$

/3 $\Pr(F|E) = \dfrac{\Pr(F \text{ and } E)}{\Pr(E)} = \dfrac{.2}{.3} = \dfrac{2}{3}$

/3 $\Pr(F|E') = \dfrac{\Pr(F \text{ and } E')}{\Pr(E')} = \dfrac{.3}{.7} = \dfrac{3}{7}$

/2 Are events E and F independent? Explain why or why not.

No. $\Pr(F|E) \neq \Pr(F)$.

You do NOT need to simply your answers for Problem 2.

15 points   2. Consider a group of 4 persons. We are interested in their birth months. (There are 12 months. ☺)

/4 What is the probability they are all born in different months?

$\dfrac{12 \cdot 11 \cdot 10 \cdot 9}{12 \cdot 12 \cdot 12 \cdot 12}$

/4 What is the probability they are all born in July?

$\dfrac{1 \cdot 1 \cdot 1 \cdot 1}{12 \cdot 12 \cdot 12 \cdot 12}$

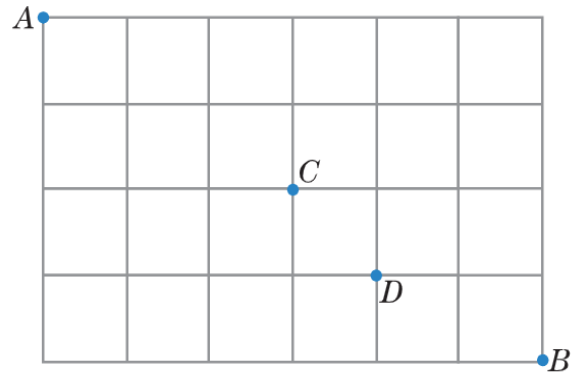/4 What is the probability they are all born in the same month?

$\dfrac{12 \cdot 1 \cdot 1 \cdot 1}{12 \cdot 12 \cdot 12 \cdot 12}$

/3 What is the probability at least two of them are born in the same month?

$1 -$

2

16 points 3. You will move from Point A to Point B. You may move either right (east) or down (south). In the figure at right, find the probability that you pass through:

/4 Point C: $\dfrac{C(5,3) \cdot C(5,3)}{C(10,6)}$

/4 Point D: $\dfrac{C(7,4) \cdot C(3,2)}{C(10,6)}$

/4 Point C and Point D: $\dfrac{C(5,3) \cdot C(2,1) \cdot C(3,2)}{C(10,6)}$

A to C to D to B

/4 Point C or Point D:

$Pr(C \text{ or } D) = Pr(c) + Pr(D) - P(C \text{ and } D)$

11 points 4. There is a group of 15 children: 10 boys and 5 girls. 4 students are chosen. Find the probability that:

/4 No girls are chosen: $\dfrac{10 \cdot 9 \cdot 8 \cdot 7}{15 \cdot 14 \cdot 13 \cdot 12} = \left(\dfrac{10}{15}\right)\left(\dfrac{9}{14}\right)\left(\dfrac{8}{13}\right)\left(\dfrac{7}{12}\right) = \dfrac{C(10,4)}{C(15,4)}$

/4 The first 2 children chosen are girls:
$\dfrac{5 \cdot 4}{15 \cdot 14} = \left(\dfrac{5}{15}\right)\left(\dfrac{4}{14}\right) = \dfrac{C(5,2) \cdot C(13,2)}{C(15,2) \cdot C(13,2)}$ } Not really needed

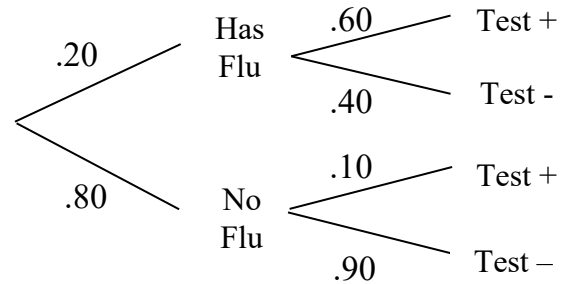/3 At least one of the children is a girl:

$1 - Pr(\text{No girls chosen})$

7 points 5. A number is chosen at random from numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15. What is the probability that the number is:

/2 Odd: $\dfrac{8}{15}$

/2 A multiple of 3: $\dfrac{5}{15}$

/3 Odd, given that it is a multiple of 3: $\dfrac{3}{5}$

3

17 points 6. Suppose in a certain group of people, 20% of them have the Flu, and suppose that a certain test which is used to determine whether a person has the Flu gives false negatives 40% of the time and false positives only 10% of the time.

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              .20    Has    .60    Test +
                     Flu
                            .40    Test -
              .80    No     .10    Test +
                     Flu
                            .90    Test –
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/7 What is the probability that someone <u>has</u> the flu if he/she tests <u>negative</u>?

$$Pr(Flu|-) = \frac{Pr(Flu \text{ and } -)}{Pr(-)} = \frac{(.20)(.40)}{(.20)(.40)+(.80)(.90)} = \frac{.08}{.80} = .10$$

/2 What is the probability that someone does <u>not</u> have the flu if he/she tests <u>negative</u>?

$$1 - .10 = .90$$

/8 What is the probability that someone would test <u>positive</u> if he/she has already tested <u>negative</u> once?

$$Pr(+|-) = \frac{Pr(- \text{ then } +)}{Pr(-)} = \frac{(.20)(.60)(.40) + (.80)(.10)(.90)}{(.20)(.40) + (.80)(.90)}$$

$$= .15$$

8 points 7. Given the information at right, if a student received an A, what is the probability he/she is from Gryffindor?

House	Proportion of all students	Fraction of group who received an A
Gryffindor	.10	.50
Hufflepuff	.30	.30
Ravenclaw	.20	.50
Slytherin	.40	.10

$$Pr(G|A) = \frac{Pr(G \text{ and } A)}{Pr(A)} = \frac{(.10)(.50)}{(.10)(.50) + (.30)(.30) + (.20)(.50) + (.40)(.10)}$$

$$= \frac{.05}{.28} \approx .18$$

4

8 points   8. Each of three bowls contains five pieces of fruit:
- Bowl 1 contains:
  Two Apples.
  Three Oranges.
- Bowl 2 contains:
  Three Apples.
  Two Oranges.
- Bowl 3 contains:
  Four Apples.
  One Orange.
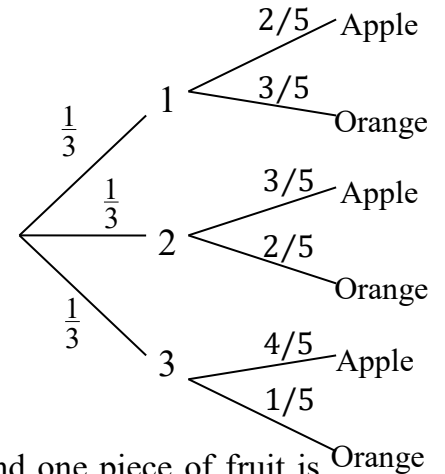
A fruit bowl is randomly selected (from three fruit bowls) and one piece of fruit is randomly selected from that bowl. Suppose the first piece fruit is an Apple (which we do <u>not</u> put back into the box). If we then choose a second piece of fruit from the <u>same</u> bowl, what is the probability that the other fruit we choose is also an Apple?

Tree diagram:
- $\frac{1}{3}$ → 1 → 2/5 Apple, 3/5 Orange
- $\frac{1}{3}$ → 2 → 3/5 Apple, 2/5 Orange
- $\frac{1}{3}$ → 3 → 4/5 Apple, 1/5 Orange

$$Pr(Apple\ 2\ |\ Apple\ 1) = \frac{Pr(Both\ Apples)}{Pr(Apple\ 1)}$$

$$= \frac{(\frac{1}{3})(\frac{2}{5})(\frac{1}{4}) + (\frac{1}{3})(\frac{3}{5})(\frac{2}{4}) + (\frac{1}{3})(\frac{4}{5})(\frac{3}{4})}{(\frac{1}{3})(\frac{2}{5}) + (\frac{1}{3})(\frac{3}{5}) + (\frac{1}{3})(\frac{4}{5})} = \frac{\frac{20}{60}}{\frac{9}{15}} = \frac{\frac{1}{3}}{\frac{3}{5}} = \frac{1}{3} \cdot \frac{5}{3} = \frac{5}{9}$$

8 points   9. Consider the hair color and eye color of 500 persons.

Are eye color and hair color dependent or independent? Show (do a bit of math—you might have to use a formula or two) to support your conclusion. For example, is the likelihood of having Light Hair dependent or independent of having Brown Eyes?

Eyes

Hair	Brown	Blue	Total
Light	20	80	100
Dark	80	320	400
Total	100	400	500

$$Pr(LH) = \frac{100}{500} = .20$$

$$Pr(LH|BE) = \frac{20}{100} = .20$$

So LH and BE are independent.